Higher-Order Radiative Corrections to Eikonal Functions in Massive Electrodynamics at Very High Energy*

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We show that the assumptions of an impact-parameter representation and energy-independent $d\sigma/dt$ for fermion-fermion scattering at high energy imply that the amplitude is proportional to square of the vector form factors, multiplied by the bare eikonal functions, to all orders in radiative corrections without vacuum polarization. This is done in the framework of massive electrodynamics. We explicitly demonstrate that these two assumptions are satisfied for some sets of graphs in the two-photon exchanged amplitude with fourth-order radiative correction, where the radiative particle is taken to be a pseudoscalar in order to simplify the algebra.

I. INTRODUCTION

THIS is a continuation of our study of effects of radiative corrections on the eikonal functions in massive electrodynamics at very high energy. It was shown that[1] if lowest-order (second-order) radiative effects are included, then the eikonal function for $e(p_1)+e'(p_2)\rightarrow e(p_3)+e'(p_4)$,

$$E(k^2)=\int d^2x_\perp e^{-ik\cdot x_\perp}\times\left\{\exp\left[-iee'\int\frac{d^2q}{(2\pi)^2}e^{iq\cdot x_\perp}\frac{1}{q^2+\mu^2-i\epsilon}\right]-1\right\},$$

is changed into

$$E(k^2)\rightarrow[\delta_{\lambda_1\lambda_3}F_1(k^2)+i\chi_{\lambda_3}(\boldsymbol{\sigma}\times\mathbf{k})_3\chi_{\lambda_1}F_2(k^2)]\times[\delta_{\lambda_2\lambda_4}F_1(k^2)+i\chi_{\lambda_4}(\boldsymbol{\sigma}\times\mathbf{k})_3\chi_{\lambda_2}F_2(k^2)]E(k^2).\quad(1)$$

In the above, $k=p_3-p_1$ is the finite momentum transfer, and μ is the mass of the vector meson. $F_1(k^2)$ and $F_2(k^2)$ are the vector form factors conventionally defined. The λ's are the spin indices, and the χ_λ's are their corresponding two-component wave functions.

When it comes to higher-order radiative corrections, opinions differ. In particular, it has been pointed out by this author that,[2] in the absence of vacuum polarization, the leading term of the amplitude for n-photon absorption (emission) in certain sets of diagrams is proportional to $p^{\alpha_n}\cdots p^{\alpha_1}$; i.e.,

$$M_n{}^{\alpha_n\cdots\alpha_1}=X_np^{\alpha_n}\cdots p^{\alpha_1}.\quad(2)$$

It then follows simply from Ward's identity that Eq. (1) will ensue.

It is the purpose of this paper to argue that in fact all diagrams without vacuum polarization possess the property displayed in Eq. (2). More precisely, we shall show that in order to have an impact-parameter representation and a constant differential cross section[3] $d\sigma/dt(t=k^2)$ in the leading order at high energy, Eq. (2) is the natural requirement. We shall explicitly verify that these two conditions are satisfied for some subsets of fourth-order radiative graphs.

Effects of high-order radiative corrections were also considered by Cheng and Wu[4] and by Chang.[5] The methods used by them are quite similar to each other. It was concluded that if Z graphs were neglected, then Eq. (1) would be the result. Cheng and Wu furthermore claimed that if Z graphs were included, then one would no longer have such a simple conclusion. In fact, there would result in what they called a hierarchy of form factors.[4,6] Our conclusion therefore is at variance with theirs, or perhaps we may say that effects of their Z graphs cancel out completely.

In Sec. II, we present a general argument that Eq. (2) is necessary in order to have an impact-parameter representation and a constant differential cross section at very high energy.

In Sec. III, to support our argument, we explicitly analyze a subset of graphs with fourth-order radiative correction and two exchanged photons. A brief conclusion is given in Sec. IV.

II. A SIMPLE ARGUMENT

Unless otherwise specified, effects of vacuum polarization will be neglected in this section and in Sec. III.

What we want to do in this section is to show that the n-photon absorption (emission) amplitude necessarily has to have the property of Eq. (2) in order that the scattering amplitude for $e+e'\rightarrow e+e'$ satisfies an impact-parameter representation and that its differential cross section $d\sigma/dt$ becomes constant at very high energy. The best way to present our argument is by way of examples. We shall use some simple cases to illustrate

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[1] Y. P. Yao, Phys. Rev. D 1, 2316 (1970). The kinematics used here is the same as those in the reference. Thus $p_1=p-\frac{1}{2}k$, $p_3=p+\frac{1}{2}k$, $p_2=p'+\frac{1}{2}k$, $p_4=p'-\frac{1}{2}k$, with $p^\mu=((p^2+\frac{1}{4}k^2+m^2)^{1/2},0,0,p)$, $p'^\mu=((p^2+\frac{1}{4}k^2+m^2)^{1/2},0,0,-p)$, and $k^\mu=(0,k_1,k_2,0)$. The Dirac equation is $(m+\gamma\cdot p)u(p)=0$, with $\gamma^0=\gamma^{0\dagger}$, $\gamma_k=-\gamma_k{}^\dagger$, $\gamma_5=\gamma^0\gamma^1\gamma^2\gamma^3=-\gamma_5{}^\dagger$, $\{\gamma^\mu,\gamma^\nu\}=-2g^{\mu\nu}$, $g^{\mu\nu}=(-1,1,1,1)$, and $\epsilon^{0123}=1$.

[2] Y. P. Yao, Phys. Rev. D 1, 2971 (1970).

[3] These two properties are also predicted by the prescription given by H. Cheng and T. T. Wu (see Ref. 4).

[4] H. Cheng and T. T. Wu, Phys. Rev. D 1, 1069 (1970); 1, 1083 (1970).

[5] S. J. Chang, Phys. Rev. D 1, 2977 (1970).

[6] H. Cheng and T. T. Wu, Phys. Rev. 184, 1868 (1969).

how eikonal representation and constant differential cross section emerge in massive electrodynamics. Then we shall argue for the general case.

Admittedly, a more satisfying approach should be to prove the two assumptions stated above. The author as yet does not know how to go about this, except by means of perturbation. This we shall do to fourth order in Sec. III.

To begin, let us consider fermion-fermion scattering with two photons exchanged but without radiative correction. As is familiar by now, a labor-saving way to obtain the leading term of the amplitude is to consider first the partial amplitudes of two-photon absorption (emission). Figure 1(a) gives

$$(M_2{}^0)^{\alpha_2\alpha_1}=e^2\bar{u}(p_3)\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^{\alpha_1}u(p_1)$$

$$\cong e^2\frac{p^{\alpha_2}p^{\alpha_1}}{m}\frac{1}{p\cdot k_1-i\epsilon}\delta_{\lambda_1\lambda_3}. \tag{3a}$$

Similarly, Fig. 1(b) yields

$$(M_2{}^0)'^{\alpha_2\alpha_1}\cong e^2\frac{p'^{\alpha_2}p'^{\alpha_1}}{m}\frac{1}{-p'\cdot k_1-i\epsilon}\delta_{\lambda_2\lambda_4}. \tag{3b}$$

The scattering amplitude which corresponds to Fig. 2 is

$$T_2{}^0\cong i\delta_{\lambda_1\lambda_3}\delta_{\lambda_2\lambda_4}\int\frac{d^4k_1}{(2\pi)^4}\frac{-i}{(k-k_1)^2+\mu^2-i\epsilon}\,\frac{-i}{k_1{}^2+\mu^2-i\epsilon}$$

$$\times\tfrac{1}{2}e^2\frac{p^{\alpha_2}p^{\alpha_1}}{m}\left(\frac{1}{p\cdot k_1-i\epsilon}+\frac{1}{-p\cdot k_1-i\epsilon}\right)$$

$$\times e^2\frac{p_{\alpha_2}'p_{\alpha_1}'}{m}\left(\frac{1}{p'\cdot k_1-i\epsilon}+\frac{1}{-p'\cdot k_1-i\epsilon}\right).$$

Upon using the identities

$$\frac{1}{p\cdot k_1-i\epsilon}+\frac{1}{-p\cdot k_1-i\epsilon}\cong\frac{1}{p}\,{-2\pi i}\delta(k^3-k^0) \tag{4a}$$

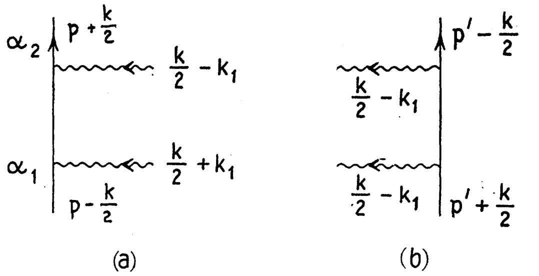

FIG. 1. Two-photon absorption and emission amplitudes without radiative correction.

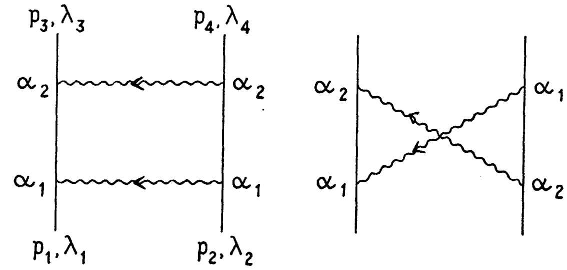

FIG. 2. Uncrossed and crossed graphs for fermion-fermion scattering with two exchanged photons.

and

$$\frac{1}{p'\cdot k_1-i\epsilon}+\frac{1}{-p'\cdot k_1-i\epsilon}\cong\frac{1}{p}\,{-2\pi i}\delta(k^3+k^0), \tag{4b}$$

we finally obtain

$$T_2{}^0\cong ie^4\frac{p\cdot p'}{m^2}\int d^2x_\perp\, e^{-ik\cdot x_\perp}$$

$$\times\left[\frac{1}{2}\int\frac{d^2r_1}{(2\pi)^2}e^{ir_1\cdot x_\perp}\frac{-i}{r_1{}^2+\mu^2-i\epsilon}\right.$$

$$\left.\times\int\frac{d^2r_2}{(2\pi)^2}e^{ir_2\cdot x_\perp}\frac{-i}{r_2{}^2+\mu^2-i\epsilon}\right], \tag{5}$$

where r_1 and r_2 are transverse, i.e., they have only x and y components. There are two salient features worth emphasizing: (i) In Eqs. (3a) and (3b), the numerators are one power higher in p or p' than the corresponding denominators, and (ii) the existence of factors $1/(p\cdot k_1-i\epsilon)$ and $1/(-p'\cdot k_1-i\epsilon)$ in Eqs. (3a) and (3b).

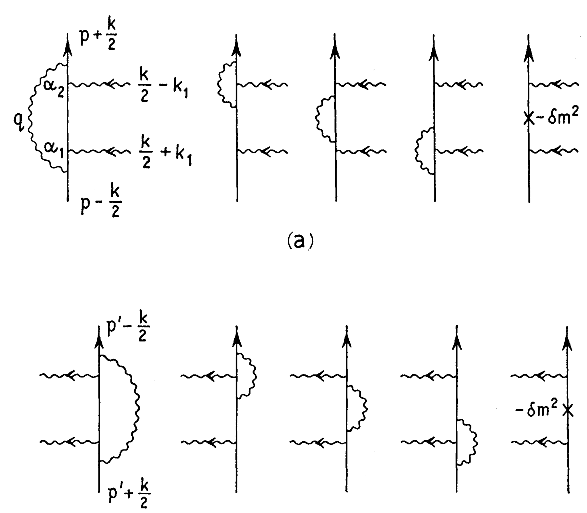

FIG. 3. Two-photon absorption and emission amplitudes with second-order radiative corrections.

Property (i) ensures that Eq. (5) is proportional to $p\cdot p'\cong-\frac{1}{2}s$, which in turn leads to constant $d\sigma/dt$. On the other hand, property (ii) is crucial in that it allows us to use identities (4a) and (4b) to make the integration with respect to the exchanged photons become two-dimensional. This is the impact-parameter representation.

These features are again borne out in the case when effects of second-order radiative correction are included. The absorption and emission amplitudes here include the sets of graphs in Fig. 3(a) and 3(b), respectively. In fact, individual graphs in each set have properties (i) up to powers of $\ln(p)$ and (ii). These $\ln(p)$ factors cancel themselves in each set.

For the sake of completeness, let us illustrate how one can obtain the corresponding term in Eq. (1), upon showing that the absorption amplitude possesses the property of Eq. (2). Figure 3(a) gives

$$\begin{aligned}(M_2{}^2)^{\alpha_2\alpha_1}=e^4\bar{u}(p_3)\int\frac{d^4q}{(2\pi)^4i}\Big[&\gamma^\mu\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-q)-i\epsilon}\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon}\gamma^{\alpha_1}\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-q)-i\epsilon}\gamma_\mu\\&+\gamma^\mu\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-q)-i\epsilon}\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon}\gamma_\mu\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^{\alpha_1}\\&+\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^\mu\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon}\gamma_\mu\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^{\alpha_1}\\&+\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^\mu\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon}\gamma^{\alpha_1}\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-q)-i\epsilon}\gamma_\mu\\&+\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}(-\delta m^2)\frac{1}{m+\gamma\cdot(p+k_1)-i\epsilon}\gamma^{\alpha_1}\Big]\frac{1}{q^2+\mu^2-i\epsilon}u(p_1),\end{aligned}\tag{6}$$

where δm^2 is the second-order mass counterterm. Now, if we accept

$$(M_2{}^2)^{\alpha_2\alpha_1}\cong X^{\alpha_2}p^{\alpha_1},$$

then

$$(M_2{}^2)^{\alpha_2\alpha_1}=(M_2{}^2)^{\alpha_2\mu}(k_1+\tfrac{1}{2}k)_\mu\frac{p^{\alpha_1}}{(p\cdot k_1-i\epsilon)}.\tag{7}$$

To be more accurate, to have Eq. (7) as a true identity we should show that we have not dropped terms proportional to $\delta(p\cdot k_1)$. This was done in Ref. 1. Rewriting the right-hand side of Eq. (6) in the form of Eq. (7), and using the Ward identity

$$\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon}\gamma\cdot(\tfrac{1}{2}k+k_1)\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-q)-i\epsilon}=\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-q)-i\epsilon}-\frac{1}{m+\gamma\cdot(p+k_1-q)-i\epsilon},\tag{8}$$

we see a great amount of cancellation; at the end we have

$$(M_2{}^2)^{\alpha_2\alpha_1}\cong e^2\bar{u}(p_3)[\Lambda^2(k)]^{\alpha_2}u(p_1)\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon},\tag{9}$$

where

$$[\Lambda^2(k)]^{\alpha_2}=e^2\int\frac{d^4q}{(2\pi)^4i}\gamma^\mu\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-q)-i\epsilon}\gamma^{\alpha_2}\times\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-q)-i\epsilon}\gamma_\mu\frac{1}{q^2+\mu^2-i\epsilon}$$

is the second-order electromagnetic vertex. It is clear from the derivation that renormalization is not a problem at all. Also, the radiative particle can just as well be a neutral scalar or a neutral pseudoscalar.

In exactly the same way, we can show that the two-photon emission amplitude with second-order radiative correction is

$$(M_2{}^2)'^{\alpha_2\alpha_1}\cong e^2\bar{u}(p_4)[\Lambda^2(k)]^{\alpha_2}u(p_2)\frac{p'^{\alpha_1}}{-p'\cdot k_1-i\epsilon}.\tag{10}$$

Equations (9) and (10) then trivially lead to the corresponding term in Eq. (1). The major merit of Eq. (7) is that it allows us to cancel terms at the beginning of a calculation. This fact must be appreciated if one realizes the difficulty in extracting the first few leading-order terms of an amplitude (not just the leading term).

With this experience, we proceed to ask ourselves: What can be expected in general of a two-photon absorption amplitude which should possess properties (i) and (ii)? We want to construct a tensor of second rank, with indices α_2 and α_1. We shall first overcome the spin complication. The following high-energy approximations are very useful:

$$\bar{u}_{\lambda_3}(p_3)\gamma^{\alpha_i}u_{\lambda_1}(p_1)\cong\delta_{\lambda_1\lambda_3}\frac{p^{\alpha_i}}{m},\tag{11a}$$

$$\bar{u}_{\lambda_3}(p_3)\gamma^{\alpha_i}\gamma_5u_{\lambda_1}(p_1)\cong-i\frac{p^{\alpha_i}}{m}\chi_{\lambda_3}\sigma_3\chi_{\lambda_1},\tag{11b}$$

$$\bar{u}_{\lambda_3}(p_3)\sigma^{\alpha_i\mu}a_\mu u_{\lambda_1}(p_1) \cong -\frac{p^{\alpha_i}}{m}\chi_{\lambda_3}(\boldsymbol{\sigma}\times\mathbf{a})_3\chi_{\lambda_1}, \quad (11c)$$

where $i=1$, 2 and a is a finite four-vector. If $a_\mu = p_\mu$, then we can reduce by using the Dirac equations. Finally, we note that from experience no two exchanged photon indices should stand next to each other; terms of such arrangement will be of order $1/p$ smaller than the others, i.e.,

$$\text{terms with } \sigma^{\alpha_2\alpha_1} \text{ or } g^{\alpha_2\alpha_1} \text{ are negligible.} \quad (11d)$$

Because of the Dirac-matrix considerations just presented, and because the vector $\epsilon^{\alpha_1\mu\nu\lambda}p_\mu(k_1)_\nu k_\lambda$ can always be reduced by the Dirac equation and the identity $\epsilon^{\alpha_1\mu\nu\lambda} = -\gamma_5 \text{ antisym}(\gamma^{\alpha_1}\gamma^\mu\gamma^\nu\gamma^\lambda)$, we conclude that the vectors available to carry the indices α_1 and α_2 are k_1, k, and p. Besides, only one scalar of order p can be formed, namely, $p\cdot k_1$. (We remind ourselves that the kinematics is so chosen that $p\cdot k=0$.) For example, to have a numerator of order p^2, we must choose from $p^{\alpha_{1,2}}$, $(p\cdot k_1)k^{\alpha_{1,2}}$, and $(p\cdot k_1)k_1{}^{\alpha_{1,2}}$. Clearly, property (ii) makes the denominator proportional to $p\cdot k_1$. The only combination of the numerator which can give us properties (i) and (ii) is $p^{\alpha_2}p^{\alpha_1}$. All the other choices for the numerator will cancel the denominator factor $p\cdot k_1 - i\epsilon$ and, consequently, will not lead to an impact-parameter representation.

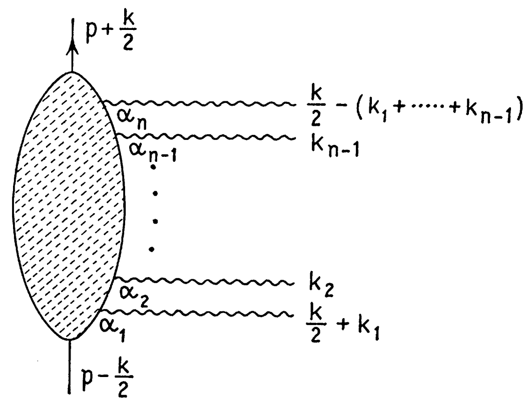

FIG. 4. N-photon absorption amplitudes with general radiative corrections.

We thus prove that to satisfy properties (i) and (ii), the two-photon absorption (emission) amplitude must be proportional to $p^{\alpha_2}p^{\alpha_1}$. We shall verify this explicitly for some parts of fourth-order radiative correction in Sec. III.

Now, for the n-photon absorption amplitude of Fig.

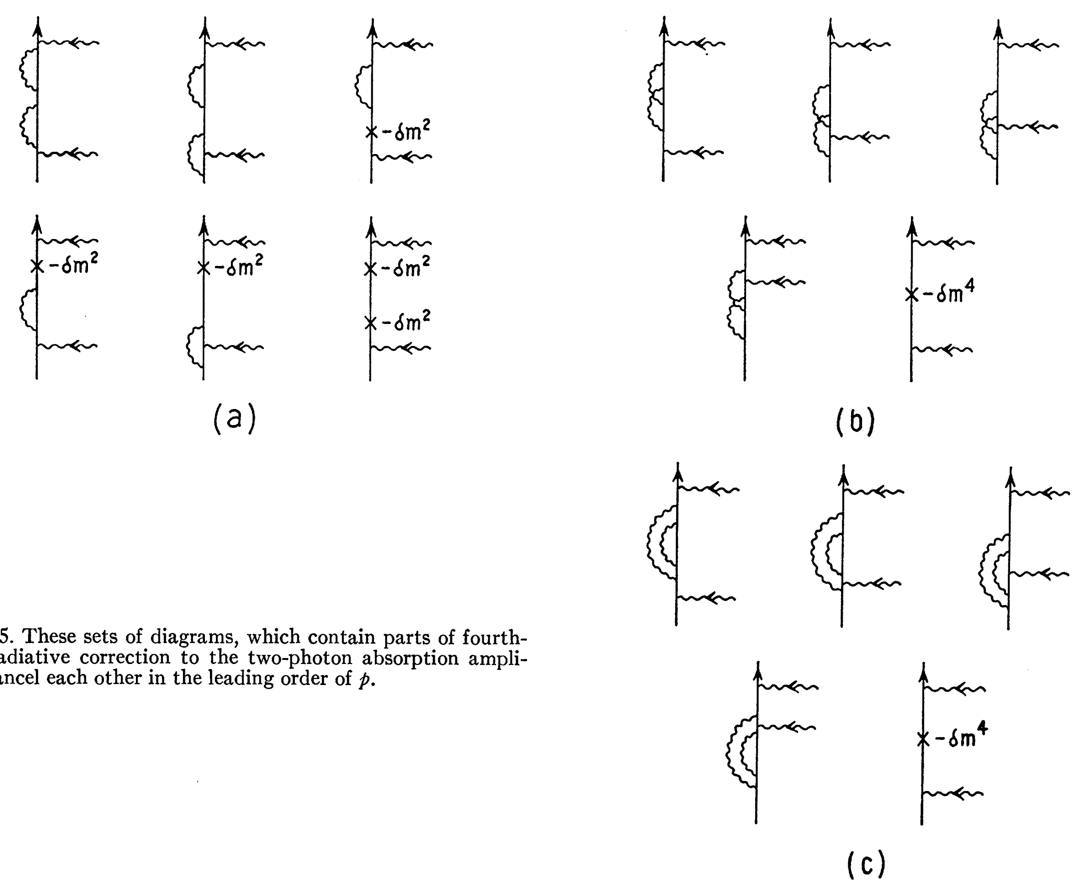

FIG. 5. These sets of diagrams, which contain parts of fourth-order radiative correction to the two-photon absorption amplitude, cancel each other in the leading order of p.

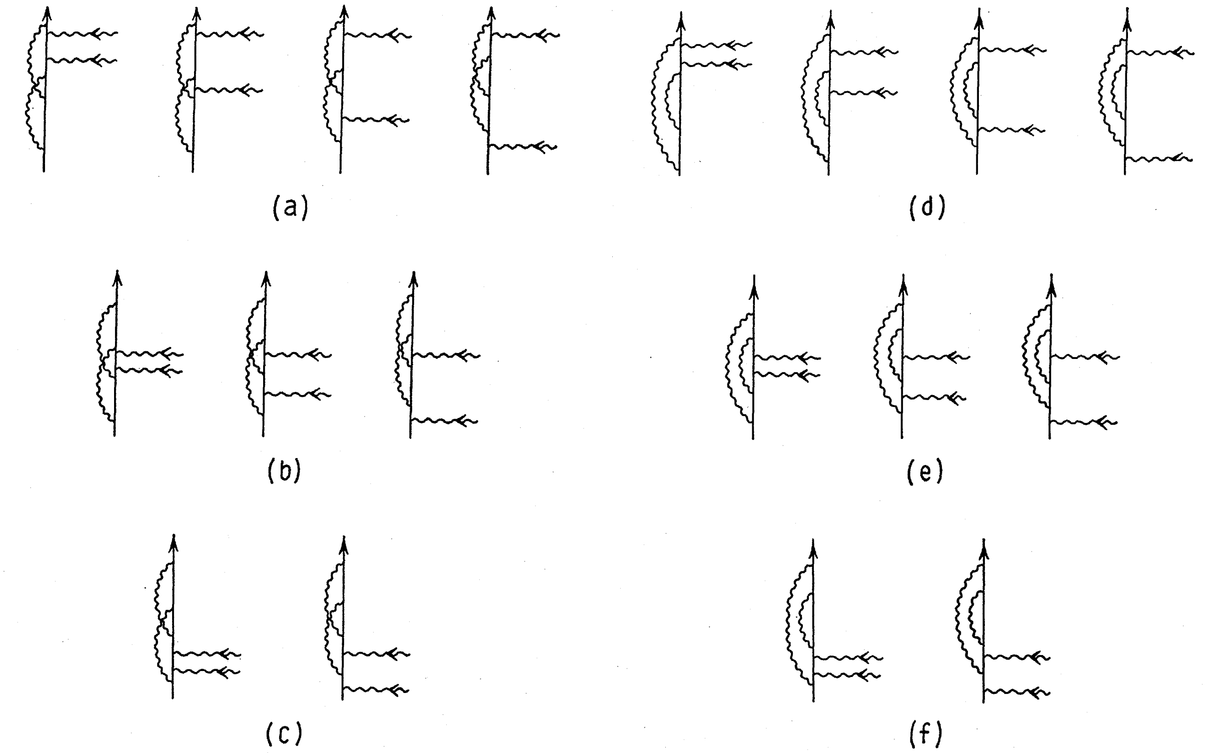

FIG. 6. Diagrams which give nontrivial fourth-order radiative correction to the two-photon absorption amplitude.

4, property (i) remains. Property (ii) becomes (ii′) the existence of the factor

$$\frac{1}{p\cdot k_1-i\epsilon}\cdots\frac{1}{p\cdot k_{n-1}-i\epsilon}$$

in the amplitude. This allows us to use the identity

$$\sum_{\text{per}}^{n}\delta[p\cdot(r_1+\cdots+r_n)]\frac{1}{p\cdot r_{\nu_1}-i\epsilon}\cdots\frac{1}{p\cdot r_{\nu_1}+\cdots+p\cdot r_{\nu_{n-1}}-i\epsilon}$$
$$=(2\pi i)^{n-1}\delta(p\cdot r_1)\cdots\delta(p\cdot r_n)\,, \quad (12)$$

where per means permuting the $n-1$ $r_{\nu_1},\ldots,r_{\nu_{n-1}}$ variables $n(n-1)$ ways among $r_1,\ldots,r_n$. This identity makes the fermion-fermion scattering amplitude satisfy an impact-parameter representation.

One can then follow the same argument for the case when $n=2$ and conclude that the n-photon absorption (emission) amplitude must be proportional to $p^{\alpha_n}\cdots p^{\alpha_1}$. After the usual manipulation, we will be led to Eq. (1).

III. FOURTH-ORDER RADIATIVE CORRECTION

We explicitly verify in this section that the two requirements of impact representation and constant differential cross section and, consequently, the property of Eq. (2), are satisfied by (at least) some of the graphs with fourth-order radiative correction. Another purpose of this tedious calculation is to show that in applying Eq. (7), we will not inadvertently pick up or drop extra terms proportional to $\delta(p\cdot k_1)$.

There are two types of graphs[7] which need to be considered for the two-photon absorption amplitude with fourth-order radiative correction (Figs. 6 and 7). It is not difficult to show the leading order $O(p)$ of the first type of graph shown in Figs. 5(a)–5(c) cancels out completely. The second type will be of principal concern to us. They are drawn in Figs. 6(a)–6(f). We have grouped them in such a way that they will lead to vertex corrections shown in Figs. 7(a)–7(f), respectively. The essential difference between Figs. 6(a)–6(c) and Figs. 6(d)–6(f) is that the latter are planar. We can show that the property of Eq. (2) is possessed by Figs. 6(d)–6(f) and, consequently,

$$(M_2{}^4)_i{}^{\alpha_2\alpha_1}\cong e^2\bar{u}(p_3)[\Lambda^4(k)]_i{}^{\alpha_2}u(p_1)\frac{p^{\alpha_1}}{(p\cdot k_1-i\epsilon)}\,,$$
$$i=d,\,e,\,f. \quad (13)$$

The graphs that require much more effort to tackle are those in Figs. 6(a)–6(c). We shall analyze explicitly graphs shown in Fig. 6(b).[8] To begin, let us redraw

[7] We have not drawn the diagrams which require mass renormalization on the external fermion lines. They reduce to the lower-order cases we considered before in Ref. 1.

[8] Figure 8(b) was singled out by Cheng and Wu in Ref. 6; they claimed it led to a hierarchy of form factors.

them in Figs. 8(a)–8(c) and label their momenta and Feynman parameters. Substantial amounts of algebra will be saved by assuming that the radiative particles are pseudoscalars with coupling $g\bar{\psi}\gamma_5\psi\phi$ and mass λ. One can convince oneself that this does not change the essential points of our argument. Figure 8(a) gives

$$M_a{}^{\alpha_2\alpha_1}=g^4\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\times 6!\int dx_1\cdots dx_5dy_1dy_2 \times\delta(1-x_1-\cdots-y_1-y_2)\frac{N_a{}^{\alpha_2\alpha_1}}{D_a{}^7}, \quad (14)$$

where

$$N_a{}^{\alpha_2\alpha_1}=\bar{u}(p_3)\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2} \times(m-\gamma\cdot r_3)\gamma^{\alpha_1}(m-\gamma\cdot r_2)\gamma_5(m-\gamma\cdot r_1)\gamma_5u(p_1)$$

and

$$D_a=y_1(s_1{}^2+\lambda^2)+y_2(s_2{}^2+\lambda^2)+\sum_{i=1}^{5}x^i(r_i{}^2+m^2)-i\epsilon.$$

We introduce the following variables:

$$r_1=t_1+l_1,\quad r_2=t_2+l_1+l_2,\quad r_3=t_3+l_1+l_2,$$
$$r_4=t_4+l_1+l_2,\quad r_5=t_5+l_2,$$
$$s_1=t_6-l_1,\quad s_2=t_7-l_2,$$

with

$$t_1=\Delta_a{}^{-1}\{p[y_1(y_2+x_2+x_3+x_4+x_5)+x_5(x_2+x_3+x_4)] -k_1[x_3(y_2+x_5)]-\tfrac{1}{2}k[(y_1+x_3+2x_4) \times(y_2+x_2+x_3+x_4+x_5)-(x_3+2x_4+x_5) \times(x_2+x_3+x_4)]\}. \quad (15a)$$

$$t_2=\Delta_a{}^{-1}\{p(y_1y_2-x_1x_5)-k_1[x_3(y_1+y_2+x_1+x_5)] -\tfrac{1}{2}k[(y_2+x_5)(y_1+x_3+2x_4)+(y_1+x_1) \times(x_3+2x_4+x_5)]\}, \quad (15b)$$

$$t_3=\Delta_a{}^{-1}\{p(y_1y_2-x_1x_5)+k_1[(y_1+x_1)(y_2+x_2+x_4+x_5) +(y_2+x_5)(x_2+x_4)]+\tfrac{1}{2}k[(y_1+x_1)(y_2+x_2-x_4) +(y_2+x_5)(x_2-y_1-x_4)]\}, \quad (15c)$$

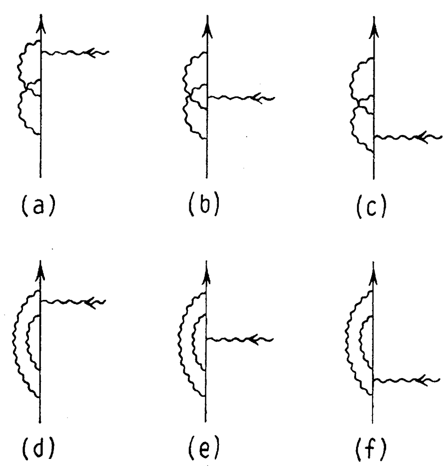

FIG. 7. Fourth-order vertex correction.

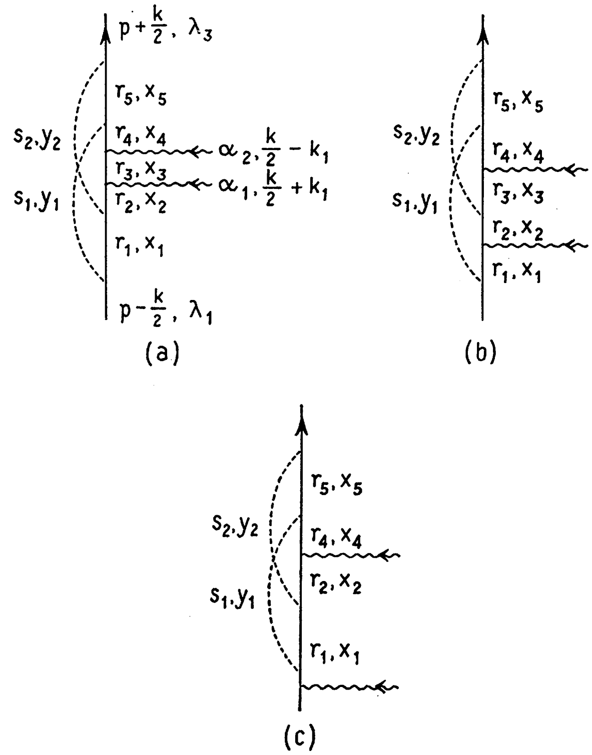

FIG. 8. Redrawing of Fig. 6(b). The radiative particle (dashed line) here is taken to be a pseudoscalar in order to simplify the algebra.

$$t_4=\Delta_a{}^{-1}\{p(y_1y_2-x_1x_5)-k_1[x_3(y_1+y_2+x_1+x_5)] +\tfrac{1}{2}k[(y_1+x_1)(y_2+2x_2+x_3)+(y_2+x_5) \times(x_1+2x_2+x_3)]\}, \quad (15d)$$

$$t_5=\Delta_a{}^{-1}\{p[y_2(y_1+x_1+x_2+x_3+x_4)+x_1(x_2+x_3+x_4)] -k_1[x_3(y_1+x_1)]+\tfrac{1}{2}k[(y_2+2x_2+x_3) \times(y_1+x_1+x_2+x_3+x_4)-(x_1+2x_2+x_3) \times(x_2+x_3+x_4)]\}, \quad (15e)$$

and

$$\Delta_a=(y_1+x_1)(y_2+x_2+x_3+x_4+x_5) +(y_2+x_5)(x_2+x_3+x_4).$$

There are similar expressions for t_6 and t_7, but since they will not appear later on, we do not write them down. In terms of these variables, the denominator function can be written as

$$D_a=(y_1+x_1+x_2+x_3+x_4)l_1{}^2+(y_2+x_2+x_3+x_4+x_5)l_2{}^2 +2(x_2+x_3+x_4)l_1\cdot l_2+2p\cdot k_1 \times\Delta_a{}^{-1}x_3(y_1y_2-x_1x_5)+f_a-i\epsilon, \quad (16)$$

where f_a is a function of the masses, k, and k_1, independent of $p\cdot k_1$.

We shall add and subtract terms to write

$$M_a{}^{\alpha_2\alpha_1}=g^4\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\times 6!\int dx_1\cdots dx_5 dx_1 dy_2$$
$$\times\delta(1-x_1-\cdots-y_1-y_2)\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\frac{N_a{}^{\alpha_2}}{D_a{}^7}+S_a{}^{\alpha_2\alpha_1},\quad(17)$$

where

$$N_a{}^{\alpha_2}=\bar{u}(p_3)\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2}\times(m-\gamma\cdot r_3)\gamma\cdot(k_1+\tfrac{1}{2}k)(m-\gamma\cdot r_2)\gamma_5\times(m-\gamma\cdot r_1)\gamma_5 u(p_1);$$

in words, the first term of $M_a{}^{\alpha_2\alpha_1}$ is obtained from Eq. (14) by replacing γ^{α_1} by

$$\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\gamma\cdot(k_1+\tfrac{1}{2}k).$$

Also,

$$S_a{}^{\alpha_2\alpha_1}=g^4\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\times 6!\int dx_1\cdots dx_5 dy_1 dy_2$$
$$\times\delta(1-x_1-\cdots-y_1-y_2)\frac{1}{p\cdot k_1-i\epsilon}\frac{N_a{}'^{\alpha_2\alpha_1}}{D_a{}^7},\quad(18)$$

where

$$N_a{}'^{\alpha_2\alpha_1}=\bar{u}(p_3)\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2}\times(m-\gamma\cdot r_3)[\gamma^{\alpha_1}p\cdot k_1-p^{\alpha_1}\gamma\cdot(k_1+\tfrac{1}{2}k)]\times(m-\gamma\cdot r_2)\gamma_5(m-\gamma\cdot r_1)\gamma_5 u(p_1).$$

In the following, our effort is to show that $S_a{}^{\alpha_2\alpha_1}$ vanishes to order $O(\ln p)$. Now, the coefficient of $2p\cdot k_1$ in D_a tells us that the best high-energy behavior ($p\to\infty$) of $S_a{}^{\alpha_2\alpha_1}$ is obtained by setting $x_3\cong 0$.[9] Let us then inspect the structure of $N_a{}'^{\alpha_2\alpha_1}$, bearing this in mind. We first look at terms without internal integration variables. By pushing $\gamma\cdot p$ to act on the Dirac spinors, we can easily show that

$$\begin{aligned}&\bar{u}(p_3)(m+\gamma\cdot t_5)(m-\gamma\cdot t_4)\gamma^{\alpha_2}(m-\gamma\cdot t_3)p^{\alpha_1}\gamma\cdot(k_1+\tfrac{1}{2}k)(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)\\
&\cong\bar{u}(p_3)[(m-ma_5+c_5\gamma\cdot k)(m-c_4\gamma\cdot k)+a_4m(m-ma_5-c_5\gamma\cdot k)+2a_4b_5p\cdot k_1]\\
&\quad\times\gamma^{\alpha_2}2a_3p\cdot k_1p^{\alpha_1}[(m-c_2\gamma\cdot k)(m-ma_1+c_1\gamma\cdot k)+a_2m(m-ma_1-c_1\gamma\cdot k)+2a_2b_1p\cdot k_1]u(p_1)\\
&\cong\bar{u}(p_3)(m+\gamma\cdot t_5)(m-\gamma\cdot t_4)\gamma^{\alpha_2}p\cdot k_1(m-\gamma\cdot t_3)\times\gamma^{\alpha_1}(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)+O(p^2),\end{aligned}$$

in which we have introduced the simplified notations

$$\begin{aligned}t_1&=a_1(p-\tfrac{1}{2}k)+b_1k_1+c_1k,\\
t_2&=a_2(p-\tfrac{1}{2}k)+b_2k_1+c_2k,\\
t_3&=a_3p+b_3k_1+c_3k,\\
t_4&=a_4(p+\tfrac{1}{2}k)+b_4k_1+c_4k,\\
t_5&=a_5(p+\tfrac{1}{2}k)+b_5k_1+c_5k.\end{aligned}$$

Note that $a_2b_1p\cdot k_1$ and $a_4b_5p\cdot k_1$ are effectively of order $O(1)$ because the combination $a_2b_1\cong x_3(y_1y_2-x_1x_5)\cong a_4b_5$ will reduce the power by $1/p$. The terms in $N_a{}'^{\alpha_2\alpha_1}$ independent of l^2 are therefore of order $O(p^2)$; that is, the order $O(p^3)$ terms cancel out completely. Taking into account the factor $1/(p\cdot k_1-i\epsilon)$ in front of $1/D_a{}^7$ and that $1/D_a{}^7$ itself will provide us another power of $1/(p\cdot k_1-i\epsilon)$ when we apply the standard technique to extract the high-energy behavior, we see that their contribution to $S_a{}^{\alpha_2\alpha_1}$ is of order $O(\ln p)$, which can be discarded.

To look into terms with internal integration (l) dependence, we shall perform the rotation

$$\begin{pmatrix}l_1\\l_2\end{pmatrix}=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}l_1'\\l_2'\end{pmatrix}$$

and

$$\begin{pmatrix}y_1+x_1+x_2+x_3+x_4&x_2+x_3+x_4\\x_2+x_3+x_4&y_2+x_2+x_3+x_4+x_5\end{pmatrix}\times\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}=\begin{pmatrix}\lambda_1\cos\theta&\lambda_2\sin\theta\\-\lambda_1\sin\theta&\lambda_2\cos\theta\end{pmatrix}.$$

We do not write down the eigenvalues λ_1 and λ_2, since we do not need them for the following discussion. Then, for example,

$$\begin{aligned}&-\bar{u}(p_3)\gamma\cdot l_2\gamma\cdot(l_1+l_2)\gamma^{\alpha_2}(m-\gamma\cdot t_3)\gamma^{\alpha_1}p\cdot k_1(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)\\
&\cong[-\sin\theta(\cos\theta-\sin\theta)]l_1'^2+\cos\theta(\cos\theta+\sin\theta)l_2'^2]\\
&\quad\times 2a_3p^{\alpha_1}p\cdot k_1\bar{u}(p_3)\gamma^{\alpha_2}[(m-c_2\gamma\cdot k)(m-ma_1+c_1\gamma\cdot k)+ma_2(m-ma_1-c_1\gamma\cdot k)+2a_2b_1p\cdot k_1]u(p_1)+O(p^2)\\
&\cong-\bar{u}(p_3)\gamma\cdot l_2\gamma\cdot(l_1+l_2)\gamma^{\alpha_2}(m-\gamma\cdot t_3)p^{\alpha_1}\gamma\cdot(k_1+\tfrac{1}{2}k)\times(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1).\end{aligned}$$

As another example,

$$\begin{aligned}&-\bar{u}(p_3)\gamma\cdot l_2(m-\gamma\cdot t_4)\gamma^{\alpha_2}\gamma\cdot(l_1+l_2)\gamma^{\alpha_1}p\cdot k_1(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)\\
&\cong[-\sin\theta(\cos\theta-\sin\theta)l_1'^2+\cos\theta(\cos\theta+\sin\theta)l_2'^2]\\
&\quad\times a_4p^{\alpha_2}p\cdot k_1\bar{u}(p_3)\gamma^{\alpha_1}[m^2(1-a_1)+mc_1\gamma\cdot k-m(1-a_1)c_2\gamma\cdot k+c_1c_2k^2+a_2m(m-ma_1-c_1\gamma\cdot k)+2a_2b_1p\cdot k_1]u(p_1)\end{aligned}$$

[9] To extract the high-energy behavior of an amplitude, we follow the work of P. G. Federbush and M. T. Grisaru, Ann. Phys. (N. Y.) **22**, 263 (1963); **22**, 299 (1963); J. C. Polkinghorne, J. Math. Phys. **4**, 503 (1963); **4**, 1396 (1963); and G. Tiktopoulos, Phys. Rev. **131**, 480 (1963); **131**, 2373 (1963).

and

$$-\bar{u}(p_3)\gamma\cdot l_2(m-\gamma\cdot t_4)\gamma^{\alpha_2}\gamma\cdot(l_1+l_2)p^{\alpha_1}\gamma\cdot(k_1+\tfrac{1}{2}k)(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)$$
$$\cong[-\sin\theta(\cos\theta-\sin\theta)l_1'^2+\cos\theta(\cos\theta+\sin\theta)l_2'^2]\times a_4p^{\alpha_2}p^{\alpha_1}\bar{u}(p_3)\gamma\cdot(k_1+\tfrac{1}{2}k)[m^2(1-a_1)+mc_1\gamma\cdot k-m(1-a_1)c_2\gamma\cdot k+c_1c_2k^2+a_2m(m-ma_1-c_1\gamma\cdot k)+2a_2b_1p\cdot k_1]u(p_1).$$

Using the high-energy approximations of Eqs. (11a)–(11d), we can easily show that

$$p^{\alpha_1}\bar{u}(p_3)\gamma\cdot(k_1+\tfrac{1}{2}k)u(p_1)\cong p\cdot k_1u(p_3)\gamma^{\alpha_1}u(p_1)$$

and

$$p^{\alpha_1}\bar{u}(p_3)\gamma\cdot(k_1+\tfrac{1}{2}k)\gamma\cdot ku(p_1)=p\cdot k_1\bar{u}(p_3)\gamma^{\alpha_1}\gamma\cdot ku(p_1).$$

As a result,

$$-\bar{u}(p_3)\gamma\cdot l_2(m-\gamma\cdot t_4)\gamma^{\alpha_2}\gamma\cdot(l_1+l_2)\gamma^{\alpha_1}p\cdot k_1(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1)$$
$$\cong-\bar{u}(p_3)\gamma\cdot l_2(m-\gamma\cdot t_4)\gamma^{\alpha_2}\gamma\cdot(l_1+l_2)p^{\alpha_1}\gamma\cdot(k_1+\tfrac{1}{2}k)\times(m-\gamma\cdot t_2)(m+\gamma\cdot t_1)u(p_1).$$

In an entirely similar fashion, we can show that all the other terms of $N_a'^{\alpha_2\alpha_1}$ with internal integration dependence cancel to order $O(p^2)$, i.e.,

$$N_a'^{\alpha_2\alpha_1}=O(p^2),$$

and therefore

$$S_a^{\alpha_2\alpha_1}=O(\ln p),$$

which can be discarded. It is also clear that

$$M_a^{\alpha_2\alpha_1}=O(p\ln p). \tag{19}$$

In the following, the procedure of extracting the high-energy behavior of a graph is almost the same as above. In order not to tax the reader's patience, we shall not present the details.

We shall analyze Fig. 8(c) next:

$$M_c^{\alpha_2\alpha_1}=g^4\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\times5!\int dx_1\cdots dx_5dy_1dy_2\times\delta(1-x_1-\cdots-x_5-y_1-y_2)\frac{N_c^{\alpha_2\alpha_1}}{D_c^6}\frac{1}{2p\cdot k_1-i\epsilon}, \tag{20}$$

in which

$$N_c^{\alpha_2\alpha_1}=\bar{u}(p_3)\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2}(m-\gamma\cdot r_2)\times r_5(m-\gamma\cdot r_1)\gamma_5[m-\gamma\cdot(p+k_1)]\gamma^{\alpha_1}u(p_1)$$

and

$$D_c=y_1(s_1^2+\lambda^2)+y_2(s_2^2+\lambda^2)+\sum x_i(r_i^2+m^2)-i\epsilon,\quad i=1,2,4,5.$$

The following changes of variables are introduced:

$$r_1=t_1+l_1,\quad r_2=t_2+l_1+l_2,$$
$$r_4=t_4+l_1+l_2,\quad r_5=t_5+l_2,$$
$$s_1=t_6-l_1,\quad s_2=t_7-l_2,$$

with

$$t_1=\Delta_c^{-1}\{p[y_1(y_2+x_2+x_4+x_5)+x_5(x_2+x_4)]+k_1[y_1(y_2+x_2+x_4+x_5)+x_4(y_2+x_5)]-\tfrac{1}{2}k(x_4y_2-x_2x_5)\}, \tag{21a}$$

$$t_2=\Delta_c^{-1}\{p(y_1y_2-x_1x_5)+k_1[(y_1+x_4)(y_2+x_5)+x_4(y_1+x_1)]-\tfrac{1}{2}k[x_4(y_2+x_5)+(x_4+x_5)(y_1+x_1)]\}, \tag{21b}$$

$$t_4=\Delta_c^{-1}\{p(y_1y_2-x_1x_5)+k_1[(y_1-x_2)(y_2+x_5)-(y_1+x_1)(y_2+x_2+x_5)]+\tfrac{1}{2}k[(y_1+x_1)(y_2+x_2)+(y_2+x_5)x_2]\}, \tag{21c}$$

$$t_5=\Delta_c^{-1}\{p[y_2(y_1+x_1+x_2+x_4)+x_1(x_2+x_4)]-k_1(y_1x_2-x_1x_4)-\tfrac{1}{2}k[(y_1+x_1)(y_2+x_2)+y_2(x_2+x_4)]\}, \tag{21d}$$

and

$$\Delta_c=(y_1+x_1)(y_2+x_2+x_4+x_5)+(y_2+x_5)(x_2+x_4).$$

The denominator can be expressed as

$$D_c=(y_1+x_1+x_2+x_4)l_1^2+(y_2+x_2+x_4+x_5)l_2^2+2(x_2+x_4)l_1\cdot l_2+2p\cdot k_1\Delta_c^{-1}[x_1y_1(y_2+x_2+x_4+x_5)+x_2y_1y_2+x_1x_4x_5]+f_c-i\epsilon, \tag{22}$$

where, again, f_c is a function of the masses, k_1, and k. It is obvious that as $p\to\infty$, the dominant behavior comes from end points.

Defining

$$N_c^{\alpha_2}=\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2}(m-\gamma\cdot r_2)\times\gamma_5(m-\gamma\cdot r_1)\gamma_5, \tag{23}$$

we write

$$N_c^{\alpha_2\alpha_1}=\bar{u}(p_3)N_c^{\alpha_2}[m-\gamma\cdot(p-\tfrac{1}{2}k+k_1+\tfrac{1}{2}k)]\gamma^{\alpha_1}u(p_1)$$
$$\cong-\bar{u}(p_3)N_c^{\alpha_2}\gamma\cdot(k_1+\tfrac{1}{2}k)\gamma^{\alpha_1}u(p_1)+2p^{\alpha_1}\bar{u}(p_3)N_c^{\alpha_2}u(p_1). \tag{24}$$

The second term already has the desired property, namely, it is proportional to p^{α_1}. Our task now is to show that the first term is negligible. To facilitate subsequent discussion, we split this term into three parts:

$$-\bar{u}(p_3)N_c^{\alpha_2}\gamma\cdot(k_1+\tfrac{1}{2}k)\gamma^{\alpha_1}u(p_1)=R_1^{\alpha_2\alpha_1}+R_2^{\alpha_2\alpha_1}+R_3^{\alpha_2\alpha_1}, \tag{25}$$

where the first term has no l dependence, the second is proportional to l^2, and the last is proportional to $(l^2)^2$. It is a matter of tedious algebra to show that

$$R_1^{\alpha_2\alpha_1}=O(p^3), \tag{26a}$$

$$R_2^{\alpha_2\alpha_1}=l^2O(p^2), \tag{26b}$$

and

$$R_3^{\alpha_2\alpha_1}=(l^2)^2O(p). \tag{26c}$$

Then we look at

$$I_1=\int d^4l_1d^4l_2\int dx_1\cdots dx_5dy_1dy_2 \times\delta(1-x_1-\cdots-x_5-y_1-y_2) \times\frac{R_1{}^{\alpha_2\alpha_1}}{D_c{}^6}\frac{1}{2p\cdot k_1-i\epsilon}$$
$$=\frac{O(p^3)}{2p\cdot k_1-i\epsilon}\int dx_1\cdots dx_5dy_1dy_2 \times\delta(1-x_1-\cdots-x_5-y_1-y_2) \times\frac{1}{(2p\cdot k_1\phi_c(x,y)+f_c-i\epsilon)_2}, \quad (27)$$

where[10]

$$\phi_c(x,y)=\Delta_c{}^{-1}[x_1y_1(y_2+x_2+x_3+x_5) +x_2y_1y_2+x_1x_4x_5]. \quad (28)$$

The integral over the Feynman parameters gives $1/p^2$, up to powers of $\ln p$. To see this, one observes that one has to set at least two Feynman parameters near zero to have vanishingly small $\phi(x,y)$. Thereupon,

$$I_1=O(\ln p), \quad (29)$$

which means $R_1{}^{\alpha_2\alpha_1}$ can be neglected. We then take up $R_2{}^{\alpha_2\alpha_1}$ by defining

$$I_2=\int d^4l_1d^4l_2\int dx_1\cdots dx_5dy_1dy_2 \times\delta(1-x_1-\cdots-x_5-y_1-y_2)\frac{R_2{}^{\alpha_2\alpha_1}}{D_c{}^6}\frac{1}{2p\cdot k_1-i\epsilon}$$
$$=\frac{O(p^2)}{2p\cdot k_1-i\epsilon}\int dx_1\cdots dx_5dy_1dy_2 \times\delta(1-x_1-\cdots-x_5-y_1-y_2) \times\frac{1}{2p\cdot k_1\phi_c(x,y)+f_c-i\epsilon}. \quad (30)$$

The integral can at best be of order $1/p$, again up to powers of $\ln(p)$. As a result,

$$I_2=O(\ln p), \quad (31)$$

and $R_2{}^{\alpha_2\alpha_1}$ can be neglected.

In order to discuss $R_3{}^{\alpha_2\alpha_1}$, one should either introduce regulators or cutoffs in the l integration. It is not hard to show that again $R_3{}^{\alpha_2\alpha_1}$ gives negligible contribution.

We may mention in passing that $M_c{}^{\alpha_2\alpha_1}$ is also of order $p\ln(p)$.

In the above consideration of $M_a{}^{\alpha_2\alpha_1}$ and $M_c{}^{\alpha_2\alpha_1}$, we have not attempted to extract the exact coefficients of the terms of order $p\ln(p)$ and p. That would be in fact too complicated.

Now, we add $M_a{}^{\alpha_2\alpha_1}$ and $M_c{}^{\alpha_2\alpha_1}$:

$$M_a{}^{\alpha_2\alpha_1}+M_c{}^{\alpha_2\alpha_1}\cong\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2\bar{u}(p_3)}{(2\pi)^4i}\gamma_5\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_2)-i\epsilon}\gamma_5\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_1-l_2)-i\epsilon}\gamma^{\alpha_2}$$
$$\times\frac{1}{m+\gamma\cdot(p+k_1-l_1-l_2)-i\epsilon}\gamma\cdot(k_1+\tfrac{1}{2}k)\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-l_1-l_2)-i\epsilon}\gamma_5\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-l_1)-i\epsilon}$$
$$\times\frac{1}{l_1{}^2+\lambda^2+i\epsilon}\frac{1}{l_2{}^2+\lambda^2-i\epsilon}+\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\gamma_5\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_2)-i\epsilon}\gamma_5$$
$$\times\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_1-l_2)-i\epsilon}\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1-l_1-l_2)-i\epsilon}\gamma_5\frac{1}{m+\gamma\cdot(p+k_1-l_1)-i\epsilon}\gamma_5up_1$$
$$\times\frac{1}{l_1{}^2+\lambda^2-i\epsilon}\frac{1}{l_2{}^2+\lambda^2-i\epsilon}+O(\ln p),$$

in which we have written $M_a{}^{\alpha_2\alpha_1}$ and $M_c{}^{\alpha_2\alpha_1}$ both in momentum space. Now, we employ the Ward identity, Eq. (8), twice to rearrange terms and write

$$M_a{}^{\alpha_2\alpha_1}+M_c{}^{\alpha_2\alpha_1}\cong\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\bar{u}(p_3)[\Lambda_4(k)]_b{}^{\alpha_2}u(p_1)-\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\bar{u}(p_3)\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\gamma_5$$
$$\times\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_2)-i\epsilon}\gamma_5\frac{1}{m+\gamma\cdot(p+\frac{1}{2}k-l_1-l_2)-i\epsilon}\gamma^{\alpha_2}\frac{1}{m+\gamma\cdot(p+k_1-l_1-l_2)-i\epsilon}\gamma_5u(p_1)$$
$$\times\frac{1}{m+\gamma\cdot(p+k_1-l_1)-i\epsilon}\gamma\cdot(k_1+\tfrac{1}{2}k)\frac{1}{m+\gamma\cdot(p-\frac{1}{2}k-l_1)-i\epsilon}u(p_1)\frac{1}{l_1{}^2+\lambda^2-i\epsilon}\frac{1}{l_2{}^2+\lambda^2-i\epsilon}. \quad (32)$$

[10] The last equation is evidently symbolic. There are extra factors of the Feynman parameters in the integrand. We have been careful to ascertain that, after scaling and integrating with respect to certain sets of variables to extract the leading behavior, the remaining integrals are finite.

Then

$$(M_2{}^4)_b{}^{\alpha_2\alpha_1}=M_a{}^{\alpha_2\alpha_1}+M_b{}^{\alpha_2\alpha_1}+M_c{}^{\alpha_2\alpha_1}$$
$$\cong\frac{p^{\alpha_1}}{p\cdot k_1-i\epsilon}\bar{u}(p_3)[\Lambda_4(k)]_b{}^{\alpha_2}u(p_1)+S^{\alpha_2\alpha_1},$$

where

$$S^{\alpha_2\alpha_1}=\frac{1}{p\cdot k_1-i\epsilon}g^4\int\frac{d^4l_1}{(2\pi)^4i}\frac{d^4l_2}{(2\pi)^4i}\times 6!\int dx_1\cdots$$
$$\times dx_5dy_1dy_2\delta(1-x-\cdots-x_5-y_1-y_2)\frac{N^{\alpha_2\alpha_1}}{D^7},\quad(33)$$

with

$$N^{\alpha_2\alpha_1}=\bar{u}(p_3)\gamma_5(m-\gamma\cdot r_5)\gamma_5(m-\gamma\cdot r_4)\gamma^{\alpha_2}(m-\gamma\cdot r_3)$$
$$\times\gamma_5(m-\gamma\cdot r_2)[p\cdot(k_1+\tfrac{1}{2}k)\gamma^{\alpha_1}-p^{\alpha_1}\gamma\cdot(k+\tfrac{1}{2}k)]$$
$$\times(m-\gamma\cdot r_1)\gamma_5u(p_1)$$

and

$$D=y_1(s_1{}^2+\lambda^2)+y_2(s_2{}^2+\lambda^2)+\sum_{i=1}^{5}x_i(r_i{}^2+m^2)-i\epsilon.$$

The momenta and Feynman parameters are those labeled in Fig. 8(b). $[\Lambda_4(k)]_b{}^{\alpha_2}$ is the fourth-order vertex function of Fig. 7(b). We shall show that $S^{\alpha_2\alpha_1}$ on the right-hand side of Eq. (33) is small compared to the first term. As usual, we make a change of variables:

$$r_1=t_1+l_1,\quad r_2=t_2+l_2,\quad r_3=t_3+l_1+l_2,$$
$$r_4=t_4+l_1+l_2,\quad r_5=t_5+l_2,$$
$$s_1=t_6-l_1,\quad s_2=t_7-l_2,$$

where

$$t_1=\Delta^{-1}\{p[y_1(y_2+x_3+x_4+x_5)+x_5(x_3+x_4)]$$
$$-k_1[x_2(y_2+x_3+x_4+x_5)+x_3(y_2+x_5)]$$
$$-\tfrac{1}{2}k[(y_1+x_2+x_4)(y_2+x_3+x_4+x_5)$$
$$+(y_2-x_4)(x_3+x_4)]\},\quad(34a)$$

$$t_2=\Delta^{-1}\{p[y_1(y_2+x_3+x_4+x_5)+x_5(x_3+x_4)]$$
$$+k_1[(y_1+x_1)(y_2+x_3+x_4+x_5)+x_4(y_2+x_5)]$$
$$+\tfrac{1}{2}k[x_1(y_2+x_3+x_4+x_5)+x_3x_5-y_2x_4]\},\quad(34b)$$

$$t_3=\Delta^{-1}\{p[y_1y_2-x_5(x_1+x_2)]+k_1[(y_1+x_1+x_4)(y_2+x_5)$$
$$+x_4(y_1+x_1+x_2)]+\tfrac{1}{2}k[(y_1+x_1+x_2)(y_2-x_4)$$
$$-(y_1+x_2+x_4)(y_2+x_5)]\},\quad(34c)$$

$$t_4=\Delta^{-1}\{p[y_1y_2-x_5(x_1+x_2)]-k_1[(x_2+x_3)(y_2+x_5)$$
$$+x_3(y_1+x_1+x_2)]+\tfrac{1}{2}k[(x_1+x_3)(y_2+x_5)$$
$$+(y_2+x_3)(y_1+x_1+x_2)]\},\quad(34d)$$

$$t_5=\Delta^{-1}\{p[(y_1+x_1+x_2)y_2+(x_3+x_4)(y_2+x_1+x_2)]$$
$$-k_1[x_3(y_1+x_1)-x_2x_4]+\tfrac{1}{2}k[(y_2-x_1)(x_3+x_4)$$
$$+(y_1+x_1+x_2)(y_2+x_3)]\},\quad(34e)$$

$$\Delta=(y_2+x_5)(x_3+x_4)+(y_2+x_3+x_4+x_5)(y_1+x_1+x_2),$$

and similar expressions for t_6 and t_7. The denominator function becomes

$$D=(y_1+x_1+x_2+x_3+x_4)l_1{}^2+(y_2+x_3+x_4+x_5)l_2{}^2$$
$$+2(x_3+x_4)l_1\cdot l_2+2p\cdot k_1\Delta^{-1}\phi(x,y)+f-i\epsilon,\quad(35)$$

where

$$\phi(x,y)=x_3(y_1y_2-x_1x_5)$$
$$+x_2[y_1(y_2+x_3+x_4+x_5)+x_4x_5]\quad(36)$$

and f is again independent of $p\cdot k_1$ and the l's.

As before, we separate the numerator function into three parts:

$$N^{\alpha_2\alpha_1}=\bar{R}_1{}^{\alpha_2\alpha_1}+\bar{R}_2{}^{\alpha_2\alpha_1}+\bar{R}_3{}^{\alpha_2\alpha_1},\quad(37)$$

where $\bar{R}_1{}^{\alpha_2\alpha_1}$ has no l dependence, $\bar{R}_2{}^{\alpha_2\alpha_1}$ is proportional to l^2, and $\bar{R}_3{}^{\alpha_2\alpha_1}$ is proportional to $(l^2)^2$. A substantial amount of algebra shows that[11,12]

$$\bar{R}_1{}^{\alpha_2\alpha_1}=[y_1y_2-x_5(x_1+x_2)][x_3(y_1+x_1)-x_2x_4]O(p^4)$$
$$+O(p^3),\quad(38a)$$

$$\bar{R}_2{}^{\alpha_2\alpha_1}=l^2O(p^3),\quad(38b)$$

and

$$\bar{R}_3{}^{\alpha_2\alpha_1}=(l^2)^2O(p^2).\quad(38c)$$

One can then show that the end-point contribution ($x_2\cong 0$, $x_3\cong 0$) and/or ($y_1\cong 0$, $x_5\cong 0$) will give

$$S^{\alpha_2\alpha_1}(\text{end points})=O(\ln(p)),\quad(39)$$

which is negligible. Note that the combination of the coefficients in front of $O(p^4)$ in Eq. (38a) effectively reduces its contribution by order $1/p$. We do not need regulators to make the contribution due to $\bar{R}_3{}^{\alpha_2\alpha_1}$ finite here.

Our major concern at this stage is whether the leading contribution to $S^{\alpha_2\alpha_1}$ may come from some interior points of the Feynman parameter space since $\phi(x,y)$ is not positive semidefinite. In other words, we are worried whether on some hypersurfaces we can factor

$$\phi(x,y)=\phi_1(x,y)\phi_2(x,y)\quad(40)$$

such that $\phi_1(x,y)$ and $\phi_2(x,y)$ vanish simultaneously and that they both change signs in the neighborhood of these surfaces. This will give rise to pinches in the integration contour and will enhance the high-energy behavior of an amplitude.[9] A necessary condition for for this to occur is

$$\frac{\partial}{\partial\alpha_i}\phi(x,y)=0,\quad(41)$$

where $\alpha_i=x$'s or y's.

The conditions

$$\frac{\partial}{\partial x_4}\phi(x,y)=(y_1+x_5)x_2=0$$

[11] Since we can always use the Dirac equations to reduce the $\gamma\cdot p$ factors in $\gamma\cdot t_5$ and $\gamma\cdot t_1$, we see that the leading behavior of $N^{\alpha_2\alpha_1}$ must be due to the p dependence of t_2, t_3, and t_4. The coefficients of these p factors [Eqs. (34b)–(34d)] are all proportional to y_1 and x_5. Therefore we can conclude that $x_2\cong 0$ and $x_3\cong 0$ will give the best high-energy behavior of $S^{\alpha_2\alpha_1}$.

[12] It may be of some interest to mention that had we not combined $p\cdot(k_1+\frac{1}{2}k)\gamma^{\alpha_1}$ and $p^{\alpha_1}\gamma\cdot(k_1+\frac{1}{2}k)$ in the definition of $N^{\alpha_2\alpha_1}$, each would have given terms of order $[y_1y_2-x_5(x_1+x_2)]\times[x_3(y_1+x_1)-x_2x_4]O(p^4)+O(p^4)$, $O(p^4)l^2$, and $O(p^3)(l^2)^2$.

and

$$\frac{\partial}{\partial y_2}\phi(x,y)=y_1(x_2+x_3)=0$$

will imply ($y_1 \cong 0$, $x_5 \cong 0$) and/or ($x_2 \cong 0$, $x_3 \cong 0$). However, this is just the case we investigated before, which leads to Eq. (39). There is no pinch there.

One may also argue that $\phi(x,y)$ cannot produce pinches because we know of no way to draw Fig. 8(b) as a product of two nonplanar graphs, which is essentially the content of Eq. (40).

We then conclude that

$$(M_2{}^4)_b{}^{\alpha_2\alpha_1}=\bar{u}(p_3)[\Lambda_4(k)]_b{}^{\alpha_2}u(p_1)\frac{p^{\alpha_1}}{(p\cdot k_1-i\epsilon)}. \quad (42)$$

There is in principle no difficulty in following this line of reasoning to analyze Figs. 6(a) and 6(b). Since the algebra is involved and, above all, since our intention is only to illustrate that nothing unexpected occurs along the way as to invalidate our argument of Sec. II, we shall not pursue this task any further.

Our method of analysis here is based on the conventional way of extracting the high-energy behavior of graphs. The extra essential ingredient we put in is to combine terms in such a way that the unwanted terms (e.g., $S_a{}^{\alpha_2\alpha_1}$ and $S^{\alpha_2\alpha_1}$) vanish in the leading order. On the other hand, Cheng and Wu[4] used "old-fashioned" perturbation techniques and came to a different conclusion. We have not subjected the same set of graphs (Fig. 8) to an analysis in their framework to see how cancellation occurs. Interested readers are invited to undertake this task.

IV. CONCLUSION

We have argued that, for massive electrodynamics at very high energy without vacuum polarization effects, if the assumptions of an impact-parameter representation and energy-independent $d\sigma/dt$ for fermion-fermion scattering are accepted, then the scattering amplitude will be proportional to the square of the vector form factors,[13] multiplied by the eikonal function [Eq. (1)]. We have explicitly demonstrated that these assumptions are fulfilled in certain sets of diagrams, in particular those in Fig. 6(b). These assumptions can also be realized if one accepts the prescriptions of Cheng and Wu.[4]

As we mentioned in Sec. II, a more challenging problem is to prove these assumptions by some different means. However, it is also the author's conviction that since they have been proved up to fourth-order in radiative correction, it is unlikely that the situation will go awry in higher orders.

We applied the implication of Eq. (1) to the p-p scattering data,[14] and qualitative agreement was reached. The vector meson was taken to be the ω meson, since it is the lowest-mass particle with the quantum numbers we want. A natural query is how important are the vacuum polarization effects? Work along this line was carried out by Cheng and Wu[15] and Chang and Fishbane.[16] The unfortunate aspect here is that we do not know how to relate the quantities appearing in their formulas to experimentally available quantities, whereas the possibility of a pure theoretical calculation seems to be rather remote. In view of this remark, we are inclined to take Eq. (1) as a first approximation to the physical reality.

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