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S-Matrix Interpretation of Quantum Theory*

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The usual interpretation of quantum theory is clouded by the following points. (1) Invalid classical concepts are ascribed fundamental status. (2) The process of measurement is not describable within the framework of the theory. (3) The subject-object distinction is blurred. (4) The observed system is required to be isolated in order to be defined, yet interacting in order to be observed. These problems are resolved in a straightforward way by the unequivocal acceptance of a thoroughly pragmatic S-matrix viewpoint. This interpretation is described, and it is explained how and why the full physical content of quantum theory resides in the S matrix.

I. INTRODUCTION

IT is often claimed that S-matrix theory is intrinsically incomplete, in comparison with general quantum theory, because its predictions refer only to asymptotic observables, whereas those of general quantum theory refer to observables at finite times. This argument is incorrect, for S-matrix theory makes predictions about observables at finite times, and general quantum theory makes no predictions beyond those made by S-matrix theory. One aim of this work is to explain why this is so. A second is to give a coherent interpretation of quantum theory itself.

The interpretation of quantum theory to be given here is an elaboration upon the Copenhagen interpretation. That interpretation is based on four basic precepts. (1) To apply quantum theory, the physical world must be separated into two parts, called the observed and observing systems. (2) The observing system is described in terms of classical concepts. (3) The observed system is represented by a probability function. (4) This probability function describes the probabilities of the various possible responses of the observing system to various possible measurement-type interactions with the observed system.

This format has allowed physicists to apply quantum theory successfully to many practical problems. However, more than forty years of effort to clarify the basic logical structure of the scheme has left considerable doubt about its underlying logical coherence.[1]

The problem, basically, is that to apply quantum theory, one must divide the fundamentally unified physical world into two idealized parts, the observed and observing system, but the theory gives no adequate description of the connection between these two parts. The probability function is a function of the degrees of freedom of the microscopic observed system, whereas the probabilities it defines are probabilities of responses of macroscopic measuring devices, and these responses are described in terms of quite different degrees of freedom. Yet there is no realistic theory of actual measuring devices, and hence no theoretical connection between the quantum description of the microscopic observed system and the classical description of the macroscopic observing system.

In practice the required connection between these two levels of description is established empirically: Measuring devices are calibrated experimentally. This empirical procedure is accepted in what follows as a fundamental element in the interpretation of the theory: The connection between the two logically different levels of description, i.e., between probability functions and the actualities to which they refer, is considered to be determined in principle by empirical methods.

In regard to S-matrix theory, the main point is the following: A meaningful separation of the physical world into observed and observing systems requires that the (important) correlations between the devices that prepare and subsequently measure the observed system must be expressible in terms of the degrees of freedom of the observed system. In imposing this requirement we adhere to Bohr's dictum that the whole experimental arrangement must be taken into account: The microscopic observed system must be viewed in the context of the actual macroscopic situation to which it refers.

This requirement that the correlations between macroscopic preparing and measuring devices be expressible in terms of the degrees of freedom of the (microscopic) observed system is a stringent condition. Physically, one can expect it to be satisfied only if the preparing and measuring devices are physically separated, and the over-all situation is compatible with the idea that the observed system travels from the space-time region of the preparation to the space-time region of the measurement. Viewed from the standpoint of the entire system, there must be a long-range interaction between the preparing and measuring parts, and the effect of this long-range interaction must be expressible in terms of the degrees of freedom of the observed system.

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[1] For a sampling of recent comments on the Copenhagen interpretation, see *Quantum Theory and Reality*, edited by M. Bunge (Springer, Berlin, 1967) (see especially the articles by K. R. Popper and M. Bunge); *Quantum Theory and Beyond*, edited by E. Bastin (Cambridge U. P., Cambridge, England 1970); A. Shimony, Am. J. Phys. **31**, 755 (1963); J. Earman and A. Shimony, Nuovo Cimento **54B**, 332 (1968); H. P. Stapp, LRL Report No. UCRL-19729, 1970 (unpublished).

Long-range correlations between observables are controlled, both in S-matrix theory and field theory, by physical-region singularities of the S matrix (or its field-theoretic counterpart). Pole singularities in the physical region correspond to stable physical particles, which are the mediators of the true long-range interactions. Pole singularities slightly removed from the physical region correspond to slightly unstable physical particles. These are mediators of almost-long-range interactions. Nonpole singularities correspond to more complex structures. They mediate intermediate-range interactions.

A pole contribution to the long-range correlation between observables has the important property that it can be expressed in terms of the degrees of freedom of the corresponding physical particle. The equation of motion that governs the propagation of this effect is precisely the equation of motion of the freely moving physical particle. Thus, if the interaction between the preparing and measuring devices can be approximated by the asymptotically dominant pole contribution, then the condition for a meaningful separation of the physical world into observed and observing systems can be met. The observed system is identified as the particle associated with the pole. This particle is, in effect, the effect of that pole.

In the example just cited, the separation between the observed and observing system is achieved by extracting from the full interaction between the preparing and measuring devices an asymptotically dominant contribution. In order to observe this contribution, the preparing and the measuring devices must be placed far enough apart so that the main interaction between them is actually due to this contribution. If the preparing and measuring devices are not sufficiently far apart, then various short-range effects will mask the effect of the system that one is trying to observe, and the notion of an observed system breaks down.

A separation between the observed and observing systems is essential to the interpretation of quantum theory. It is always achieved, as in the above example, by effectively extracting from the full interaction between the preparing and measuring devices an asymptotically dominant contribution. This identification of the observed system with an asymptotic contribution is required both in practice and in principle.

In practice, the calibration of the measuring devices depends upon the observed system's having only a few degrees of freedom. However, the phenomenon of particle creation always entails that the representation of the full interaction between the preparing and measuring devices will have an infinite number of degrees of freedom. Experimental calibration is consequently precluded, because an infinite set of variables cannot be fixed by empirical methods. The required reduction to a few degrees of freedom is invariably achieved, in practice, by neglecting all but the long-range component of the interaction.

In principle, the representation of the observed system by a quantum probability function is possible only insofar as that system is free from the external disturbances associated with the processes of observation. This means that the observed system must be effectively free from these disturbances during some interval between its preparation and subsequent measurement. But if the observed system is effectively free of these disturbances during some such interval, then the contribution corresponding to this system will have asymptotic effects, and can be expressed by folding the empirically determined probability functions corresponding to the preparation and measurements into appropriate S-matrix elements. Consequently, the full physical content of the theory resides in the S matrix. These arguments are developed in detail in the following sections.

II. PRAGMATIC DESCRIPTION OF QUANTUM THEORY

A brief pragmatic description of quantum theory will serve as a basis for the ensuing discussion.

A. Description of Physical Objects

Physical theories predict correlations among things that can be known. A physical object is known by the traces it leaves—it is known by its durable effects on the world about it. A description of an object in terms of its effects on the world about it can be contrasted with an "intrinsic description" of the object, which is a description of an object considered as an independent entity.

Consider, for example, a piece of machinery and an engineer's description of it. This description is not a catalogue of the 10^{23} variables that might be sufficient to describe this object as an independent entity. It is essentially a description of the results of measurements that have been made on the object. It is a description of the object in terms of its effects on various measuring devices, such as calipers, etc. It is a description of some information about the object that has been transferred to its environment, and which resides there in a moderately static and durable form.

This description can be regarded as a description of a preparation of the object. From this description, a scientist conversant with the principles of classical physics can form expectations about the results of certain future measurements.

Or consider a system of billiard balls. From an engineer's description of the preparation of this system, a scientist can form expectations about the results of future measurements on the system.

The descriptions of the preparation and of the measurement can be called "extrinsic descriptions," to emphasize that they are descriptions of the effects of the system on its environment, and that the information resides in the environment.

B. Transition Probabilities

The engineer's description of the preparation of the system is interpreted as information about the system itself. From this information predictions are formed about the results of subsequent possible measurements on the system.

These predictions are of a statistical nature. The information residing in the engineer's description of the preparation is not sufficient, in general, to allow the actual result of a subsequent measurement to be predicted with certainty. Only the probabilities of the various possible results are predicted.

The procedure is as follows. Let A represent the engineer's description of the preparation of the system, and let B represent an engineer's description of a possible subsequent measurement on the system, together with a possible result of that measurement. Let x be a set of coordinates of a theoretical model of the measured system—for example, the coordinates of the centers of the billiard balls—and let p be the corresponding set of canonical momentum variables. To apply classical theory, the scientist transforms the description A of the preparation of the system into a function $\rho_A(x,p)$, which is regarded as the probability density function of the measured system. He also transforms the description B of the possible subsequent measurement and its result into a function $\rho_B(x,p)$, which is regarded as the detection efficiency function. (ρ_B could be written δ_B to emphasize that it is a detection efficiency. The subscript B, which represents the description of a final measurement, or detection, serves the same purpose.) The predicted transition probability is then

$$\mathcal{P}(A,B) \equiv \int dx dp\, \rho_A(x,p) \rho_B(x,p) .$$

This is the predicted probability that a measurement specified by the description B will give a result specified by B if the preparation is specified by A.

To apply quantum theory, the scientist transforms the description A of the preparation into a function $\rho_A(x';x'')$, and the description B of the measurement and its result into a function $\rho_B(x'';x')$, and writes the predicted transition probability as

$$\mathcal{P}(A,B) = \int dx' dx'' \rho_A(x';x'') \rho_B(x'';x') .$$

Each density function $\rho(x';x'')$ can be expressed in the form $\sum w_i \psi_i(x') \psi_i^*(x'')$, where the w_i are positive statistical weights, and the $\psi_i(x)$ are normalized wave functions of the measured system.

The complex density functions $\rho(x';x'')$ can be transformed, if desired, into real density functions $\rho(x,p)$. In terms of these, the quantum-mechanical formula for the transition probability becomes identical to the classical one. These real $\rho(x,p)$ are analogs of the classical ones in the sense that the two can be equated in certain (classical) limits.[2] But the $\rho(x,p)$ derived from the quantum-mechanical $\rho(x';x'')$ are not always positive, and hence they cannot be interpreted as ordinary probability densities and detection efficiencies.

C. Dynamical Theories

The calculation of ρ_A has two parts. First the description A of the preparation is transformed into a description ρ_A' of the prepared system. Then a dynamical rule is used to transform ρ_A' into its image ρ_A in the space of the measured system.

In classical theory the rule of propagation of ρ_A is derived from the rule that determines the possible motions of the classical model by considering statistical ensembles of these models. But since the measured system is, for practical purposes, represented by the density function ρ_A, the practical form of the dynamical theory is the rule that transforms this density function from the time of the preparation to the time of the measurement. If this rule is known, then no further dynamical description of the measured system is needed.

In quantum theory the rule that transforms ρ_A' into ρ_A is simpler than in classical theory: Instead of decomposing ρ into an equivalent statistical ensemble, and then following the motions of the individuals, and finally recombining them, one simply transforms the whole function ρ_A by a certain linear transformation.

D. Summary

Physical theories allow scientists to make statistical predictions about results of possible measurements performed on systems prepared in specified ways. To make such a prediction, the scientist transforms the descriptions A and B of the preparation and possible subsequent measurement into functions ρ_A' and ρ_B defined on the variables of some theoretical model of the prepared and measured systems. That is, he transcribes the extrinsic descriptions A and B into intrinsic descriptions ρ_A' and ρ_B. Then he uses some dynamical rule to transform the description ρ_A' to its image ρ_A in the space of the measured system. The predicted transition probability is obtained by folding ρ_B into ρ_A.

A crucial point is the factorization property. The information associated with the preparation A and with the measurement B appear in separate factors in the transition probability formula. These factors are, moreover, in practice, functions of a set of degrees of freedom that would not be sufficient, or appropriate, for a representation of the entire system of preparing and measuring devices. This factorization and reduction allow one to introduce the concept of an "observed system." It allows one to imagine that there is some "observed system" that carries information from the

[2] E. P. Wigner, Phys. Rev. **40**, 749 (1932); D. Iagolnitzer, J. Math. Phys. **10**, 1241 (1969).

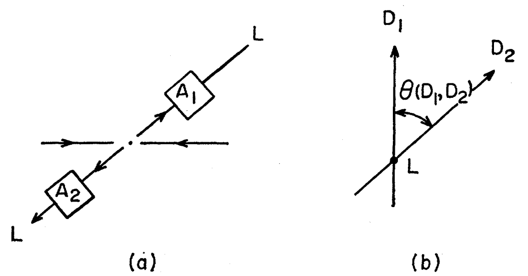

FIG. 1. (a) Two neutrons emerge from a collision. One passes through the Stern-Gerlach device A_1, the other passes through A_2. The line L is the c.m. line of flight. (b) The axes D_1 and D_2 of A_1 and A_2 are normal to L, and $\theta(D_1,D_2)$ is the angle between them. The particle is deflected either in the direction of the axis, or in the opposite direction.

preparation to the measurement. It allows one to think that the preparation is a preparation *of* something, and that the subsequent measurement is a measurement of that same thing.

Quantum theory applies only to situations in which this idea of an observed system is applicable. However, no representation of the observed system beyond that given by their density functions occurs in quantum theory. And these density functions are ascribed no physical meaning beyond what follows from their roles in the transition probability formulas.

The fact that the complete quantum-theoretical description of the observed system is given by density functions that describe only the probabilities of responses of other systems raises the question of whether, and in what sense, quantum theory can be considered complete. This is discussed next.

III. COMPLETENESS OF QUANTUM THEORY

Quantum theory predicts the probabilities of the various possible results of a measurement, not the individual results themselves. The question therefore arises whether a more complete theory is possible: Is it possible to have a theory that agrees with the predictions of quantum theory but predicts the individual results themselves, instead of merely their probabilities?

The answer is a qualified no. This conclusion is contained in the following theorem, which is based on some recent work by Bell.[3]

Theorem. No theory can (a) give contingent general predictions of the individual results of measurements, (b) be compatible with the statistical predictions of quantum theory (to within say 5%), (c) satisfy "local causes." These conditions must be explained.

A prediction of an individual result is, for example, the prediction of whether an individual particle in a Stern-Gerlach device will be deflected up or down. Quantum theory predicts the *probabilities* of these two alternatives, but not, in general, the individual result itself.

The word general in condition (a) specifies, in particular, that the individual results of Stern-Gerlach-type measurements are to be predicted by the theory.

The axis of a Stern-Gerlach device can be rotated: It can have different alternative possible settings. The word "contingent" in condition (a) means that the theory gives predictions for various possible alternative settings. It does not merely give predictions only for the one unique setting that is actually chosen by the experimenter.

Consider, specifically, an experiment in which two low-energy neutrons are made to collide, and in which each of the two scattered particles passes through a Stern-Gerlach-type device, as is shown in Fig. 1(a). The axes of the two Stern-Gerlach devices are denoted by D_1 and D_2. They are both normal to the c.m. line of flight, and $\theta(D_1,D_2)$ is the angle between them, as shown in Fig. 1(b).

Two different settings, D_1' and D_1'', of D_1 are considered. And two different settings, D_2' and D_2'', of D_2 are considered. Thus altogether four alternative combinations of settings are considered. Let j label the individual experiment, i.e., a single pair of scattered particles. Then let $n_{1j}(D_1,D_2)$ be defined to be plus one or minus one, according to whether the theory predicts that the particle from the jth pair that passes through A_1 is deflected up or down, when the settings of the axes are D_1 and D_2. The numbers $n_{2j}(D_1,D_2)$ are defined analogously for the other particle. In this example condition (a) means precisely that, for each individual pair j, the numbers $n_{1j}(D_1,D_2)$ and $n_{2j}(D_1,D_2)$ are defined for all four combinations of the arguments D_1 and D_2.

There is absolutely no restriction upon what factors other than D_1 and D_2 the numbers $n_{1j}(D_1,D_2)$ and $n_{2j}(D_1,D_2)$ depend. In particular, there is no requirement that the predictions be based only on information that can be known to a real or idealized observer. Nor is the theory required to have any preconceived type of structure. Condition (a) requires only that the eight numbers $n_{ij}(D_1,D_2)$ be defined for each experiment j. Of these eight numbers only two can be compared directly to experiment. The other six correspond to the three alternative experiments that could have been performed but were not.

If one is willing to accept that the three alternative experiments that could have been performed, but were not, would have had certain definite results if they had been performed, then the n's can be defined to be the results that those experiments would have had if they had been performed. In other words, condition (a) can be converted from a requirement that a contingent nonstatistical theory—or law of nature—exists, to the requirement that it is possible to assume that an

[3] J. S. Bell, Physics **1**, 195 (1964). See also J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Letters **23**, 880 (1969); Eugene P. Wigner, Am. J. Phys. **38**, 1005 (1970).

unperformed measurement would have had some definite result if it had been performed.

Stated more forcefully, condition (a) requires only that if the experimenters had actually adjusted the mechanical devices to give the alternative experimental setup, then these alternative experiments would have had certain definite results. For then the numbers $n_{1j}(D_1,D_2)$ and $n_{2j}(D_1,D_2)$ can be defined by the results that the experiments would have had if they had actually been performed.

According to quantum theory, the following relationship should hold with increasing accuracy as N increases:

$$\frac{1}{N}\sum_{j=1}^{N} n_{1j}(D_1,D_2)n_{2j}(D_1,D_2) = -\cos\theta(D_1,D_2). \quad (1)$$

(This result follows from the fact that the two neutrons are in a singlet spin state.) Condition (b), in this case, is simply the condition that this equation holds (to within, say, 5%) for each of the four alternative settings.

The devices D_1 and D_2 can be placed in far-apart laboratories. The requirement (c), of local causes, is that the deflection of the particle going through the first device should not depend appreciably on whether the setting of the second device is D_2' or D_2'', and that the deflection of the particle going through the second device should not depend appreciably on whether the setting of the first device is D_1' or D_1''. The setting of D_1 can be made just before the first particle arrives at A_1, and the setting of D_2 can be made just before the second particle arrives at A_2. In this case any large dependence of $n_{1j}(D_1,D_2)$ on D_2, or of $n_{2j}(D_1,D_2)$ on D_1, would require a large and almost instantaneous effect of a far-away cause.

The three assumptions all seem plausible. Yet they are mutually incompatible.

To see this, let the directions of D_1', D_1'', D_2', and D_2'' be chosen so that

$$\cos\theta(D_1',D_2') = 1, \quad (2a)$$

$$\cos\theta(D_1',D_2'') = 0, \quad (2b)$$

$$\cos\theta(D_1'',D_2') = -1/\sqrt{2}, \quad (2c)$$

$$\cos\theta(D_1'',D_2'') = 1/\sqrt{2}. \quad (2d)$$

The assumptions (a) and (b) of the theorem require that there be functions $n_{1j}(D_1,D_2)$ and $n_{2j}(D_1,D_2)$ such that the equations

$$(1/N)\sum_j n_{1j}(D_1,D_2)n_{2j}(D_1,D_2) = -\cos\theta(D_1,D_2) \quad (3)$$

are approximately satisfied for all four combinations of D_1 and D_2. Assumption (c) says that

$$n_{1j}(D_1',D_2') = n_{1j}(D_1',D_2'') \equiv n_{1j}', \quad (4a)$$

$$n_{1j}(D_1'',D_2') = n_{1j}(D_1'',D_2'') \equiv n_{1j}'', \quad (4b)$$

$$n_{2j}(D_1',D_2') = n_{2j}(D_1'',D_2') \equiv n_{2j}', \quad (4c)$$

$$n_{2j}(D_1',D_2'') = n_{2j}(D_1'',D_2'') \equiv n_{2j}''. \quad (4d)$$

Inserting (2) and (4) into (3), one obtains

$$(1/N)\sum n_{1j}'n_{2j}' = -1, \quad (5a)$$

$$(1/N)\sum n_{1j}'n_{2j}'' = 0, \quad (5b)$$

$$(1/N)\sum n_{1j}''n_{2j}' = 1/\sqrt{2}, \quad (5c)$$

$$(1/N)\sum n_{1j}''n_{2j}'' = -1/\sqrt{2}. \quad (5d)$$

From (5a) one concludes that

$$n_{1j}' = -n_{2j}'. \quad (6)$$

This combines with (5b) to give

$$(1/N)\sum n_{2j}'n_{2j}'' = 0. \quad (7)$$

Subtraction of (5d) from (5c) gives

$$(1/N)\sum n_{1j}''(n_{2j}' - n_{2j}'') = \sqrt{2}. \quad (8)$$

Using the fact that $n_{2j}''n_{2j}'' = 1$, and the fact that the absolute value of a sum is less than or equal to the sum of the absolute values, one obtains from (8)

$$\begin{aligned}\sqrt{2} &= (1/N)\sum n_{1j}''n_{2j}''(n_{2j}''n_{2j}' - 1)\\ &\leq (1/N)\sum |n_{2j}''n_{2j}' - 1|\\ &\leq (1/N)\sum(1 - n_{2j}''n_{2j}')\\ &\leq 1 - (1/N)\sum n_{2j}''n_{2j}'\\ &\leq 1,\end{aligned}$$

where (7) is used to obtain the last line. Squaring both sides one obtains the false result that two is less than one. Small variations (say 5%) in (5a) to (5d) cannot remove the contradiction. Q.E.D.

A conclusion that can be drawn from this theorem is that the demands of causality, locality, and individuality cannot be simultaneously maintained in the description of nature. Causality demands contingent predictions; locality demands local causes of localized results; individuality demands the specification of individual results, not merely their probabilities.

In quantum theory the extrinsic description describes individual localized results, but it has no causal content. The intrinsic description is a causal space-time description, but it predicts the probabilities of results, rather than the individual results. Thus the dilemma posed by Bell's theorem is circumvented, in quantum theory, by the use of two complementary descriptions, one of which omits the causality requirement, and the other of which omits the individuality requirement.

Condition (a) of Bell's theorem is essentially the requirement that nature be fundamentally lawful, in the sense that the *individual* results are specified by contingent rules.

Condition (c) is the condition that the world be fundamentally separable into independent parts, on the macroscopic space-time level. For if a cause can have a large instantaneous effect far away, then far-apart macroscopic objects can no longer be considered separate and distinct in the usual sense. Space itself loses its status.

If the world is fundamentally lawful, in the above sense, then these laws are presumably compatible (to within 5%) with the statistical laws of quantum theory. Thus the theorem of Bell proves, in effect, the profound truth that the world is either fundamentally lawless or fundamentally inseparable (see Secs. X and XII).

Science deals traditionally with rules that connect observable phenomena. Observable phenomena are essentially localized. Thus the rules of science must evidently be rules that apply to localized, essentially separate, parts of the universe. In view of the limitations on lawfulness described above, it is reasonable to believe that quantum theory is the basic scientific theory dealing with space-time relationships between results of local measurements, even though it predicts only probabilities of the results, rather than the individual results themselves.

The need for a dualistic description of nature is deduced here from correlation effects. These effects are manifestations of an element of order in nature. They can hardly be understood in terms of disorder, microscopic uncertainties, or uncontrollable elements. What is needed is more order or unity than classical ideas permit, not less.

IV. OBSERVED AND OBSERVING SYSTEMS

The limitation on physical theories imposed by Bell's theorem is met in quantum theory by separating the physical world into two parts, the observed and observing systems, and treating these parts in logically different ways.

The observing system, which includes the human observers and their agencies of observation, is treated operationally. The descriptions A and B act as instructions that permit technicians to make preparations and measurements that meet certain specifications, and permit trained observers to decide whether or not the preparations, measurements, and their results meet the prescribed specifications. These descriptions are couched in a technical jargon that is an extension of everyday language.

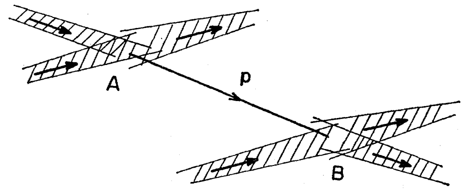

FIG. 2. Scattering process that defines two subreactions.

The observed system is treated as a carrier of dynamical predictions. It is represented by a set of contingent statistical predictions about ensembles of physical systems prepared according to certain specifications.

The observed system must be free of the dynamical disturbances associated with the process of measurement in order to develop causally. Yet it must be physically connected to the agencies of observation in order to be observed. In the words of Bohr,[4] "On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible,... . On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word."

The resolution of the disparate demands for dynamical independence and physical connection is tied to the resolution of another conflict. Quantum theory is based on a separation of the physical world into observed and observing parts. Such a separation can be made only if the observed system is a distinct system: It must be clearly distinguishable from the observing system. Yet another basic precept of quantum theory is the impossibility of any sharp separation between the observed system and the agencies of measurement.

The way in which these conflicts are resolved is exhibited in Sec. V for the case in which the observed system is a distinct physical object or entity. More complex cases are discussed later.

V. NATURE OF PHYSICAL ENTITIES

The simplest physical entities are the stable physical particles. Each such particle corresponds to a set of physical-region poles in the S matrix. These poles are direct consequences of the unitary requirement.[5] The particles are, in a sense, just manifestations of the poles: The macroscopic space-time phenomena that characterize a particle would not occur if the corresponding

[4] N. Bohr, *Atomic Theory and the Description of Nature* (Cambridge U. P., Cambridge, England, 1934), p. 53.

[5] H. P. Stapp, in *High-Energy Physics and Elementary Particles* (IAEA, Vienna, 1965), p. 41; J. Coster and H. P. Stapp, J. Math. Phys. **11**, 1441 (1970); **11**, 2743 (1970).

poles were absent, but will occur if they are present, provided the appropriate experiments are performed.[6]

Consider a scattering process of the kind shown in Fig. 2. In this process certain of the wave packets of the initial and final free particles intersect at A, and the rest intersect at B. Suppose the momentum-energy defect of the subreaction at A is approximately equal to a vector p that satisfies the mass-shell constraint $p^2=m^2$, where m is the mass of a physical particle. Then the transition amplitude for the reaction can, by virtue of the pole-factorization property of the S matrix,[5] be written in the form[6]

$$S_{AB}=\int d^3x\,\psi_B^*(x)\psi_A(x)+R\,,$$

where R goes to zero more rapidly than any inverse power of the distance between A and B. The wave functions $\psi_A(x)$ and $\psi_B(x)$ are both wave functions for a free particle of mass m.

The function $\psi_A(x)$ in this formula depends only on variables associated with the subreaction A, and $\psi_B(x)$ depends only on variables associated with subreaction B. Thus to the extent that R can be neglected, the factorization property is satisfied, and one can interpret the over-all process in terms of the idea that some intermediate system is prepared at A and subsequently measured at B.

This intermediate system has the following properties:

(a) It exists and has meaning only in a context, not as an isolated system.
(b) It develops in time as an isolated system.
(c) It has no exact physical counterpart.

Property (a) means that the intermediate system is defined in terms of correlations of other things: It has no meaning as an isolated entity. In particular it exists only to the extent that the preparation that defines it exists, and has no meaning except in terms of the subsequent measurement.

Property (b) means that the time development of the wave functions ψ_A and ψ_B (or of ρ_A and ρ_B) is not influenced by the preparing or measuring devices. The time development for all times is given by a linear transformation that does not depend on the way in which the system is prepared or measured.

Property (c) means that the wave function acquires physical meaning only in an asymptotic limit that can never be physically realized.

Macroscopic physical objects are, in a similar way, manifestations of *collections* of poles in the S matrix. Consider, for example, a dumbbell. Its coordinates include a center-of-mass variable, and also some cyclic angular variables. Quantization of the angular variables leads to a series of discrete eigenvalues, and the S matrix of a larger system that prepares and later measures this object will have poles corresponding to these eigenvalues.

More generally, any physical object (i.e., any essentially isolated physical system that is effectively confined to some finite region) will have a discrete spectrum of energy eigenvalues. And the S matrix for a system that prepares and later measures this object will have poles corresponding to these eigenvalues.

A preparation and measurement of such an object corresponds to specifications on the density functions of the preparing and measuring devices. Various poles among those associated with the object will generally fall in the energy range defined by these external specifications. The coherence of the superpositions that represent the external specifications entails the coherence of the contributions corresponding to the various poles. Thus the differences in the energies of these various poles will produce time-dependent interference effects, which will manifest themselves as time dependences in the correlations between the preparations and measurements.

These time dependences in the correlations between preparations and measurements can perhaps be repre-

[6] H. P. Stapp, Phys. Rev. **139B**, 257 (1965); D. Iagolnitzer and H. P. Stapp, Commun. Math. Phys. **14**, 15 (1969). D. Iagolnitzer, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Gordon and Breach, New York, 1969), Vol. XI D, p. 221; C. Chandler and H. P. Stapp, J. Math. Phys. **10**, 826 (1969). The main point of these papers is to show that the physical-region analyticity properties assumed in S-matrix theory are equivalent to a space-time property, called macrocausality, the physical content of which is as follows: Energy momentum is transferred over macroscopic distances only by physical particles; any transfer of energy-momentum over macroscopic distances that cannot be ascribed to a network of physical particles moving in accordance with the constraints of classical physics has a probability that falls off exponentially under space-time dilation. This property guarantees that the observed phenomena will have the space-time structure that is the basis of our intuitive ideas about causality. Other works on the question of space-time structure from the S-matrix viewpoint are D. Iagolnitzer, J. Math. Phys. **10**, 1241 (1968); M. Froissart, M. L. Goldberger, and K. M. Watson, Phys. Rev. **131**, 2820 (1963); R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Ann. Phys. (N.Y.) **10**, 62 (1960); J. Charap and S. Fubini, Ref. 12; H. P. Stapp (unpublished). The work of Iagolnitzer gives the relationship between S-matrix quantities and the quantities occurring in the classical space-time description. The work of Froissart, Goldberger, and Watson shows how the S matrix determines the space-time trajectory of particles moving through not too dense matter. The work of Blankenbecler *et al.* shows how the usual Schrödinger-theory results can be recovered from S-matrix principles in the nonrelativistic approximation. The work of Charap and Fubini shows how the Schrödinger theory results can be recovered also in relativistic cases, provided certain approximations, expressing the long-range nature of the forces and the nonimportance of particle creation, are made. The unpublished work of the present author shows how external classical electromagnetic fields are introduced into S-matrix theory, and how the Schrödinger-type description of a particle moving in an external field comes directly out of the usual S-matrix principles of analyticity and unitarity, with no extra assumptions. In regard to this last point, it should be stressed that all references to solutions of the free-field equations in the present paper should be understood to be solutions of these equations in the presence of the classically describable electromagnetic fields.

The conclusion that emerges from these works is that the Schrödinger-type results come out of S-matrix theory in those cases where it is known to work. The essential point of the present work is that the connection between the observed and observing systems can be understood in the framework of a thoroughly pragmatic S-matrix framework, and indeed must be understood in this way because the observed system is well defined only in an asymptotic limit.

sented in terms of some internal motion of the object itself. This question is discussed in a later section. But in any case the characteristics of the physical phenomena that are the signature of the physical object will manifest themselves only to the extent that the correlations between the preparations and measurements are dominated by the contributions of the poles that correspond to this object. The idea that there is, in addition to the preparing and measuring devices, some independent physical entity that is prepared and later measured, and that transmits the dynamical information between the two devices, can acquire physical significance only to the extent that these pole contributions dominate. And these pole contributions will dominate only to the extent that the separation between the devices is large, on some scale.

Of course, on the submicroscopic scale of elementary particle interactions a large separation can still be microscopic. But in principle, the concept of a distinct physical entity acquires precise physical significance in the framework of quantum theory only to the extent that this entity is infinitely far away from the agencies of observation.

Distinct physical entities are, therefore, in the framework of quantum theory, asymptotic idealizations. And the wave functions that represent them acquire physical meaning only to the extent the appropriate asymptotic limits are effectively realized. To the extent that these limits are effectively realized, i.e., to the extent that the pole contributions dominate, the entity associated with the poles is both a carrier of correlations between the agencies of preparation and measurement, and yet a dynamically independent entity.

The point here is simple but important. From Newtonian physics we have inherited the idea that a physical entity is a logical unit that is either not further analyzable, in the case of an elementary particle, or is analyzable into fundamentally unanalyzable constituents or qualities. But in quantum theory a physical entity is at the same time a dynamically independent object, yet a relationship between things that are not constituents of the object itself. And an elementary particle is not an independently existing unanalyzable entity. It is, in essence, a set of relationships that reach outward to other things.

VI. NATURE OF OBSERVED SYSTEMS

The fundamental idea of quantum theory is that correlations between certain aspects of the physical world can be predicted, with useful accuracy, by transforming specifications on the preparation and measurement into functions in a space associated with the observed system. This reduction of the complex unified actual situation to a simple model system is the essence of quantum theory.

It is entirely to be expected that this drastic simplification, which allows certain features of the behavior of the complex macroscopic system to be expressed in terms of the degrees of freedom of a simple microscopic system, should occur only in certain special situations, namely, those in which the physical connection between the macroscopic devices can be considered to be physically mediated principally by that microscopic system. The dominance of such a contribution can be expected to emerge only in an appropriate asymptotic limit. For if the various devices are not physically separated, then there is no reason either physically or theoretically to expect this one contribution to dominate: The whole macroscopic system will fuse, instead, into a unified whole. Thus the notion of the observed system, and hence entire quantum-theoretical framework, rests on the emergence of simple dominant terms in asymptotic limits.

In quantum theory the asymptotically dominating terms are controlled by physical-region singularities of the S matrix (or of its field-theoretic counterpart). These singularities are associated with systems of physical particles. This fact follows in S-matrix theory from the principle of maximal analyticity.[5] And there is no indication in ordinary field theory of any other types of singularities.

The asymptotically dominating contributions to the correlations between the preparing and measuring devices are governed always by the discontinuities associated with these singularities.[6] These discontinuities are given by Cutkosky rules, which are generalizations of the pole-factorization theorem.[5] One obtains in this way[6] formulas that are natural generalizations of the transition amplitude formula of Sec. V. In particular, the specifications on the separate preparing and measuring devices are mapped into separate functions of the degrees of freedom of the appropriate initial and final particles of the observed system. These functions now occur multiplied by the S-matrix elements that connect these initial and final particles, or, alternatively, by a discontinuity of this S matrix in case the dominant singularity corresponds to a multiple-scattering process, rather than a simple one. In any case, the predicted correlation is obtained by folding the wave functions corresponding to the external specifications into a function that is determined by the S matrix. The theoretical content of the theory thus resides in the S matrix. The concrete details are given in Appendix A.

VII. CLASSICAL DESCRIPTIONS

Probabilities must be probabilities "of something." The probabilities occurring in quantum theory are probabilities of responses of the measuring devices. These responses are the "actual things" of which the quantum probabilities are probabilities.

The question is: How are these actual things to be described? The problem is that these actual things are associated with the measuring devices, which are physical systems. Yet we cannot describe them in terms of wave functions, because wave functions represent probabilities, and we want now to describe the actual things.

Bohr tells us to describe the macroscopic results in terms of classical concepts. Does this mean we are supposed to describe the position of the pointer, or of the grain of photographic emulsion, by simply stating the position of its center? Is this center its center of mass, or its center of charge, or its center with respect to some optical property that allows it to be visible? Or should the grain be considered to be a sphere of some finite size? Or should its shape also be considered? Or should the grain be described in terms of the various continuous dielectrical properties that determine its appearances?

The problem here is that any attempt to precisely describe the macroscopic results in terms of classical concepts brings one up against the fact that the grain, or pointer, cannot be precisely described in terms of the concepts of classical physics. These concepts do not conform precisely to the nature of the actual things.

However, precise descriptions of the results of the measurements never really occur in practical applications. Quantum theory always gives, in the final analysis, predictions about the probability that some variable will fall in a certain range. It never predicts that the situation at some future time will correspond exactly to some precise classical description. Nor does it say that the situation at some initial time was exactly described by some precise classical description. Instead, it says that if the initial situation falls within the range specified by A, then the probability that a measurement that falls in the range specified by B will yield a result that falls in the range specified by B is given by $\mathrm{Tr}\rho_A\rho_B$.

These specifications A and B are "classical" in the sense that they are operational: They instruct the experimenter how to decide whether the specified conditions are met. They do not correspond to any precisely defined classical system.

Since the descriptions A and B are operational in character, they effectively join the measuring devices and the scientists that use the theory into one complex system. This system is not clearly separated into distinct well-defined parts. Thus one is not called upon to give a description of the measuring device itself, considered as a distinct or isolated physical entity.

VIII. THEORY OF MEASUREMENTS

In order to apply quantum theory, a correspondence must be established between the probability functions and the actual things that the corresponding probabilities are probabilities of, namely, the responses of the devices. But how can a probability function built on the degrees of freedom of some observed system to be correlated to actual things that refer to some completely different observing system?

One line of approach would be to treat the measuring device as part of a new enlarged observed system, and thereby to study the connection between the "observables" of the original observed system and the "observables" of the new enlarged composite system consisting of the original observed system and original measuring device. The latter observables might be taken to be the dielectrical properties of the original measuring device, which would presumably be related to the appearances of that device.

This approach leads to a number of difficulties. In the first place the practical problems involved in treating the large number of degrees of freedom of real measuring devices are immense. No calculations have yet been made that come even close to giving quantitively accurate predictions of the responses of real measuring devices. Thus we are left, at the practical level, just where we started, with no correspondence between the probability functions that represent the original observed system and the actual things to which these probabilities are supposed to refer.

There are also problems of principle. In the first place, although nonrelativistic Hamiltonians that give reasonably accurate descriptions of a measuring device plus an observed system may exist, it is not at all certain that an exact Hamiltonian corresponding to the full relativistic problem exists. Even if there were an exact relativistic Hamiltonian which, with the aid of renormalization techniques, would permit the calculation of the S matrix, it is not at all clear that this Hamiltonian would also lead to a completely unambiguous determination of the exact time development of the operators of the theory. Furthermore, even if the exact time development of all the operators of the theory could be calculated, the question would arise as to the experimental significance of these operators. How, for example, does one really measure the electromagnetic field smeared over some small space-time region? (The Bohr-Rosenfeld analysis assumes, unrealistically, the existence of arbitrarily massive point test charges.) How does one really measure the localized dielectrical properties of the measuring device? What are the operators that really correspond to what is being measured when one examines the detailed time development of the measuring device?

In order to answer these questions one must, apparently, examine the process by which the measuring device itself is observed. During this new process of measurement, the wave function representing the composite system of the original measured system plus the original measuring device no longer represents the physical situation: The new measurement interaction involves going into a new enlarged space. One is led in this way into an infinite regress. The basic problem is that the quantum theory itself provides no way of computing the precise connection between the quantum-mechanical probabilities and the classically described actualities to which they are supposed to refer. Note that a classical description, as it is usually understood, involves observables that are precisely defined functions of time. The difficulties discussed above arise precisely from this notion of classical description.

From the thoroughly pragmatic point of view, the problem is to determine some of the transformations $A \rightarrow \rho_A$ and $B \rightarrow \rho_B$. These transformations are mappings of logical structures of one type onto logical structures of another type. The descriptions A and B are operational descriptions of preparations and measurements, which are couched in a technical jargon that is an extension of everyday language. The descriptions ρ_A and ρ_B are mathematical descriptions of theoretical models of isolated systems.

The dynamical laws of quantum theory cannot determine the mappings $A \rightarrow \rho_A$ and $B \rightarrow \rho_B$. These laws operate within the mathematical framework of the description of the observed system. They do not relate these descriptions to the logically different types of descriptions A and B.

By considering successively larger observed systems, the boundary between the observed and observing systems can be pushed further toward the human observer. But this merely shifts the question to higher levels, and leads ultimately to the question of the connection between the subjective experiences of human observers and certain density functions that are supposed to represent virtually the entire physical world. There appears to be no feasible way of establishing the needed connection at this level.

The level at which a practical determination of the connections $A \rightarrow \rho_A$ and $B \rightarrow \rho_B$ is possible is the opposite limit, in which the observed system is as small as possible, i.e., a single particle. At this lowest level the connections can be determined by experimental calibration.

The feasibility of experimental calibrations at this level arises from the fact that the transition probability depends on the complicated preparing and measuring devices only through the simple independent factors ρ_A and ρ_B.

Having calibrated the devices that prepare and measure the elementary particles, one can proceed to consider experimental situations in which several of these particles are prepared, and several are later measured. The observing system then becomes in effect a source and detector of free particles. And the observed system becomes an independent physical system with a set of incoming and outgoing free particles. The physical connection between these two dynamically independent physical systems is made by equating the connecting free-particle links. In this way the observed and observing systems become both dynamically independent, yet logically connected.

Preparations and measurements that do not correspond to single particle excitations can also be made. The situation is, however, basically the same as before. The specifications on the preparations and measurements are transcribed into wave functions of the prepared and measured systems by mappings that are determined by empirical calibration. The observed system is a representation of the asymptotically dominant contribution to the interaction between the parts of the observing system associated with the preparation and the measurement. Quantum-theoretical predictions acquire physical significance only to the extent that this contribution does indeed dominate. And in this case the predictions are determined by the S matrix, as shown in Appendix A.

IX. COLLAPSE OF WAVE FUNCTION

The problem of the collapse of the wave function[1,7–10] is brought into clear focus by von Neumann's lucid discussion[7] of the process of measurement. The essential idea of his discussion is as follows.

Suppose a system S is measured by a measuring device M. Suppose for simplicity that system S is either in region 1 or in region 2, and that the measurement determines which of these two regions it is in. For definiteness, we can suppose that S is a particle that has just passed through a screen with two well-separated slits, and that M consists of two counters, one placed behind each of the two slits. Suppose finally that the counters detect with 100% efficiency, so that one can determine through which slit the particle passed by determining which counter fired.

Let the wave function of S be written $\psi_S = \psi_S^1 + \psi_S^2$, where ψ_S^1 and ψ_S^2 represent the parts of ψ_S corresponding to the two different slits. And let ψ_M represent the wave function of the measuring device. The requirement that the measurement accurately determines the slit through which the particle passed means that the two partial wave functions $\psi_S^1 \otimes \psi_M$ and $\psi_S^2 \otimes \psi_M$ of the combined system before the measurement must develop into sums of wave functions of the forms $\bar{\psi}_S^1 \otimes \bar{\psi}_M^1$ and $\bar{\psi}_S^2 \otimes \bar{\psi}_M^2$, respectively, where $\bar{\psi}_M^1$ and $\bar{\psi}_M^2$ are wave functions corresponding to the two alternative counters' having fired. The superposition principle then ensures that $\psi_S \otimes \psi_M$ must develop into a superposition of wave functions for the two alternatives:

$$(\psi_S^1 + \psi_S^2) \otimes \psi_M \rightarrow \sum \bar{\psi}_S^1 \otimes \bar{\psi}_M^1 + \sum \bar{\psi}_S^2 \otimes \bar{\psi}_M^2 .$$

In particular, the superposition principle definitely pre-

[7] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton U. P., 1955), Ch. VI.

[8] M. Bunge, *Foundations of Physics* (Springer, New York, 1967), p. 274.

[9] W. Heisenberg, *Physics and Philosophy* (Harper and Row, New York, 1958), pp. 45, 46, 53, 54; also in *Niels Bohr and the Development of Physics*, edited by W. Pauli (MacMillan, New York, 1955), pp. 26, 47.

[10] J. M. Jauch, E. P. Wigner, and M. M. Yanase, Nuovo Cimento **48B**, 144 (1967); A. Shimony, Am. J. Phys. **31**, 775 (1963); L. Rosenfeld, Nucl. Phys. **A108**, 241 (1968); Progr. Theoret. Phys. (Kyoto) Suppl., 222 (1965); A. Loinger, Nucl. Phys. **A108**, 245 (1968); J. Earman and A. Shimony, Nuovo Cimento **54B**, 332 (1968); B. d'Espagnat, Orsay Report (unpublished). References to earlier works are cited in these papers.

cludes the possibility that the wave function after the interaction should have a part corresponding to one counter's having fired, but no part corresponding to the other counter's having fired. The wave function must, by virtue of the superposition principle, have terms corresponding to each of the two mutually exclusive macroscopic possibilities.

If one now includes the observer in the quantum-mechanical system, then one has the corresponding result

$$(\psi_S{}^1+\psi_S{}^2)\otimes\psi_M\otimes\psi_0 \rightarrow \sum \bar{\psi}_S{}^1\otimes\bar{\psi}_M{}^1\otimes\bar{\psi}_0{}^1 + \sum \bar{\psi}_S{}^2\otimes\bar{\psi}_M{}^2\otimes\bar{\psi}_0{}^2,$$

where the $\bar{\psi}_0{}^1$ correspond to the observer's having observed the first counter fire, and not the second, and the $\bar{\psi}_0{}^2$ correspond to the observer's having observed the second counter fire, but not the first. Thus the wave function necessarily has terms corresponding to each of these two (mutually contradictory) possibilities.

At the level of the elementary particles the wave function is regarded as the complete representation of the entity it represents. If this same viewpoint is extended to the measuring device, and thence to the observer, one seems to arrive at the conclusion that the two mutually contradictory possibilities both somehow exist in nature: Counter 1 fires, but not counter 2; and counter 2 fires, but not counter 1.

This conclusion is bravely accepted by some authors,[11] who point out that it would not involve any direct conflict with experience, since the mutually contradictory possibilities would be essentially noninterfering, i.e., the memories of individual observers could not contain cross references to the noninterfering branches of the wave function associated with the various "incompatible" possibilities, and hence no individual would be aware of more than one branch of the full objective reality.

The Copenhagen interpretation does not, I think, unequivocally reject this possibility. It effectively circumvents the question by representing the responses of the measuring devices by specifications that link the responses of the measuring devices to the experiences of the scientists that use the theory. The question of whether there are other branches of some super-reality thus becomes irrelevant. By using ordinary language one effectively identifies actuality with the aspects of things that are relevant to one's own personal experience. For the ordinary meaning of "actual" ties the actual world to what is confirmed, or at least is not ruled out, by experience.

The parts of the wave function that are not in accord with one's own experience are, in any case, discarded. This change in the wave function violates the Schrödinger equation, and the superposition principle. But there is nothing at all strange about it: Exactly analogous changes occur in classical statistical mechanics.

The wave functions of quantum theory represent the probabilities that certain measuring devices will respond in specified ways under specified circumstances. The specified circumstances correspond to situations that can be understood in terms of the notion that a certain system is prepared, and then propagates as an essentially isolated system until the interactions with the measuring devices take place. It is thus completely natural that a wave function should lose its physical significance if the specifications to which it corresponds are violated, as, for example, by the intrusion of a system that disturbs the effective isolation of the observed system during the interval between preparation and measurement. If this intrusion can be considered part of a new preparation, then there will be a new wave function corresponding to the new specifications. This change of the wave function, sometimes called the collapse of the wave function, is thus a completely natural consequence of a change in the set of specifications on the preparation of the observed system: The old specifications are no longer satisfied because the condition of isolation is violated; the new specifications correspond to a preparation of a new system. The new ρ_A is, in effect, a catalog of predictions about the results of possible measurements that might be made on this new system.

X. ONTOLOGICAL PROBLEMS

The attitude adopted above is completely pragmatic: Quantum theory is viewed as a theory that allows scientists to make predictions about what they observe. The underlying metaphysical position is that the fundamental unity of nature makes it necessary, in the construction of a useful theory about elementary-particle phenomena, to deal with representations of complementary idealizations of parts of the world, rather than with a representation of the unified physical world itself.

Yet it is reasonable to assume that the world existed before quantum scientists appeared on the scene, and that the laws formulated by quantum theorists represent aspects of certain primordial laws that do not depend in a direct way on the presence of the scientist-observer. This assumption leads one to seek an understanding of the nature of the physical world itself.

A reasonable starting point would seem to be that the macroscopic objects of everyday experience exist in roughly the way suggested by common sense. It also seems reasonable to accept that these objects might not exist as independent entities, but must, in the final analysis, be considered as parts of a greater whole. The idea of a table existing alone in the universe has a certain air of unrealness about it; a real table is constructed by certain workmen acting with certain tools on trees from a certain forest. It rests in a certain place,

[11] H. Everett, III, Rev. Mod. Phys. **29**, 454 (1957); L. N. Cooper and D. Van Vechten, Am. J. Phys. **37**, 1212 (1969).

e.g., in my study, and has certain other objects arranged about it, e.g., my chair and me. That is, the table is "that table," and "that table" is a part of the actually existing universe. Any conception of the table that isolates it from its past, its environment, or its future, is an idealization the limits of validity of which are not immediately known.

This conception of a nature as a unified whole is not only *a priori* reasonable, but it is supported by quantum theory, which seems to say that one cannot decompose the world into independently existing smallest physical parts, or primary qualities. At the atomic and elementary-particle level, the idea of independent entities dissolves; the most elementary things have meaning only in terms of their effects on other things.

Thus we begin with the not unreasonable assumption that there is a possibly unified actual world that corresponds at least roughly, on the macroscopic scale, to what we observe.

Bell's theorem tells us that this macroscopic world cannot develop according to contingent nonstatistical laws that are even approximately local.

One way to proceed is to follow the general lead of quantum theory and to accept that the actual macroscopic world is governed, in its temporal development, by statistical laws. One imagines that the actual situation determines some sort of potentialities or tendencies for the various possible future responses.

This general sort of picture is metaphysically attractive, for it appears to break the grip of absolute mathematical determinism, and to provide, therefore, some possibility for meaning in life, and for progress in the universe. However, it does not provide a way out of the problem posed by Bell's theorem. For the conversion of the potentialities into actualities cannot proceed on the basis of locally available information. If one accepts the usual ideas about how information propagates through space and time, then Bell's theorem shows that the macroscopic responses cannot be independent of faraway causes. This problem is neither resolved nor alleviated by saying that the response is determined by "pure chance." Bell's theorem proves precisely that determination of the macroscopic response must be "nonchance," at least to the extent of allowing some sort of dependence of this response upon the far-away cause.

I can see only three ways out of the metaphysical problem posed by Bell's theorem. The first is to accept, with Everett,[11] the idea that human observers are cognizant only of individual branches of the full reality of the world. This would mean that our common-sense idea of the physical reality is profoundly mistaken, even at the macroscopic level: The full physical world would contain a superposition of a myriad of interconnected physical worlds of the kind we know. An individual observer would be personally aware of only one response of a macroscopic measuring device, but a full account of reality would include all the other possible responses on an equal footing, though perhaps with unequal "weights."

The second way out is to accept that nature is basically highly nonlocal, in the sense that correlations exist that violently contradict—even at the macroscopic level—the usual ideas of the space-time propagation of information. The intuitive idea of the physical distinctness of physically well-separated macroscopic objects then becomes open to question. And the intuitive idea of space itself is placed in jeopardy. For space is intimately connected to the space-time relationships that are naturally expressed in terms of it. If there are, between far-apart macroscopic events, large instantaneous connections that do not respect spatial separation, then the significance of space would seem to arise only from the statistical relationships that do respect it.

The third way out is to deny that the measurements that "could have been performed, but were not," would have had definite results if they had been performed. This way out seems, at first, to be closest to the spirit of the Copenhagen interpretation. However, it seems to conflict with the idea of indeterminism, which is also an important element of the spirit of the Copenhagen interpretation.

To see this conflict, suppose we allow the settings of D_1 and D_2 in the Bell experiment to be determined by whether or not certain radioactive decays occur in certain time intervals. According to the usual idea of indeterminism the question of whether or not the decays will occur in the allotted time intervals is a matter of pure chance: Either possibility might occur. On the other hand, we are now supposed to accept that what would have happened in the cases that did not actually occur cannot even be defined. But how can one reconcile the claim that the other possibilities could have occurred with the claim that no numbers can represent the results that would ultimately have occurred in the other cases. The third way out seems to lead to a new type of determinism. Even the human scientist-observer is denied the freedom to choose which measurement he will perform, not because the present is determined by the past, but because the results that would ultimately come out in the other cases cannot even be conceived to be something definite. Even if the results were determined by pure chance, they would *ultimately* become something definite, and could be represented by some definite set of numbers. But to deny the validity of the first assumption of Bell's theorem, one must deny that the alternative possibilities could *ever* lead to definite results. [See the discussion above Eq. (1) in Sec. III.]

The significance that should be ascribed to such verbal arguments can of course be debated. Ordinary words are tied to our common-sense ideas about the world. Thus in situations where these common-sense ideas are in question, one must be careful about what the words are supposed to mean.

But the point of the argument is precisely to show that common-sense ideas about the world are definitely inadequate, and that, moreover, they fail already at the *macroscopic* level. The important thing about Bell's theorem is that it puts the dilemma posed by quantum phenomena clearly into the realm of macroscopic phenomena. It refers only to macroscopic events, and shows that our ordinary ideas about the world are somehow profoundly deficient even on the macroscopic level. For it apparently forces one to accept either that the real world is one in which *all* of the various possibilities permitted by quantum theory are realized, and that the testimony of individual experience is thus completely deceiving, right at the basic level, or that the world possesses some structural unity that completely transcends ordinary physical ideas, in that it either entails relationships that are totally alien to ordinary ideas about the space-time structure of causal connections, or demands a highly structured determinism that is alien both to the prevailing notion about the occurrence of chance elements in nature and also to our intuitive feeling that we are free to decide which way a piece of apparatus will be set up. According to this new determinism we are nonfree not simply because our decisions are mechanically determined by what has gone before, but by the fact that a future different from the one that will actually occur cannot even be conceived.

It is important for the understanding of quantum theory to recognize that very deep metaphysical questions do exist, and that they cannot be resolved in any way that accords with ordinary common-sense ideas about the *macroscopic* physical world. The interpretation of quantum theory described here circumvents these problems by the adoption of a thoroughly pragmatic attitude.

The consequences of trying to go beyond a purely pragmatic understanding of quantum phenomena are briefly explored in Appendix B.

XI. SPACE-TIME DESCRIPTION

In nonrelativistic quantum theory a pole in the S matrix is associated with a space-time eigenfunction. The dominance of the pole can thus be understood as a dominance of the intermediate quantum state represented by this eigenfunction.

In the nonrelativistic case the system is generally a collection of some well-defined set of subsystems. The wave function of the system is a function in the product of the spaces of the component systems, just as it should be for a function representing the probabilities associated with a composite system. This function has experimental significance to the extent that the system can be regarded as essentially a collection of the independent subsystems.

When one goes to the relativistic domain, the concept of the system as a collection of independent entities breaks down. Due to the phenomena of particle creation, and the consequent blurring of the distinction between forces and particles, the component parts of a composite system lose their identities when they get very close to each other. Thus the space-time coordinates in terms of which the composite system was described lose their meaning. Hence it is not clear, *a priori*, whether a useful microscopic space-time description can still be maintained.

Charap and Fubini[12] have shown that if the S matrix for a relativistic two-particle system has only the singularities clearly required by unitarity, and if certain approximations expressing the long-range character of the interaction and the unimportance of particle creation are valid, then the S matrix can be derived from an equivalent Schrödinger-type equation. That is, the on-mass-shell analyticity properties of the S matrix are equivalent in this approximation to the validity of a Schrödinger-type equation.

The approximations leading to this result ensure that the two particles retain, in effect, their identities in the course of the interaction: They remain "independent entities" in spite of the fact that they interact.

It is plausible that results analogous to those of Charap and Fubini should hold also for many-particle systems. One then understands in terms of mass-shell analyticity properties the relevance of Schrödinger-type wave functions in the realm of atomic physics. In this realm the interactions are long range and particle creation is unimportant.

The existence in the approximation of Charap and Fubini, and in the nonrelativistic approximation, of a unitarity transformation that generates the time development of compound systems has inspired the hope that a similar transformation exists in the full relativistic case for a general quantum system, conceived to represent the general interacting physical system. However, severe mathematical difficulties ensue. Thus it is worth emphasizing that the existence of such a transformation arises in the simple cases from approximations that are not valid in the general case, and that such a transformation has, moreover, no place in the basic logical structure of the theory. The physical predictions of the theory all reside in the S matrix. A transformation that generates a time development of the system is a useful adjunct to the theory only insofar as it aids in the calculation of the S matrix.

It might be maintained that observed macroscopic space-time causal structure of phenomena indicates, or at least suggests, the existence of a corresponding microscopic space-time causal structure. However, all known causality and locality properties of physical phenomena are guaranteed by a few simple analyticity properties of the (mass-shell) S matrix in and near the physical region.[6] The microcausality conditions give

[12] J. Charap and S. Fubini, Nuovo Cimento 14, 540 (1959). In view of its contents, this paper might more appropriately be entitled "The S-Matrix Definition of the Nuclear Potential."

analyticity far away from the physical region, and off the mass shell, yet do not give analyticity in the physical region. Thus they are not closely related to the causal structure of physical phenomena, and are certainly not required by it.

This fact, together with the fact that the generator of the time development of compound systems does not enter into the fundamental logical structure of quantum theory, casts doubt on the reasonableness of imposing, as a fundamental *a priori* requirement, a microscopic space-time condition that simply transfers to the microscopic domain certain ideas suggested by macroscopic phenomena. Since the S matrix is the fundamental element of the logical structure of quantum theory, one is led to the idea of trying to derive the S matrix—and hence also the detailed space-time structure of physical phenomena, insofar as it is predicted by quantum theory—directly from general properties of the S matrix itself. This is the S-matrix program. The general properties of the S matrix recognized today are its analyticity, unitarity, and symmetry properties.

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APPENDIX A: S-MATRIX THEORY AND SCHRÖDINGER THEORY

In the Schrödinger approach to quantum theory, the observed system is represented by a state vector that develops in time according to some equation of motion. The predicted relationships between measurements performed at different times are determined by this equation. In S-matrix theory, on the other hand, one has, at the fundamental level, only the asymptotic initial and final states, and the S matrix that connects them. There is no equation of motion that determines the detailed time development of the initial state into the final state. Thus it might seem, at first, that S-matrix theory would yield no predictions of relationships between measurements performed at finite times, and would be, therefore, merely an idealized aspect of the complete quantum theory.

For example, suppose that the prepared system were, according to Schrödinger's ideas, represented by the state vector $\psi_A(t)$, and the detected system were represented by the state vector $\psi_B(t)$. Then the transition probability would be represented by $|\langle\psi_B(t)|\psi_A(t)\rangle|^2$. And suppose that during the time interval between the preparation and measurement, the states $\psi_A(t)$ and $\psi_B(t)$ were not even approximately equal to their asymptotic forms. In such a case it might seem that S-matrix theory would yield no prediction of the transition probability.

According to S-matrix theory, the discontinuity associated with the process in question is given[5] by $S_B S_I^{-1} S_A$. Here S_A is the S matrix associated with the preparation, S_B is the S-matrix associated with the detection, and S_I is the S matrix for the observed system. The wave function $\psi_A^{\text{out}}(p)$ of the prepared system is constructed by folding S_A into the wave functions of the external particles of the preparation reaction. That is,

$$\psi_A{}^{\text{out}}(p)=\int S_A(p,p_1,\ldots;\ldots,p_n)\times\prod_{j=1}^{n}\left[\frac{d^4p_j}{(2\pi)^3}\delta^+(p_j{}^2-m_j{}^2)\phi_j{}^{\pm}(p_j)\right]. \quad \text{(A1)}$$

Here the $\pm$ sign is plus for initial particles and minus for final particles, $\phi_j{}^-(p_j)=[\phi_j(p_j)]^*$; and the variable p represents the set of mass-shell vectors of the asymptotic particles of the prepared system. Similarly,

$$\psi_B{}^{\text{in}*}(p')=\int S_B(p_1',\ldots;\ldots,p_{n'}',p')\times\prod_{j=1}^{n'}\left[\frac{d^4p^{j'}}{(2\pi)^3}\delta^+(p_j{}'^2-m_j{}'^2)\phi_j{}'^{\mp}(p_j')\right], \quad \text{(A2)}$$

where the primed quantities refer to the detection reaction.

The contribution to the transition amplitude associated with the observed system is governed by the corresponding discontinuity,[6] and is thus given by

$$\begin{aligned}\psi_B{}^{\text{in}*}S_I{}^{-1}\psi_A{}^{\text{out}}&=\psi_B{}^{\text{in}*}\psi_A{}^{\text{in}}\\ &\equiv\int\psi_B{}^{\text{in}*}(p')\psi_A{}^{\text{in}}(p')\times\prod_{i=1}^{l}\left[\frac{d^4p_i'}{(2\pi)^3}\delta^+(p_i'-m_i{}^2)\right]\\ &=\langle\psi_B(-\infty)|\psi_A|-\infty\rangle\\ &=\langle\psi_B(t)|\psi_A(t)\rangle,\end{aligned} \quad \text{(A3)}$$

where the last line comes from introducing

$$U^\dagger(t,-\infty)U(t,-\infty)=I.$$

Thus the answer given by S-matrix theory agrees with that given by Schrödinger theory, provided the latter exists.

The wave functions $\psi_A{}^{\text{out}}$ and $\psi_B{}^{\text{in}}$ depend on the wave functions of the external particles of the preparation and detection reactions, respectively, and through them on the locations of the preparation and detection. A translation by the amount Δ of the wave functions of the external particles of, say, the detection process, has the effect of multiplying $\psi_B^{\text{in}}(p')$ by $e^{iP'\Delta}$, where $P'=\sum p_i'$ is the sum of the momentum-energies of the measured system. (This is a consequence of the conservation of momentum energy by S_B.) Thus the effect is to displace the function ψ_B^{in} that represents the measurement by the amount Δ. In this way the transition probability depends upon the finite separation between the reactions that define the preparation and measurement of the observed system.

The important (though mathematically trivial) point here is that one can calculate the transition probabilities corresponding to preparations and measurements that take place at finite times in terms of asymptotic states even when these states are not relevant to the physical situation. In particular, the transition probability is determined in general by (1) the representation of the prepared system in terms of the asymptotic final state into which it will eventually evolve, (2) the representation of the measured or detected system in terms of the asymptotic initial state from which it evolves, and (3) the S matrix for the observed system. The dependence of the transition probability on the finite space-time separation between the preparing and measuring devices is determined by the known (and trivial) dependence of the asymptotic states on space and time, even though these states are not physically relevant. Thus one can deal exclusively with the asymptotic states, which are the only ones that are in principle well defined, since the prepared and measured systems are in principle fully distinguishable from the preparing and measuring devices, respectively, only in the asymptotic limit.

An important special case is that in which the observed system is a stable physical object, rather than a system that eventually decays into other systems. A stable physical object corresponds to a pole, or a collection of poles, in the S matrix of the larger process. In this case S_I^{-1} is unity, and one gets for each pole a contribution of the form given in Sec. V of the text. [Factors of $(2\omega)^{1/2}$ have been incorporated into the $\psi(x)$'s of the formula.]

The Schrödinger-theory analog of that formula is the statement that the time dependence of a physical object can be obtained by decomposing the state into energy-momentum eigenvectors, and letting each eigenstate develop according to its own eigenfrequency. The S-matrix formula permits also the superposition of eigenstates corresponding to different rest frames: The free-particle wave function represents simply the time development of a superposition of such eigenstates.

The above considerations show how the dependence of the transition probability on the times at which preparation and measurements are performed arises in S-matrix theory from the trivial space-time dependence of the noninteracting asymptotic states, rather than from an equation of motion of the observed interacting system during the interval between the preparation and measurement. However, if there were an exact equation of motion, then the results obtained using it would be the same as those given by S-matrix theory.

Since the S-matrix and Schrödinger formulations give equivalent results, in cases where the Schrödinger solution exists, the distinction between them in such cases is mainly a matter of viewpoint and terminology. The S-matrix formulation is, to be sure, logically more satisfactory, since it respects the fact that the wave function of the observed system becomes well defined only in the asymptotic region where it is free from the interactions associated with the process of measurement. However, this logical superiority is somewhat academic if the equivalent Schrödinger solution exists, since in that case the two formulations give the same results, and results are what count for most physicists.

The important question is this: Which formulation provides the appropriate basis for generalization to the relativistic domain? Here the logical considerations carry some weight: In view of the logical deficiency of the Schrödinger viewpoint, it would not be surprising that it should fail to extend into the relativistic domain.[13]

In regard to the process of measurement, the Schrödinger viewpoint is that the measurement determines certain characteristics of the observed system at the time of the measurement. Consequently, the equation of motion would, according to this view, be needed to convert the information provided by the measurement into information about the asymptotic wave function. However, the following points must be considered.

(1) The observed system cannot be represented by a wave function during the period of measurement. Therefore, the information provided by the measurement or preparation must be information about the wave function before or after the observation, respectively. This raises some question about the preciseness

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[13] In this connection it is interesting to review the words of Bohr (Ref. 4, pp. 77 and 90): "... Schrödinger's formulation of the interaction problem...involves a neglect of the finite velocity of propagation of the forces claimed by relativity theory. On the whole, it would scarcely seem justifiable, in the case of the interaction problem, to demand a visualization by means of ordinary space-time pictures. In fact, all our knowledge concerning the internal properties of atoms is derived from experiments on their radiation or collision reactions, such that the interpretation of experimental facts ultimately depends on the abstractions of radiation in free space, and free material particles. Hence our whole space-time view of physical phenomena, as well as the definition of energy and momentum, depends ultimately upon these abstractions. In judging the applications of these auxiliary ideas we should only demand inner consistency, in which special consideration has to be paid to the possibilities of definition and observation."

"Already the formulation of the relativity argument implies essentially the union of the space-time coordination and the demand of causality characterizing the classical theories. In the adaptation of the relativity requirement to the quantum postulate, we must therefore be prepared to meet with a renunciation as to visualization in the ordinary sense going still further than the formulation of the quantum laws considered here."

of the idea that the measurement determines characteristics of the observed system at the time of the measurement.

(2) The closely related Schrödinger idea that there are observables that refer to instants of time becomes clouded when one attempts to envisage the sequence of measurement that might approach as a limit a measurement of some observable at an instant of time. The phenomena of particle creation seem to introduce insuperable complications: It is not at all clear that even a complete knowledge of all the solutions of the equations of motion would allow one to devise such a sequence of measurements.

(3) If the measuring device is calibrated empirically, then the mappings $A \longrightarrow \rho_A$ and $B \longrightarrow \rho_B$ are most naturally taken to be mappings onto the "in" or "out" states. Of course, any unitarily equivalent representation would do just as well, in principle, since the devices are calibrated by comparing the experimental transition probabilities to the theoretical ones $\mathrm{Tr}\rho_A\rho_B$—for a variety of values of A and B—and the trace does not depend on the representation. The times of preparation and measurement enter, of course, through the specifications A and B. But there is no need to make the *representation* of ρ_A and ρ_B also depend on these times. The natural procedure is to use a fixed representation for these operators, and the "in" or "out" representations are the natural choices.

(4) If one attempts to determine the mappings $A \longrightarrow \rho_A$ and $B \longrightarrow \rho_B$ by theoretical methods, then the standard procedure is to consider an idealized situation in which the system of measuring devices plus observed systems is isolated from the rest of the world. For it is only in this idealization that this system can be represented by wave functions. The setup of the devices before and after the measuring operations can be examined by "looking at them," i.e., by using light-beam probes. Thus what the theoretical calculation must provide is the connection between the information about the measuring devices provided by the light-beam probes, and the values of the density matrices ρ_A and ρ_B, in say the "in" representation. But these connections are all determined by the S matrix.

APPENDIX B: WORLD VIEW

The physical world was once widely believed to consist of elementary particles and radiation. Quantum theory showed that elementary particles and radiation were mere probabilities. The combination of these two ideas yields a paradox: How can the physical world consist of mere probabilities?

Today the elementary particles are recognized as manifestations of poles in the S matrix. They are understood as long-range correlations between things. This understanding relieves the paradox, for there is nothing strange in correlations being statistical. But the question becomes: Correlations of what?

Quantum theory does not answer this question by introducing a new basic ingredient. Its answer is essentially pragmatic. Quantum theory is simply a theory that makes predictions about things that are described as results of measurements.

This pragmatic attitude leads to an understanding of quantum theory itself: Quantum theory is understood in the sense that one understands how to use it. But how does one incorporate this understanding into an understanding of the world as a whole?

The problem, basically, is with the "actual things."

In the framework of quantum theory, the actual things are preparations, measurements, and their results. They are described in practical terms. These descriptions allow technicians first to set up measurements that satisfy certain specifications, and then to judge whether the results meet the prescribed specifications. Thus the actual things are formulated in terms of a complicated interconnection between descriptions, technicians, measuring devices, and their actions upon each other.

In the framework of a mechanistic world view, one would like to separate the technicians from the measuring instruments, and to understand the results of measurements in terms of the characteristics of the measuring instruments alone. To accomplish this, one evidently needs some sort of conception or description of the measuring device itself. But how is the actual measuring device itself described?

This problem seems to have no satisfactory solution within the framework of a mechanistic world view. Indeed, we know already from Bell's theorem that a local mechanistic model of the actual world cannot be reconciled with the known facts. (We take the approximate validity of quantum theory as known.) One must, apparently, seek a solution outside the framework of mechanistic world views.

Mechanistic world views are based on three categories of things: (1) a field of space-time locations, (2) primary qualities attached to these locations, and (3) relationships (laws) that hold for the space-time configurations of these primary qualities. The adequacy of mechanistic world views has, of course, been doubted for a long time by many philosophers. A major difficulty with schemes built on a mechanistic framework is that they provide no adequate understanding of the connection of the mechanical quantities to experience. The "redness" of an experience seems qualitatively different both from the oscillations of the primary qualities to which it is related, and also from the corresponding discharges of neurons in the brain of the observer. The inclusion of experiences in one's account of reality seems to demand nonmechanistic categories.

If one goes outside the mechanistic framework, then one can view the measurement and its result, not in terms of concretely existing space-time structures, but rather in terms of webs of relations. The description of the measurement and its result is expressed in terms of words. These words are parts of an enveloping web of words, called language. This web derives its meaning from the webs of experience into which it is woven.

An experience is an integral part of some web of experience. Experiences cannot be analyzed into ultimate unanalyzable entities. The component parts invariably reach out to things outside themselves. To isolate an experience from its references is to destroy its essence. In short, experiences must be viewed as parts of webs, whose parts are not defined except through their connections to the whole.

One finds, therefore, in the realm of experience, essentially the same type of structure that one finds in the realm of elementary-particle physics, namely a web structure: Analysis never yields an ultimate set of unanalyzable basic entities or qualities. The smallest elements always reach out to other things and find their meaning and ground of being in these other things.

Since this same type of structure is suitable both in the realm of mind and in the realm of matter, one is led to adopt it as the basis of an over-all world view.

This "web" philosophy, which is that the world cannot be understood as a construction built from a set of unanalyzable basic entities or qualities (or at least from basic entities or qualities that can be precisely located in space and time) is closely connected to the "bootstrap" philosophy, which asserts that the structure of nature is determined by the requirement of consistency among the relationships of the web, rather than by laws that govern some primitive substance or quality. The web or bootstrap philosophy represents the final rejection of the mechanistic ideal. This ideal was rejected at the earlier stage of quantum theory at the level of actualities, but found refuge in a quasireal realm of probabilities or potentialities. The S-matrix viewpoint banishes it altogether.

The web viewpoint provides the natural framework for the pragmatic interpretation of quantum theory described above. To go beyond a pragmatic position, one must make a more definite commitment about the nature of actual things, and the nature of their relationship to probabilities.

In this connection the following observations seem pertinent.

(1) Physical entities, such as elementary particles, correspond to probabilities.

(2) The actual things in quantum theory are responses.

(3) A response is an event (or occurrence, or process) rather than an object.

(4) The only events known to exist are mental events.

(5) A mental event links prior mental events in a particular way, and creates corresponding new possibilities for subsequent mental events.

(6) Mental events are associated with living forms. However, as natural phenomena, they should be members of a general class that includes also similar events not associated with living forms.

(7) A collision of elementary particles is similar to a mental event: It links prior collisions in a particular way, and creates corresponding new possibilities for subsequent collisions.

These observations suggest that an actual thing has the character of an event (or occurrence, or process) that links prior events in a particular way, and creates corresponding new possibilities for subsequent events. This idea is the underlying theme of the world view proposed by Whitehead.[14]

The particular formulation of this idea developed by Whitehead has been examined by Shimony,[15] who finds it not fully compatible with quantum theory. The main difficulty arises from Whitehead's association of actual entities with localized space-time regions. The present work bears on that point.

The ontological analysis of Sec. X evidently forces one to choose between three unattractive possibilities: Either all possible worlds exist, or no alternatives exist, or our usual ideas about the propagation of causes through space-time are incorrect. The first two possibilities are unattractive because they contradict the direct testimony of experience, which assures us that we are free and able to affect the course of events. Any world view that denies us this freedom contradicts experience, and also reduces our lives to meaningless shams. It is doubtful that any theoretical construct could be secure enough to warrant acceptance at this price. The fate of classical physics should be sufficient warning.

On the other hand, the validity of simple ideas about the propagation of causes through space and time is hardly of comparable certainty. These ideas arise from theoretical analysis, and there is absolutely no evidence that they apply to actual causes, as opposed to their statistical effects. Moreover, simple ideas about space and time are already discredited by the theory of relativity.

These considerations suggest that the space-time continuum of physical theories is a construct based on statistical relationships between the actual things of nature, rather than simply a primitive container of these things. This possibility is in harmony with the web philosophy. It is also in harmony with the intuitive notion that ideas, which certainly qualify as real things, do not, strictly speaking, have spatial extension. The fundamental world process is, according to this suggestion, the growth of a system of relationships which

[14] Alfred North Whitehead, *Process and Reality* (MacMillan, New York, 1929). For a short account of Whitehead's ideas see J. M. Burgers, Rev. Mod. Phys. **35**, 145 (1963).

[15] Abner Shimony, *Quantum Physics and the Philosophy of Whitehead* (Humanities, New York, 1956).

do not themselves exist in space-time but which, taken as a whole, exhibit features that can be mapped onto a space-time continuum. The growth of the fundamental system of actual relationships would not map onto a smoothly developing space-time image.

This picture of the fundamental nature of things is to be contrasted with the picture inherited from classical physics, in which a persisting spatial structure changes in the course of time. This classical picture is, of course, not in harmony with the theory of relativity.

The speculative nature of the considerations that have arisen on this excursion outside the pragmatic domain emphasizes the need for a sharp distinction between pragmatic understandings of nature, and ones that claim to be more. The main aim of this paper is to show that quantum theory can be understood in a clear and straightforward way if a thoroughly pragmatic S-matrix viewpoint is adopted. Any attempt to go beyond a pragmatic understanding raises deep metaphysical questions that are still unresolved. However, the demands of quantum theory, relativity theory, and human freedom seem definitely incompatible with the idea that the actual things of nature reside in the space-time continuum of classical physics. It is probably the retention of this fundamentally incorrect metaphysical assumption that is the origin of the conceptual difficulties that arise in naive attempts to beyond a pragmatic understanding of quantum theory.

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Formal Restrictions on Schwinger Terms in Commutators Containing J_μ and/or $\Theta_{\mu\nu}$*

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The minimally coupled electromagnetic current J_μ and the symmetric energy-momentum tensor $\Theta_{\mu\nu}$ of the strong interactions are written as the sources of the electromagnetic and gravitational fields, respectively. In a *formal* sense the canonical commutators for these fields restrict the Schwinger terms in commutators containing J_μ and/or $\Theta_{\mu\nu}$. The model-independent results are ($x^0=0$): $S_T(n\geq 2)[J_\mu(x), J_\nu(0)]=0$, $S_T(n\geq 4)[\Theta_{\mu\nu}(x), \Theta_{\rho\lambda}(0)]=0$, where $S_T(n)$ denotes the nth-order Schwinger term (term with the nth derivative of a δ function). In addition, in low-spin models it can be shown that (for spin≤ 1) $S_T(n\geq 4)\times[J_\lambda(x), \Theta_{\mu\nu}(0)]=0$, and that (for spin $\leq\frac{1}{2}$) $S_T(n\geq 3)[J_\lambda(x), \Theta_{\mu\nu}(0)]=0$. These conditions apply when $\Theta_{\mu\nu}$ is defined in the usual way or in the manner prescribed by Callan, Coleman, and Jackiw. Stronger conditions can be derived for a more narrowly defined $\Theta_{\mu\nu}$ and for models with restricted forms of interactions. The formal significance of these results is discussed.

I. INTRODUCTION

THIS paper presents an investigation of maximal Schwinger terms in formally calculated commutators of J_μ (electromagnetic current) and/or $\Theta_{\mu\nu}$ (symmetric energy-momentum tensor). The methods are formal in the sense that they exploit the "naive" assumptions of canonical field theory: for instance, nonsingular products of local operators, and the Jacobi identity for local operators. It is known that these assumptions break down in some cases.[1] Therefore, our results are not rigorous. On the other hand, formal methods give us considerable control over the calculations; it is possible to derive a hierarchy of conditions on maximal Schwinger terms, beginning with weak but completely model-independent restrictions and ending with strong but model-dependent restrictions. Results of this kind can be used to uncover violations of our formal assumptions; this can be done by comparing the consequences of those assumptions with the consequences of a rigorous principle such as positivity of the energy. Formal restrictions on the commutators of $\Theta_{\mu\nu}$ may also have some heuristic value, especially in regard to scale-invariance arguments.[2]

Section II contains the statement of a "Lorentz-transformation theorem" which will be used in most calculations. The model-independent conditions on[3] $[J_\mu,J_\nu]$ and $[\Theta_{\mu\nu},\Theta_{\rho\lambda}]$ are discussed in Secs. III and IV, respectively. In Sec. V we develop the analogous restrictions on $[J_\lambda,\Theta_{\mu\nu}]$ in low-spin models. Section VI is a sketch of some stronger conditions which hold in models with more specific types of interactions. The formal significance of these results is discussed in Sec. VII.

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[1] For a review, see R. Jackiw, in Trieste Conference on Renormalization Theory, 1969 (unpublished).

[2] Reference 11 describes a symmetric energy-momentum tensor ($\Theta_{\mu\nu}^{[CCJ]}$) for renormalizable theories which can be used to write the scale-invariance current ($\mathfrak{D}_\mu$) in the form $\mathfrak{D}_\mu=x^\nu\Theta_{\mu\nu}^{[CCJ]}$.

[3] We use the notation in which Latin and Greek indices are summed from 1 to 3 and 0 to 3, respectively.