Phenomenological Predictions for Deep-Inelastic Electron Scattering*

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With preliminary information from colliding beams relative to the coupling of photons with "high-mass" hadron states, we have extended the 1960 Drell model for photons interacting with nucleons to estimate off-the-mass-shell photon cross sections. Based on kinematic considerations of the domain in which the propagator of the virtual intermediate particle is large, one can predict cross sections and configurations for the final produced events.

THE theory for inelastic scattering off nucleons can be formulated in terms of $\sigma_t(K,Q^2)$ and $\sigma_0(K,Q^2)$, the total cross sections for transverse and longitudinal or scalar off-mass-shell photons interacting with nucleons.[1] K is the energy of an on-mass-shell photon that produces the same c.m. energy of the final hadron system, and Q^2 is the negative of the four-momentum transfer squared.

The most natural class of theories to account for these cross sections has been the vector-dominance theories based on the assumption that the ρ^0, ω, and ϕ mesons saturate the photon coupling.[2] The rough prediction of vector-dominance models is

$$\sigma_t(K,Q^2)\approx\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)^2\sigma_t(K,0)\,, \tag{1}$$

whereas the experimental observation is[3]

$$Q_t(K,Q^2)\approx\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)\sigma_t(K,0)\,. \tag{2}$$

In view of the fact that vector dominance is approximately true on the mass shell, it would be surprising if this theory did not remain approximately valid out to Q^2 of the order of m_ρ^2. At high Q^2, however, the contribution to the cross sections from the ρ, ω, and ϕ mesons should have become small, and it is necessary to look elsewhere to account for the experimental results.

In the belief that the photon coupling was almost completely saturated by the vector mesons, most theories have postulated a new form of fundamental interactions involving partons or constituentlike substructures of the nucleon.[4-6] These theories are decoupled from other strong-interaction theories and involve a number of implicit as well as explicit assumptions. For example, it is implicitly assumed that the "effective mass" of a constituent does not change after a collision, and for those theories which involve fractionally charged, or spin-$\frac{1}{2}$, constituents it is assumed that these constituents are prevented by some mechanism from appearing in the lab. However, with the successful operation of the Adone e^+e^- colliding-beam storage ring, we now know that the photon does indeed couple strongly to "high-mass" hadron states other than the ρ, ω, and ϕ mesons.[7] The preliminary observations suggest that multipion final states are produced with cross sections of the order of those expected for "point" Dirac particles.

The purpose of this paper is to explore whether by using this fact we can explain inelastic electron scattering in conventional terms. We shall write the total cross section in the form

$$\sigma_t(K,Q^2)\approx\sigma_t(K,0)\left[0.75\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)^2+0.25f(K,Q^2)\right]. \tag{3}$$

The first term on the right-hand side of Eq. (3) is assumed to be the vector-dominance contribution, and the second term is that due to the higher-mass states coupled to the photon. The factors 0.75 and 0.25 are estimates, based on the present tests, of the extent to which vector dominance holds. The function $f(K,Q^2)$ is evaluated below in the spirit of the 1960 Drell model[8] for evaluating total photon-nucleon cross sections.

Figure 1 shows the Drell model, diagrammatically. In this model, on-mass-shell photons couple to a pion pair, of which one member is real, and the other virtual but close to the mass shell. This virtual pion then interacts with the nucleon. This theory assumed that the pion pair interacted with the photon via a pointlike cross section (vertex coupling of e), and predicted

$$\sigma_{\rm tot}(\gamma p,\,Q^2=0)\sim e^2\sigma_{\rm tot}(\pi p)\,. \tag{4}$$

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[1] L. N. Hand, Phys. Rev. 129, 1834 (1963).
[2] J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969).
[3] For a summary of the present experimental data, see R. E. Taylor, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 51.
[4] For a general discussion of the literature, see F. J. Gilman, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 177.
[5] J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
[6] S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1969).
[7] Report of V. Silvestrini at the Conference on Phenomenological Interactions, Naples, Italy, 1970 (unpublished).
[8] S. D. Drell, Phys. Rev. Letters 5, 278 (1960).

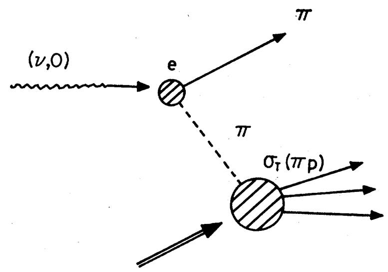

FIG. 1. Diagrammatic representation of the interaction of a photon (on mass shell) with a nucleon to give "anything," as in Ref. 8. The cross-section contributions are assumed to come predominantly from $t<m_\pi^2$, where t is the exchanged four-momentum squared.

Figure 2 shows the analogous process for an off-mass-shell photon. The photon is assumed to interact with a group of bosons with an effective coupling constant of e. One of the bosons is virtual and proceeds to the second vertex, where it interacts with the nucleon. The photon has energy and four-momentum squared (ν,Q^2), the group of produced bosons has energy $(1-r)\nu$, and the intermediate particle has energy and four-momentum squared $(r\nu,t)$ and mass μ. As has been pointed out by West,[9] the domain for which this diagram gives an appreciable contribution should be determined by the kinematic region for which the propagator of the virtual particle is large. At large values of Q^2 and ν, the minimum value of the four-momentum squared, $t_{\min}$, is given by[9]

$$|t_{\min}|\approx[x/(1-x)][(M_n^2-M^2)+xM^2]\,, \qquad (5)$$

where the parameter x is the conventional $x=Q^2/2M\nu$, M_n is the residual mass of the products at the nucleon vertex, and M is the nucleon mass.

In terms of the energy partition factor r, we have

$$M_n^2=2r\nu M+M^2+t_{\min} \qquad (6)$$

and, by substitution into Eq. (5),

$$t_{\min}=-x(2r\nu M+xM^2)\,. \qquad (7)$$

If $|t_{\min}|<\mu^2$, the intermediate particle is close to the mass shell. If $|t_{\min}|>\mu^2$, the propagator of the intermediate particle will be small, and the contribution to the cross section should be small. We first consider values of $x<\mu/M$. The range of r values which lead to $|t_{\min}|\leq\mu^2$ and which thus contribute appreciably to the cross section is then given by

$$r\leq r_{\max}=\left(\frac{\mu^2}{2x\nu M}-\frac{xM}{2\nu}\right), \qquad (8)$$

with

$$r_{\max}\approx\mu^2/2x\nu M=\mu^2/Q^2\,.$$

To estimate the cross section σ_t' due to states other than the ρ, ω, and ϕ, we assume a one-dimensional phase-space distribution for r of the form $(\langle n\rangle-1)(1-r)^{\langle n\rangle-2}dr$, where $\langle n\rangle$ is the average multiplicity. Then, if the contribution to the cross section comes predominantly from $|t_{\min}|\leq\mu^2$, we have[10]

$$\frac{\sigma_t'(K,Q^2)}{\sigma_t'(K,0)}\approx\int_0^{r_{\max}}(1-r)^{\langle n\rangle-2}dr\bigg/\int_0^1(1-r)^{\langle n\rangle-2}dr$$
$$\approx(\langle n\rangle-1)r_{\max}\,, \qquad (9)$$

and substituting from Eq. (8) into Eq. (9),

$$\sigma_t'(K,Q^2)\approx\sigma_t'(K,0)(\langle n\rangle-1)\mu^2/Q^2\,. \qquad (10)$$

If $\langle n\rangle\sim5$ and $\mu^2\sim m_\rho^2$, substitution of the result of Eq. (10) into Eq. (3) gives, for large Q^2,

$$\sigma_t(K,Q^2)\approx\sigma_t(K,0)m_\rho^2/Q^2\,, \qquad (11)$$

which is indeed what is observed experimentally.[3] The essential point to note is that the restriction that the dominant contribution to the cross section comes from four-momentum transfers squared less than μ^2 has modified the on-mass-shell cross section by the factor m_ρ^2/Q^2.

Because the r values contributing to the cross section are small, the average event will consist of a collimated group of high-energy bosons carrying most of the energy and a low-energy boson-nucleon collision with energy E_{eff} of the order of $r_{\max}\nu/2$, or

$$E_{\text{eff}}\approx\frac{\mu^2\nu}{2Q^2}=\frac{M}{x}\left(\frac{\mu^2}{4M^2}\right). \qquad (12)$$

Using a value of μ^2 of the order of 0.5 GeV^2, one obtains $E_{\text{eff}}<2$ GeV for values of $x>0.06$. Therefore, for $x>0.06$ there is no reason to expect identical cross sections for neutrons and protons. For $x<0.06$, neutron and proton cross sections should be approximately the same, and the over-all behavior should be close to that predicted by "diffraction" models.[11]

For the kinematic region in which $x>\mu/M$, $|t_{\min}|$ is always greater than μ^2, but is minimized when $M_n=M$, i.e.,

$$t_{\min}=-[x^2/(1-x)]M^2\,. \qquad (13)$$

As x increases beyond μ/M, the cross section will be sharply cut into by the propagator of the intermediate virtual particle. This is indeed observed experimentally, σ_t dropping drastically for $x>0.3$. If it is assumed that the major contribution to the cross section comes from the neighborhood where $|t|$ is minimized, the typical event configuration will consist of a collimated jet of bosons and a recoiling nucleon carrying kinetic energy

[9] G. West, Phys. Rev. Letters **24**, 1207 (1970).

[10] Implicit in these estimates is the assumption that there are no Regge "damping" factors $s^{\alpha(t)}$ for the intermediate particles. Considerations advanced by R. Feynman [Phys. Rev. Letters **23**, 1415 (1969)] suggest that such factors should appear for any specific final channel ("inclusive state") but that summed over all final states ("exclusive state") they should not appear.

[11] Cf., e.g., H. Harari, Phys. Rev. Letters **20**, 1395 (1968).

along the direction of the momentum of the virtual photon of the order of $x^2M/2(1-x)$. There is no reason to expect the neutron and proton cross sections to be the same.

By analogy with the theory for vector-meson photoproduction at low energies, we would expect the form for $\sigma_t'(K,Q^2)$ where the minimum momentum transfer is large to be[12]

$$\sigma_t'(K,Q^2)\sim\sigma_t'(K,0)e^{Bt_{\min}}. \tag{14}$$

In order to fit the experimentally observed cross-section dropoff for inelastic electron scattering with increasing x, the quantity B should be ~ 3 $(\text{GeV}/c)^{-2}$. An interpolation formula which connects Eqs. (3), (11), and (14) so that they are correct in their respective regions of validity is

$$\sigma_t(K,Q^2)\approx\sigma_t(K,0)\times\left[0.75\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)^2+\left(\frac{m_\rho^2}{4m_\rho^2+Q^2}\right)e^{3t_{\min}}\right], \tag{15}$$

where $t_{\min}=-[x^2/(1-x)]M^2$. The first term corresponds to the contribution from vector dominance through the ρ, ω, and ϕ mesons, the second term corresponds to the contribution from the higher-mass states with an assumed final-state multiplicity of 5, and the exponent multiplying the second term allows for the large minimum-momentum-transfer region. We believe that this equation should closely represent the actual physical situation, and that event configurations should be as predicted by us above.[13]

[12] Cf., e.g., S. D. Drell and J. Trefil, Phys. Rev. Letters 16, 552 (1966).

[13] Our predicted event configurations are very different from those given by the interesting field-theoretic model of Ref. 6, which expects most of the four-momentum transfer to be given to the recoiling nucleon. We note that, in contradistinction to our model, this model would predict small cross sections for inelastic electron scattering from a pion target ($e+\pi\rightarrow e+\pi+$anything), and that by crossing, it would then predict much smaller cross sections for e^+e^- annihilation into hadrons than are observed experimentally (see Ref. 7).

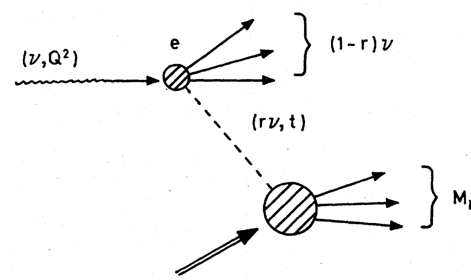

FIG. 2. Diagrammatic representation of an off-mass-shell photon interacting with a nucleon to give "anything." The particles at the top vertex take off energy $(1-r)\nu$; the c.m. energy of the products of the lower vertex is M_n. The effective coupling of the top vertex is assumed to be of the order of e. The cross-section contributions are assumed to come predominantly from $|t|<\mu^2$, where μ is the effective mass of the exchanged particle. It follows that values of r less than μ^2/Q^2 give the dominant contribution to the cross section.

It is clear, if the foregoing is correct, that experiments in different kinematic regions of x and Q^2 will check very different physical situations, and no single experiment will unequivocally select a "correct" theory.

One further result follows from these considerations. On the basis of this model, inelastic electron scattering should connect to e^+e^- annihilation into protons or antiprotons plus anything by crossing, in the range where $|t_{\min}|>4M^2$ or for $x>0.8$. Accordingly, the cross sections should be connected in the region for which νW_2 as determined from inelastic electron scattering[3] is less than 0.01; we would therefore expect asymptotic cross sections for e^+e^- colliding-beam machines to produce nucleons of the order of 10^{-2} of the point Dirac cross section and therefore of the order of 10^{-2} of the actually experimentally observed e^+e^- colliding-beam cross sections for multipion production.

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