physically reasonable (in view of high-energy scattering data) range $-2.9 \leq \alpha_r(t) \leq 0.7$, it is not possible to maintain $\alpha_i^2(t) \geq 0$ between $t=0$ and $t=-1.8$. So, we are led to conclude that additional Regge structure, very likely the fixed poles of Ref. 5, is still necessary in the $B^{(-)}$ amplitude. Then, without further specific assumptions, the FESR do not provide sufficient information to compute $\alpha(t)$.

Note added in proof. From Eq. (3) we obtain at once

$$S_n(t)=N^{\alpha_r}\left|\frac{\beta\alpha}{\alpha+n}\right| \cos(\theta-\psi_n).$$

If we attempt to satisfy $S_1=S_3=S_5=0$ at $t\approx -0.4$ $(\mathrm{BeV})^2$ by means of a phase condition, we find

$$\theta-\psi_n=(2m+1)\pi/2,$$

and hence

$$\psi_n-\psi_{n+2}=k\pi,$$

for some integers m and k. Thus,

$$\frac{\alpha_i}{\alpha_r+n}=\tan\psi_n=\tan(\psi_{n+2}+k\pi)=\tan\psi_{n+2}=\frac{\alpha_i}{\alpha_r+n+2},$$

showing that $\alpha_i(-0.4)=0$. This disagrees with the experimental fits to πp charge exchange,[6] which yield $\alpha_i(-0.4)\approx 0.1$. Therefore, we must assume that S_1, S_3, and S_5 vanish near $t=0.4(\mathrm{BeV})^2$, but at slightly different points. The experimental errors[5] clearly permit this. Remembering that the phases depend on t and that $\psi_n \ll 1$ for $n\geq 1$, we see at once that this situation will occur if $\theta(-0.4)\approx \pi/2$.

Since the assumption $\alpha(-0.4)=0$ was not actually used in this paper, all the equations and conclusions remain unaltered.

I should like to thank Dr. L. Girardello for drawing my attention to Ref. 3.

[6] B. P. Desai, P. Kaus, R. T. Park, and F. Zachariasen, Phys. Rev. Letters **25**, 1389 (1970); **25**, 1686(E) (1970).

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Duality in Backward π^+p Elastic Scattering*

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A duality-inspired model for the scattering amplitude is used to explain backward π^+p elastic-scattering data from 2.18 GeV/c (incident pion laboratory momentum) on up. The quantitative success of this model gives support to the assumption that the dip at $u\sim -0.15$ GeV^2 is due to a wrong-signature nonsense zero in the N_α Regge-pole exchange amplitude. A particularly interesting polarization prediction (which can soon be compared with experimental data) is also presented.

In an earlier paper,[1] we proposed a model for the scattering amplitude that is consistent with duality[2] and particularly applicable to backward πN elastic scattering at intermediate energies.[3] Following directly from the Veneziano model,[4] we identify[1] the sum of direct-channel resonances with the signatured part of the Regge amplitude, leaving the purely real nonsignatured part as an interfering background. With such a prescription for expressing the scattering amplitude, Regge parameters determined at high energies are used in conjunction with resonance parameters determined at low energies to account for its behavior in between. Therefore, there should be relatively few additional parameters[5] needed for our description of the scattering amplitude at intermediate energies.

A wealth of backward π^+p elastic-scattering differential-cross-section data is now available at both high[6] and intermediate[7] energies, the most prominent feature of which is the occurrence of a dip at all energies with u roughly somewhere between -0.1 and -0.2 GeV^2. We interpret this as the result of a wrong-signature nonsense zero in that part of the Regge-pole amplitude due to N_α exchange,[8] and fit the experimental data at high energies[6] with just the N_α and Δ_δ Regge poles.[9] The

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[1] P. W. Coulter, E. S. Ma, and G. L. Shaw, Phys. Rev. Letters **23**, 106 (1969).
[2] In the sense of finite-energy sum rules; see R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).
[3] We have used this model to obtain a good fit to the 180° π^-p elastic-scattering differential cross-section data of Kormanyos *et al.*, Phys. Rev. **164**, 1661 (1967); also, see Ref. 1.
[4] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).
[5] Presumably, yet undiscovered resonances—with small, but perhaps significant, contributions to the scattering amplitude—are present at these energies.
[6] D. P. Owen *et al.*, Phys. Rev. **181**, 1794 (1969).
[7] J. P. Chandler *et al.*, Phys. Rev. Letters **23**, 186 (1969).
[8] C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).
[9] There is undoubtedly some small but not so easily defined contribution from the N_γ also.

TABLE I. Parameters for the 13 resonances used in our fit.

Resonance (mass in MeV)	Spin-parity (J^P)	Width (MeV)	Elasticity (%)
$\Delta_\delta(1236)$	$(\frac{3}{2})^+$	120	99.4
$\Delta_\beta(1650)$	$(\frac{1}{2})^-$	150	27
$\Delta_\gamma(1670)$	$(\frac{3}{2})^-$	240	13
$\Delta_\delta(1690)$	$(\frac{3}{2})^+$	265	9
$\Delta_\alpha(1890)$	$(\frac{5}{2})^+$	275	17
$\Delta_\alpha(1910)$	$(\frac{1}{2})^+$	325	25
$\Delta_\delta(1950)$	$(\frac{7}{2})^+$	200	44
$\Delta_\beta(1960)$	$(\frac{5}{2})^-$	355	14
$\Delta_\delta(2160)$	$(\frac{3}{2})^+$	260	25
$\Delta_\beta(2215)$	$(\frac{5}{2})^-$	265	20
$\Delta_\delta(2445)$	$(\frac{11}{2})^+$	350	10
$\Delta_\delta(2880)$	$(\frac{15}{2})^+$	245	2.5
$\Delta_\delta(3260)$	$(19/2)^+$	265	2.5

values of the parameters so obtained are then used as fixed inputs to our calculation at intermediate energies.

We take a MacDowell-symmetric Regge-pole amplitude[10] and require that the two (N_α,Δ_δ) trajectories extrapolate approximately to their respective mass spectra. Moreover, the residue functions γ are taken to be of the form $\beta(1+\delta\sqrt{u})$,[11] and together with the scale factors (S_N,S_Δ), make up our parametrization of the scattering amplitude at high energies. A good fit to the backward $\pi^\pm p$ elastic-scattering data[6] is then obtained with the following values of our parameters[12]:

$$\alpha_N=-0.35-0.09\sqrt{u}+1.07u\,,$$
$$\gamma_N=-11.7(1+1.25\sqrt{u})\ \text{GeV}^{-1},$$
$$S_N=1.0\ \text{GeV}^2;$$

and

$$\alpha_\Delta=0.14+0.89u\,,$$
$$\gamma_\Delta=0.186(1+1.6\sqrt{u})\ \text{GeV}^{-1},$$
$$S_\Delta=1.7\ \text{GeV}^2.$$

For our model fit to the experimental data at intermediate energies,[7] we extrapolate the purely real nonsignatured part of our Regge solution down to these energies and add to it a number of resonances, each of which is represented by a constant-width, constant-elasticity Breit-Wigner amplitude in the c.m. energy and is made MacDowell symmetric. As fixed inputs, we include in our amplitude all known $I=\frac{3}{2}$ resonances[13] of masses <2200 MeV with their appropriate parameters as given by those averages on p. 166 of Ref. 13. The three more massive resonances listed[13] are assumed to be members of the Δ_δ family for their spin and parity assignments. Their masses, widths, and elasticities are adjusted somewhat to fit the data. Finally, a new D_{35} resonance[14] at 2215 MeV is introduced to complete our analysis.

The resonance parameters used in our fit to the backward π^+p elastic-scattering differential-cross-section data[7] are listed in Table I. The first nine sets of values are given by Ref. 13 and the other four by our fit. One must keep in mind that, in principle, there must be

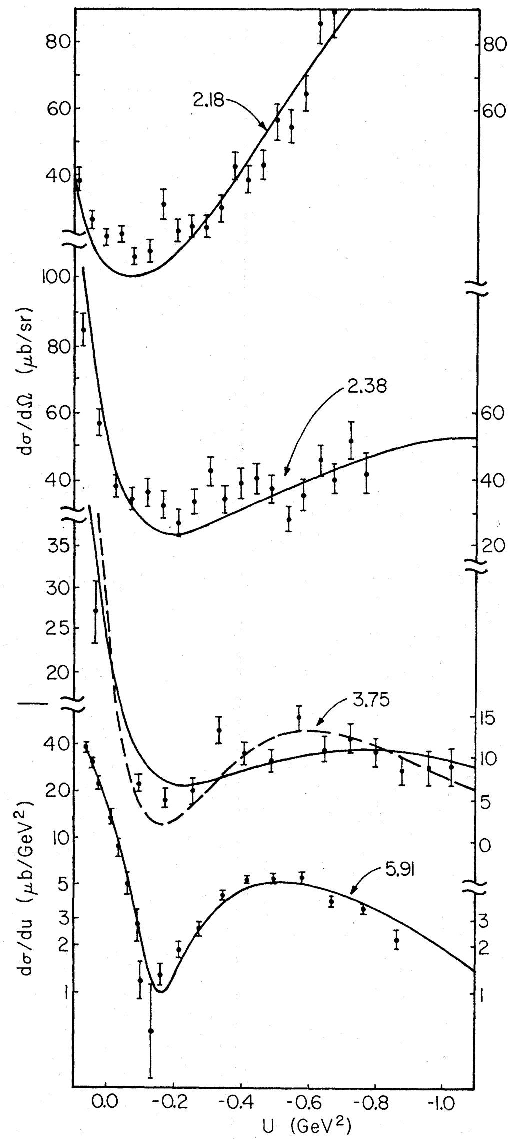

FIG. 1. Theoretical curves, as described in the text, for the angular distributions in backward π^+p elastic scattering. At $P_{\text{lab}}=5.91$ GeV/c, the logarithm of $d\sigma/du$ in μb/GeV^2 is plotted as a function of u. At all other energies, $d\sigma/d\Omega$ in μb/sr is plotted. Data points are taken from Refs. 6 and 7.

[10] V. Barger and D. Cline, Phys. Rev. **155**, 1792 (1967).

[11] Chosen as a matter of simplicity, in spite of its apparent inconsistency with asymptotics at fixed t; see E. L. Berger and G. C. Fox, Phys. Rev. **188**, 2120 (1969).

[12] The Δ_δ values are exclusively those obtained by fitting the π^-p data alone. They are then used as fixed inputs to the π^+p solution.

[13] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

[14] As the second member of a Δ_β family, of which the $S_{31}(1650)$ is the first, this seems to be a rather plausible choice. In any case, the addition of our $D_{35}(2215)$ is the only one needed for an excellent description of the data below $P_{\text{lab}}=3.0$ GeV/c.

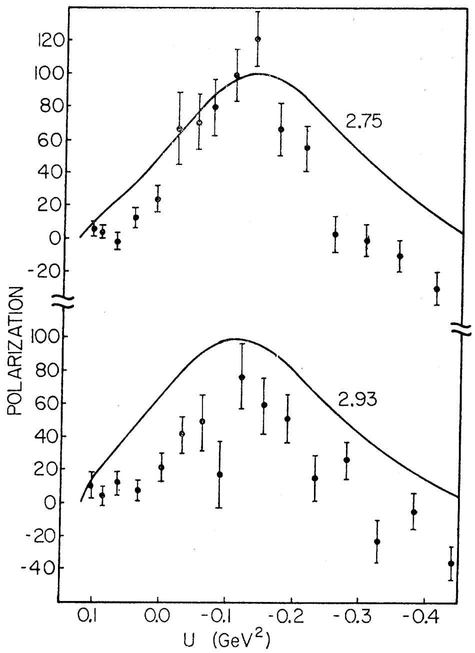

FIG. 2. Backward π^+p polarization prediction at $P_{\mathrm{lab}}=2.75$ and 2.93 GeV/c. Data points are taken from Ref. 18.

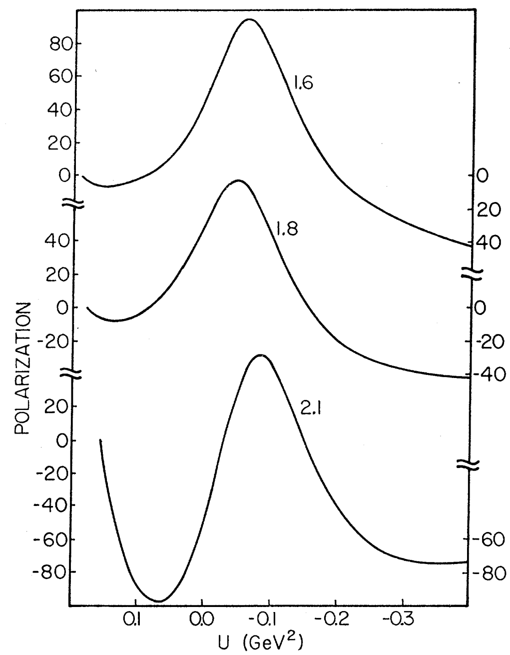

FIG. 3. Backward π^+p polarization prediction at $P_{\mathrm{lab}}=1.6$, 1.8, and 2.1 GeV/c.

many more resonances[15] at these energies; therefore, our determination of these parameters is only an effective one. With these numbers, however, a reasonably good over-all fit is obtained—with especially good quantitative agreement at the lower energies.

In summary, our simple Regge amplitude describes the data quite accurately at high energies ($P_{\mathrm{lab}}\geq 5.91$ GeV/c), whereas for $P_{\mathrm{lab}}\leq 3.0$ GeV/c, it is our duality model which gives a quantitative fit. In the region $3.25\leq P_{\mathrm{lab}}\leq 5.0$ GeV/c, the extrapolated Regge solution agrees moderately well with the data, especially at the higher energies, while the duality model, however limited by our ignorance[5] of all the necessary resonances, tends to give a roughly equivalent fit as well.

Representative curves of our fit at both high and intermediate energies are shown in Fig. 1. At $P_{\mathrm{lab}}=5.91$ GeV/c, the curve is that of our pure Regge fit. At $P_{\mathrm{lab}}=3.75$ GeV/c, the dashed curve is the extrapolation of our high-energy Regge solution and the solid one comes from our model fit. They should be, and are, roughly equivalent. At $P_{\mathrm{lab}}=2.18$ and 2.38 GeV/c, we show only curves of our model calculation.[16]

It is seen, therefore, that we have an adequate phenomenological description of the present data in terms of this model, and the fact that predetermined information from both high and low energies is used in our calculation should lend support to our particular interpretation of the concept of duality. It must be strongly emphasized that, in this model, the occurrence of the dip, which is, in principle, attributed to a wrong-signature nonsense zero in the N_α Regge-pole exchange amplitude, is, in fact, also accomplished by a nontrivial cancellation between the sum of resonances and the non-signatured part of the Regge amplitude which does not have a zero. In a strong-cut model,[17] it is not clear how one would incorporate this kind of duality at intermediate energies.

Finally, we have not constrained our solution to fit the recent polarization measurements.[18] However, as Fig. 2 shows, the prediction of our model is in good agreement with the data at $P_{\mathrm{lab}}=2.75$ and 2.93 GeV/c. We consider this as added support for our approach.

In addition, a very interesting result is obtained at $P_{\mathrm{lab}}=2.1$ GeV/c (see Fig. 3), where the prediction is for a largely negative polarization which is a drastic reversal from those at only slightly higher energies. Surprising as it is, early indications[18] are that the experimental results (soon to be available) do tend to agree with it.

Details of the calculation and complete results are being prepared for future publication. We would also like to thank the Aspen Center for Physics, where part of this work was done, for its kind hospitality.

[15] As is necessary in practice for a pure resonance model; see R. R. Crittenden, R. M. Heinz, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D 1, 169 (1970).

[16] The Regge extrapolations at these energies are far from being in agreement with experimental data.

[17] F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, Phys. Rev. **182**, 1579 (1969); R. L. Kelly, G. L. Kane, and F. Henyey, Phys. Rev. Letters **24**, 1511 (1970).

[18] R. J. Esterling *et al.*, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 348; A. Yokosawa (private communication); see also R. Miller and A. Yokosawa, Argonne National Laboratory Report No. ANL/HEP 7001, 1970 (unpublished).