Finite-Energy Sum Rules with Complex Conjugate Regge Poles in πN Charge Exchange*

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A pair of complex conjugate Regge poles is used to parametrize the contribution of the ρ trajectory and of the cut arising from its coupling with the Pomeranchuk trajectory to the $B^{(-)}$ amplitude in πN charge exchange. It is argued that the previous result of Aviv and Horn, that finite-energy sum rules require additional Regge structure in this amplitude, still holds.

IT has been suggested,[1] and verified in a model,[2] that all Regge trajectories $\alpha(t)$ acquire an imaginary part for $t<0$ owing to their collision at $t=0$ with the cut they generate by interference with the Pomeranchuk trajectory. In the model, it was shown that such a collision produces a pair of complex conjugate Regge poles for $t<0$, depending on the dynamics involved, on either the physical or unphysical angular momentum sheet.[2] More recently, it was proposed that in the region $t<0$, until very high energies are reached, the pole-cut combination is well represented by this pair of poles.[3] We shall attempt to determine whether the three lowest-moment right-signature finite-energy sum rules (FESR)[4] for the $B^{(-)}$ amplitude in πN scattering can be saturated by such a pair of poles, and, if so, what restrictions are imposed on the poles and their residues.

The Regge expansion of $B^{(-)}$ is[5]

$$B^{(-)}(\nu)\to\sum_i \frac{\beta_i(t)\alpha_i\nu^{\alpha_i-1}}{\Gamma(1+\alpha_i)\sin\pi\alpha_i}(1-e^{-i\pi\alpha_i}). \tag{1}$$

Then the FESR read[5]

$$S_n(t)\equiv\frac{1}{N^n}\int_0^N d\nu\,\nu^n\,\mathrm{Im}B^{(-)}(\nu)=\sum_i\frac{\beta_i(t)N^{\alpha_i}}{(\alpha_i+n)\Gamma(\alpha_i)}. \tag{2}$$

In Ref. 5, Aviv and Horn calculate and plot S_1, S_3, and S_5 for $-1.8\ \mathrm{BeV}^2\leq t\leq 0$. They conclude on the basis of Eq. (2) that, in addition to the leading ρ trajectory, because $|S_1|>|3S_3|$, $|5S_5|$, there is very likely a fixed pole at $\alpha=-1$, and the nonvanishing of S_3 and S_5 at $\alpha_\rho(t)=-1$ probably implies the presence of fixed poles at $\alpha=-3$ and $\alpha=-5$. Let us now replace the ρ trajectory by a pair of complex conjugate Regge poles $\alpha(t)$ and $\alpha^*(t)$. Equation (2) will no longer necessarily require the fixed poles at -3 and -5 since the imaginary part of $\alpha(t)$ need not vanish when the real part is equal to -1. We may also hope to be able to arrange the proper relative magnitudes for the S_n.

From Ref. 3 it is easy to see that if the Regge partial-wave amplitude $T(t,l)$ is a real analytic function of l for $t\leq 0$, then the residues corresponding to the complex conjugate poles must themselves be complex conjugates. Hence, absorbing a factor $2/\Gamma(\alpha+1)$ into the residue, we have the FESR

$$S_n(t)=\mathrm{Re}\left[\frac{\beta(t)\alpha(t)N^{\alpha(t)}}{\alpha(t)+n}\right]. \tag{3}$$

In order to agree with[5] $S_1\approx S_3\approx S_5=0$ at $t\approx -0.4\ \mathrm{BeV}^2$, it is necessary that $\alpha(-0.4)=0$. With the definitions

$$\alpha(t)=\alpha_r(t)+i\alpha_i(t),\quad \beta(t)=|\beta(t)|e^{i\varphi(t)},$$
$$\tan\psi_n=\alpha_i/(\alpha_r+n),\qquad \theta=\varphi+\alpha_i\ln N+\psi_0, \tag{4}$$

we find

$$A_m{}^n(t)\equiv\frac{\alpha_r+n}{\alpha_r+m}\frac{S_n}{S_m}=\frac{\cos^2\psi_n(1+\tan\theta\tan\psi_n)}{\cos^2\psi_m(1+\tan\theta\tan\psi_m)}. \tag{5}$$

Observe that $A_m{}^n=1$ in the limit of vanishing phases, as it must. From Eq. (5), with straightforward algebra, we can derive the following formula for $[\alpha_i(t)]^2$:

$$\begin{aligned}\alpha_i{}^2=&-(\alpha_r+m)(\alpha_r+n)(\alpha_r+p)[(m-n)(\alpha_r+p)A_m{}^n\\&+(n-p)(\alpha_r+m)A_p{}^n+(p-m)\\&\times(\alpha_r+n)A_m{}^nA_p{}^n]/[(m-n)(\alpha_r+m)\\&\times(\alpha_r+n)A_m{}^n+(n-p)(\alpha_r+n)(\alpha_r+p)A_p{}^n\\&+(p-m)(\alpha_r+m)(\alpha_r+p)A_m{}^nA_p{}^n].\end{aligned} \tag{6}$$

The right-hand side of Eq. (6) must, and does, vanish when all the A's are set equal to 1, but it is not positive definite. Putting $n=1$, $m=3$, and $p=5$, Eq. (6) gives $\alpha_i(t)$ as a function of $\alpha_r(t)$ and $S_1(t)$, $S_3(t)$, and $S_5(t)$. At any t in the interval $-1.8\leq t\leq 0$, we can calculate $\alpha_i{}^2(t)$ by taking the S's from Ref. 5 and assuming various values of $\alpha_r(t)$. It turns out that restricting $\alpha_r(t)$ to the

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physically reasonable (in view of high-energy scattering data) range $-2.9 \leq \alpha_r(t) \leq 0.7$, it is not possible to maintain $\alpha_i^2(t) \geq 0$ between $t=0$ and $t=-1.8$. So, we are led to conclude that additional Regge structure, very likely the fixed poles of Ref. 5, is still necessary in the $B^{(-)}$ amplitude. Then, without further specific assumptions, the FESR do not provide sufficient information to compute $\alpha(t)$.

Note added in proof. From Eq. (3) we obtain at once

$$S_n(t) = N^{\alpha_r} \left| \frac{\beta\alpha}{\alpha+n} \right| \cos(\theta - \psi_n).$$

If we attempt to satisfy $S_1 = S_3 = S_5 = 0$ at $t \approx -0.4$ (BeV)^2 by means of a phase condition, we find

$$\theta - \psi_n = (2m+1)\pi/2,$$

and hence

$$\psi_n - \psi_{n+2} = k\pi,$$

for some integers m and k. Thus,

$$\frac{\alpha_i}{\alpha_r + n} = \tan\psi_n = \tan(\psi_{n+2} + k\pi) = \tan\psi_{n+2} = \frac{\alpha_i}{\alpha_r + n + 2},$$

showing that $\alpha_i(-0.4) = 0$. This disagrees with the experimental fits to πp charge exchange,[6] which yield $\alpha_i(-0.4) \approx 0.1$. Therefore, we must assume that S_1, S_3, and S_5 vanish near $t = 0.4$(BeV)^2, but at slightly different points. The experimental errors[5] clearly permit this. Remembering that the phases depend on t and that $\psi_n \ll 1$ for $n \geq 1$, we see at once that this situation will occur if $\theta(-0.4) \approx \pi/2$.

Since the assumption $\alpha(-0.4) = 0$ was not actually used in this paper, all the equations and conclusions remain unaltered.

I should like to thank Dr. L. Girardello for drawing my attention to Ref. 3.

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Duality in Backward $\pi^+ p$ Elastic Scattering*

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A duality-inspired model for the scattering amplitude is used to explain backward $\pi^+ p$ elastic-scattering data from 2.18 GeV/c (incident pion laboratory momentum) on up. The quantitative success of this model gives support to the assumption that the dip at $u \sim -0.15$ GeV^2 is due to a wrong-signature nonsense zero in the N_α Regge-pole exchange amplitude. A particularly interesting polarization prediction (which can soon be compared with experimental data) is also presented.

In an earlier paper,[1] we proposed a model for the scattering amplitude that is consistent with duality[2] and particularly applicable to backward πN elastic scattering at intermediate energies.[3] Following directly from the Veneziano model,[4] we identify[1] the sum of direct-channel resonances with the signatured part of the Regge amplitude, leaving the purely real nonsignatured part as an interfering background. With such a prescription for expressing the scattering amplitude, Regge parameters determined at high energies are used in conjunction with resonance parameters determined at low energies to account for its behavior in between. Therefore, there should be relatively few additional parameters[5] needed for our description of the scattering amplitude at intermediate energies.

A wealth of backward $\pi^+ p$ elastic-scattering differential-cross-section data is now available at both high[6] and intermediate[7] energies, the most prominent feature of which is the occurrence of a dip at all energies with u roughly somewhere between -0.1 and -0.2 GeV^2. We interpret this as the result of a wrong-signature nonsense zero in that part of the Regge-pole amplitude due to N_α exchange,[8] and fit the experimental data at high energies[6] with just the N_α and Δ_δ Regge poles.[9] The

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[3] We have used this model to obtain a good fit to the 180° $\pi^- p$ elastic-scattering differential cross-section data of Kormanyos *et al.*, Phys. Rev. 164, 1661 (1967); also, see Ref. 1.
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