Comment on the Forward pp Dispersion Relation*

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It is argued that the unphysical region and the spin-dependence problem in forward pp scattering can be done away with at high energy, maintaining the phenomenological usefulness of the forward pp dispersion relation.

UNLIKE forward πN scattering,[1,2] measurement and evaluation of the real part of forward NN scattering[3] have been plagued with the problem of spin dependence and with the existence of an unphysical region,[4] respectively.

In the high-energy region, the unphysical integral is expected to be small,[5] owing to a large energy denominator in the dispersion integral. It remains to investigate the problem of spin dependence.

Writing the Feynman amplitudes for NN scattering in the Fermi form[6,7]

$$T=\sum_i \bar{u}(k')O_i u(k)\bar{u}(p')O_i u(p)F_i\,, \tag{1}$$

$$[O_S=1\,,\quad O_P=\gamma_5\,,\quad O_V=\gamma_\mu\,,\quad O_A=i\gamma_5\gamma_\mu\,,\quad O_T=\sigma_{\mu\nu}/\sqrt{2}]\,,$$

the five independent helicity amplitudes in the c.m. system read (in the notation $\langle k',p'|T|k,p\rangle$)

$$\begin{aligned}
\langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle
&=\cos^2(\tfrac{1}{2}\theta)\ \{F_S-F_T-[(E^2+k^2)/m^2](F_A-F_V)\}\\
&\quad-2\sin^2(\tfrac{1}{2}\theta)\ \{F_T+(E^2/m^2)F_A-(k^2/m^2)F_V\},\\
\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle
&=\cos^2(\tfrac{1}{2}\theta)\ \{F_S+F_T+[(E^2+k^2)/m^2](F_A+F_V)\}\,,\\
\langle-\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle
&=2\cos^2(\tfrac{1}{2}\theta)\ \{[(E^2+k^2)/m^2]F_T+F_A\}\\
&\quad+\sin^2(\tfrac{1}{2}\theta)\ \{-(E^2/m^2)F_S+[(E^2+k^2)/m^2]F_T+F_A-F_V-(k^2/m^2)F_p\},\\
\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle
&=\cos(\tfrac{1}{2}\theta)\sin(\tfrac{1}{2}\theta)\ (E/m)[F_S+F_T+F_A+F_V]\,,\\
\langle-\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle
&=\sin^2(\tfrac{1}{2}\theta)\ [(E^2/m^2)F_S+F_T+F_A+F_V-(k^2/m^2)F_p]\,.
\end{aligned}\tag{2}$$

In the forward direction, only three amplitudes can contribute[8]:

$$\begin{aligned}
\langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle&=F_S-F_T-(\nu/m)(F_A-F_V)=(4\pi/m)(\alpha-\epsilon)\,,\\
\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle&=F_S+F_T+(\nu/m)(F_A+F_V)=(4\pi/m)(\alpha+\epsilon),\\
\langle-\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle&=2[(\nu/m)F_T+F_A]=(4\pi/m)(\beta+\delta)=(4\pi/m)2\beta\,,
\end{aligned}\tag{3}$$

where $\nu=(E^2+k^2)/m=(s-2m^2)/2m$.

The usual forward amplitude for pp scattering refers to the combination

$$\begin{aligned}
C&=\tfrac{1}{2}[\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle+\langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle]\\
&=F_S+(\nu/m)F_V=(4\pi/m)\alpha\,,
\end{aligned}\tag{4}$$

which has a simple relation to the unpolarized total cross section: $\mathrm{Im}C=(k_{\mathrm{lab}}/2m)\sigma_{\mathrm{tot}}(pp)$ [and a similar relation for the antiproton scattering: $\mathrm{Im}\bar{C}=(k_{\mathrm{lab}}/2m)\sigma_{\mathrm{tot}}(\bar{p}p)$]. However, the experimental real part of C is to be extracted from the measurement of the differential cross section[1,9] (barring a Coulomb term)

$$\begin{aligned}
d\sigma(0)/d\Omega&=(m^4/4\pi^2s)\tfrac{1}{4}[|\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle+\langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle|^2\\
&\quad+|\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle-\langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle|^2\\
&\quad+2|\langle-\tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle|^2]\\
&=(4m^2/s)[|\alpha|^2+|\epsilon|^2+2|\beta|^2]\,.
\end{aligned}\tag{5}$$

This is possible only if one can measure β and ϵ independently, or if $|\beta|^2$ and $|\epsilon|^2$ are much smaller than $|\alpha|^2$.

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[1] K. J. Foley *et al.*, Phys. Rev. **181**, 1775 (1969).

[2] However, since the unexpected results of IHEC–CERN collaboration on total $\pi^{\pm}p$ cross sections have appeared [J. V. Allaby *et al.*, Phys. Letters **30B**, 500 (1969); and Kiev Conference, 1970 (unpublished)], the high-energy end of the dispersion integral must be reevaluated. See D. Horn and A. Yahil, Phys. Rev. D **1**, 2610 (1970); G. Höhler, F. Steiner, and R. Strauss, Z. Physik **233**, 430 (1970); V. Barger and R. J. N. Phillips, Phys. Rev. D **2**, 1871 (1970).

[3] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 857 (1967), and references therein.

[4] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N.Y.) **2**, 226 (1957); J. Hamilton, Phys. Rev. **114**, 1170 (1959).

[5] Y. C. Liu, Phys. Rev. **178**, 2243 (1969).

[6] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960); D. Amati, E. Leader, and B. Vitale, Nuovo Cimento **17**, 68 (1960).

[7] $k(p)$ and $k'(p')$ are the initial and final four-momenta of the projectile (target) nucleon. In the c.m. system, $k=(0,0,k,iE)$, $k'=(k\sin\theta,0,k\cos\theta,iE)$, $E^2=k^2+m^2$. The invariant amplitudes F_i do not satisfy the Pauli principle (Ref. 6), but they are independent and are free of kinematic singularities.

[8] $T=(4\pi/m)[\alpha+\beta\boldsymbol{\sigma}^{(1)}\cdot\mathbf{n}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{n}+i\gamma(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{n}+\delta\boldsymbol{\sigma}^{(1)}\cdot\mathbf{m}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{m}+\epsilon\boldsymbol{\sigma}^{(1)}\cdot\mathbf{l}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{l}]$, where $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$ are unit vectors in the directions $\mathbf{k}+\mathbf{k}'$, $\mathbf{k}-\mathbf{k}'$, and $\mathbf{k}\times\mathbf{k}'$, respectively. In Eq. (7), $\Delta^2=(k-k')^2=2k^2(1-\cos\theta)$.

[9] G. G. Beznogikh *et al.*, Phys. Letters **30B**, 294 (1969). Here $s=4E^2=4(k^2+m^2)$.

The idea of asymptotic helicity conservation in diffraction scattering[10] has recently been revived. It asserts[11] that asymptotically $|\langle -\frac{1}{2}-\frac{1}{2}|T|\frac{1}{2}\frac{1}{2}\rangle|$, $|\langle \frac{1}{2}-\frac{1}{2}|T|\frac{1}{2}\frac{1}{2}\rangle|$, and $|\langle -\frac{1}{2}\frac{1}{2}|T|\frac{1}{2}-\frac{1}{2}\rangle|$ are negligible compared to $|\langle \frac{1}{2}-\frac{1}{2}|T|\frac{1}{2}-\frac{1}{2}\rangle|$ and $|\langle \frac{1}{2}\frac{1}{2}|T|\frac{1}{2}\frac{1}{2}\rangle|$. If use is made of the Regge-pole theory[12,13], i.e., $|F_S|\sim s^{\alpha}$, $|F_T|\sim s^{\alpha-1}$, $|F_A|\sim s^{\alpha-1}$, $|F_V|\sim s^{\alpha-1}$, and $|F_p|\sim s^{\alpha}$, then a closer investigation on Eq. (2) would require that F_S, F_T, and F_p be decoupled from the leading trajectory,[10] leaving F_V and F_A as the dominant amplitudes for pp scattering at high energy. Furthermore, from the obvious fact that

$$\tfrac{1}{2}\,\mathrm{Im}[\langle \tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle+\langle \tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle]=\mathrm{Im}[F_S+(\nu/m)F_V]$$

is much larger than

$$\tfrac{1}{2}\,\mathrm{Im}[\langle \tfrac{1}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle-\langle \tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle]=\mathrm{Im}[F_T+(\nu/m)F_A],$$

we may finally neglect F_A as well,[14] leaving F_V as the only asymptotic amplitude.

With only one nuclear amplitude F_V present, one no longer need feel uneasy about "measuring" the real part of α by means of usual Coulomb interference method.[9] The comparison[3,5] of the forward pp dispersion relation therefore can be conducted meaningfully and serve[15] as another method (other than the well-known πN case[1]) to verify the analytic property of the scattering amplitudes.

In practice, experiments are performed at nonzero momentum transfer ($\Delta^2\neq 0$). We have then

$$\frac{d\sigma}{d\Omega}=\frac{4m^2}{s}[|\alpha|^2+|\beta|^2+2|\gamma|^2+|\delta|^2+|\epsilon|^2] \tag{6}$$

and

$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\\ \delta\\ \epsilon\end{pmatrix}=\frac{m}{4\pi}\begin{pmatrix}(1+a\Delta^2)^2 & -b\Delta^2 & -c\Delta^2 & (\nu/m-d\Delta^2) & 0\\ -c\Delta^2 & (\nu/m-d\Delta^2) & (1+a\Delta^2)^2 & -b\Delta^2 & 0\\ e\Delta & -e\Delta & -e\Delta & -f\Delta & 0\\ 0 & (\nu/m-g\Delta^2) & 1 & 0 & -g\Delta^2\\ 0 & 1 & (\nu/m-g\Delta^2) & -g\Delta^2 & 0\end{pmatrix}\begin{pmatrix}F_S\\ F_T\\ F_A\\ F_V\\ F_P\end{pmatrix}, \tag{7}$$

where

$$a=1/4m(E+m),$$
$$b=(1/4m^2)[1+(k^2-\Delta^2/4)/(E+m)^2],$$
$$c=(1/4m^2)(k^2-\Delta^2/4)/(E+m)^2,$$
$$d=(1/4m^2)[1+2E/(E+m)]-\Delta^2/16m^2(E+m)^2,$$
$$e=[(k^2-\Delta^2/4)^{1/2}/2m(E+m)][1+\Delta^2/4m(E+m)],$$
$$f=[(k^2-\Delta^2/4)^{1/2}/2m(E+m)]\times[1+2E/m-\Delta^2/4m(E+m)],$$

and

$$g=1/4m^2.$$

For extremely small Δ, our main argument to obtain $\mathrm{Re}\alpha$ from Eq. (6) still holds. For moderate Δ, since $\gamma\to-[(\Delta\sqrt{s})/2m^2]F_V$, $\alpha\to(s/2m^2)F_V$, γ may show up in addition to α,[16] destroying the above simple picture of asymptotic spin independence. No harm should arise, however, as long as the Coulomb interference experiment is actually carried out within the small forward cone.

Parenthetically, asymptotic helicity conservation leads in πN scattering to the decoupling[17] of $A^{(+)}$ from the leading trajectories. Then all four $A^{(\pm)}$ and $B^{(\pm)}$ will satisfy fixed-t unsubtracted dispersion relations. A phenomenological and self-consistent analysis for $A^{(\pm)}$ and $B^{(\pm)}$ without any arbitrary subtraction function now is possible.[18]

[10] F. J. Gilman *et al.*, Phys. Letters **31B**, 387 (1970); R. Torgerson, Phys. Rev. **143**, 1194 (1966).

[11] Note that we are comparing the magnitudes, not the amplitudes which are complex.

[12] D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **44**, 1068 (1963) [Soviet Phys. JETP **17**, 720 (1963)]; D. H. Sharp and W. G. Wagner, Phys. Rev. **131**, 2226 (1963); F. E. Paige, Phys. Rev. D **2**, 922 (1970). Serpukov data on NN scattering are still well described by the Regge poles.

[13] Ordinary Reggeization using t-channel helicity amplitudes yields the power behavior only for several linear combinations of the invariant amplitudes (Volkov and Gribov, Ref. 12). The power behavior for each invariant amplitude F_i is obtained under the circumstance that no cancellation occurs among themselves.

[14] The assumption of asymptotic helicity conservation, namely, $\langle \frac{1}{2}-\frac{1}{2}|T|\frac{1}{2}-\frac{1}{2}\rangle=\langle \frac{1}{2}\frac{1}{2}|T|\frac{1}{2}\frac{1}{2}\rangle$, yields the same result.

[15] Now the total cross sections at high energies must be known precisely.

[16] Whether the model of H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman [Phys. Rev. **177**, 2458 (1968)] will conform with the forward pp dispersion relation is to be investigated.

[17] G. Höhler *et al.*, Ref. 2; W. K. Cheng *et al.*, Phys. Rev. D (to be published); F. Steiner *et al.*, Nucl. Phys. **B24**, 398 (1970); R. Pfeffer *et al.*, Phys. Rev. D **2**, 1965 (1970).

[18] Y. C. Liu and Ian J. McGee (unpublished).