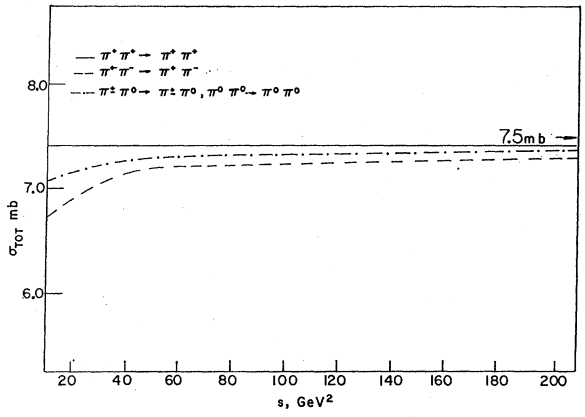

FIG. 28. Total cross section σ_T for $\pi^+\pi^+$ (solid line), and the elastic cross sections for $\pi^+\pi^-$ (dashed line), and $\pi^0\pi^0$, $\pi^\pm\pi^0$ (dash-dot line) as a function of s.

at this energy has the opposite sign, we do not have strict factorization for the ρ contribution, as we already observed in the π-π charge-exchange scattering. The asymptotic K-π total cross section is 4 mb, which can be compared with the value of ≈ 11 mb obtained from factorization using

$$\sigma_{K\pi} = \sigma_{KN}\sigma_{\pi N}/\sigma_{NN} \tag{8.10}$$

and the presently available data.[29]

IX. CONCLUSIONS

We have succeeded in finding solutions to the model for K-π scattering which are approximately unitary at low energies and satisfy s-u crossing exactly; the on-mass-shell predictions were in satisfactory agreement with the data, and the scattering lengths were close to those obtained from current algebra. Apart from a change in the over-all coupling constant, the same parameters were then used to predict the low-energy π-π scattering and the crossing-symmetric solutions were found to be approximately unitary up to 900 MeV, and satisfactory fits to the available data (see Table V) were found except in the case of the charge-exchange data; however, the latter data are still open to question due to the difficulty in measuring the cross sections for this process accurately, and further experimental information is required. The predicted on-mass-shell forward-backward asymmetry fitted the data very well, and the predicted scattering lengths at threshold were in good agreement with various analyses of the data and the results of current algebra.

The general conditions below threshold that follow from crossing symmetry and positivity were investigated and found to be well satisfied. The calculation of the high-energy π-π and K-π scattering showed that the Pomeranchukon amplitude described the low- as well as the high-energy region satisfactorily; the charge-exchange scattering at high energy displayed the "nonsense" dip correctly, and the total cross sections were found to be of the order of magnitude expected in the asymptotic region.

If we could succeed in extending the model to the resonance region at intermediate energies by some satisfactory procedure of unitarization, then we could claim to have an approximate description of π-π and K-π scattering valid in the whole energy range, consistent with the basic principles that we believe a model of strong interactions should possess.

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Contribution of Elastic Intermediate States to Polarization of the Recoil Proton in Elastic Electron-Proton Scattering

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The polarization of the recoil proton in the elastic scattering of unpolarized electrons and protons is calculated to order α^3, retaining only the elastic intermediate state in the unitarity sum that occurs. The result is therefore expected to correspond closely to the physical situation for electron laboratory energies up to the region where pion production becomes important. Using the "dipole fit" for the proton form factors G_E and G_M, the maximum value of the polarization is found to be $\sim 0.03\%$ for electron energies below 400 MeV. Above 10 GeV, the maximum elastic effect is $\sim 1\%$.

I. INTRODUCTION

IN the one-photon-exchange approximation, the scattering of unpolarized electrons by an unpolarized proton target gives no polarization of the recoil proton (see Sec. II, for example). Any nonzero polarization of the recoil proton, transverse to the scattering plane, must arise from interference of higher-order amplitudes with the one-photon amplitude. We are interested in the contribution from the two-photon-exchange amplitudes of Fig. 1. There will be such diagrams for each

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allowed intermediate hadron state (p, N^*, $N^*\pi$, etc.), and each of the corresponding amplitudes can contribute to the polarization. It is not possible at present to calculate exactly the total contribution of these diagrams, but there have been several theoretical attempts to estimate the contribution from these inelastic intermediate states.[1–8] These calculations suggest that hadronic effects do not enhance the polarization significantly above its intrinsic order of magnitude α. Despite the smallness of the effect, it is of interest to calculate, to lowest order in α, the contribution to the polarization from the elastic intermediate state, using the experimentally measured form factors G_E and G_M, in the proton electromagnetic current. This will be the only contribution to the polarization at energies below the pion threshold (~140 MeV electron energy in the laboratory), and will remain a good approximation until pion production contributes noticeably to the cross section (~340 MeV). At high energies, this elastic contribution will only be one of many similar contributions from all the possible intermediate states.

Barut and Fronsdal[9] have calculated the polarization in μ-e scattering—i.e., for two structureless "Dirac" particles—and Guerin and Piketty,[5] who calculated the contribution of two N^* intermediate states to the polarization in e-p and μ-p scattering, also quoted one numerical result for one particular case for the proton intermediate state, using Clementel-Villi-type form factors for the proton. Here we present details of the calculation of the elastic contribution to the polarization in e-p scattering for arbitrary form factors, and give results for a "structureless" proton and for a "realistic" proton with the experimentally observed dipole form factors.

Towards the end of this calculation, a paper appeared by Arafune and Shimizu[10] which was also concerned with the elastic contribution to the polarization in e-p scattering. Because of certain approximations made in their calculation, their results do not correspond directly to the physical situation. This is discussed in detail in the later sections.

We remark here that since we assume time-reversal invariance for elastic e-p scattering, measurement of the polarization of the recoil proton produced in the collision of unpolarized initial particles, is equivalent to measurement of the asymmetry in the scattering of unpolarized electrons by polarized protons.[11]

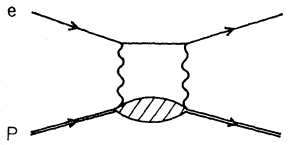

FIG. 1. Two-photon-exchange graph: shaded region indicates excited intermediate state at the nucleon vertices.

The plan of the paper is as follows. In Sec. II we give details of our covariant polarization formalism. An expression for the elastic contribution to the polarization is derived, using the unitarity relation. Section III contains details of the claculations. In Sec. III A we briefly discuss the trace calculation—the results are given in the Appendix. Section III B contains a detailed explanation of the cancellation of the infrared divergence in these polarization calculations, and Sec. III C gives details of the form factor parametrizations and also a discussion of the results of Arafune and Shimizu. Our conclusions are summarized in Sec. IV.

II. THEORETICAL FORMALISM

We use a covariant formalism similar to that used by Bilen'kii and Semikoz[12] in their calculation of the asymmetry in π-e scattering. Our notation is as follows (see Fig. 2):

$$k^2=k'^2=\mu^2,$$
$$p^2=p'^2=M^2,$$

where k and k' are the electron's initial and final four-momenta, respectively. p and p' are the proton's initial and final four-momenta, respectively. s is the covariant spin four-vector of the recoil proton: It reduces to a spatial unit vector in the rest frame. We define

$$Q=k+p, \quad \Delta=k-k',$$
$$P=p+p', \quad K=k+k'.$$

The normal to the scattering plane can be defined covariantly:

$$n^\mu=\epsilon^{\mu\nu\rho\sigma}k_\nu k_\rho' p_\sigma \quad (\epsilon_{0123}=+1), \tag{1}$$

which reduces in the lab system to

$$\mathbf{n}=M(\mathbf{k}\times\mathbf{k}'), \quad n^0=0.$$

This direction is taken to be the positive y direction, and $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{p}'$, $\mathbf{p}$ are taken all to be in the xz plane. For polarization transverse to the scattering plane, therefore,

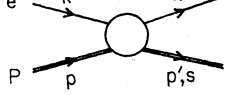

FIG. 2. Electron-proton scattering: notation

[1] S. D. Drell and M. A. Ruderman, Phys. Rev. **106**, 561 (1957).
[2] S. D. Drell and S. Fubini, Phys. Rev. **113**, 741 (1959).
[3] N. R. Werthamer and M. A. Ruderman, Phys. Rev. **123**, 1005 (1961).
[4] D. Flamm and W. Kummer, Nuovo Cimento **28**, 33 (1963).
[5] F. Guerin and C. A. Piketty, Nuovo Cimento **32**, 971 (1964).
[6] S. D. Drell and J. D. Sullivan, Phys. Letters **19**, 516 (1965).
[7] G. K. Greenhut, Phys. Rev. **184**, 1860 (1969).
[8] J. Harte, Phys. Rev. **171**, 1832 (1968). This calculation uses a specific bootstrap model for the vertex functions and suggests that two-photon effects are surprisingly large at large momentum transfer in this model.
[9] A. O. Barut and C. Fronsdal, Phys. Rev. **120**, 1871 (1960).
[10] J. Arafune and Y. Shimizu, Phys. Rev. D **1**, 3094 (1970).
[11] R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952); L. Wolfenstein and H. Ashkin, Phys. Rev. **85**, 947 (1952).
[12] S. M. Bilen'kii and V. B. Semikoz, Yadern. Fiz. **7**, 107 (1968) [Soviet J. Nucl. Phys. **7**, 79 (1968)].

we can define

$$s^\mu = n^\mu/N, \quad s^2 = -1, \quad n^2 = -N^2. \tag{2}$$

Obviously,

$$s\cdot p = s\cdot k' = s\cdot p' = s\cdot k = 0.$$

Note that this definition of the normal to the scattering plane is opposite to that of the Basel convention, according to which the normal should be defined $\boldsymbol{n}_B \propto \mathbf{k}\times\mathbf{p}'$. Here we have defined the normal using the initial and final momenta of the same particle (as used in Ref. 13). The transition matrix element is defined by

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p+k-p'-k')N_f N_i T_{fi}, \tag{3}$$

with

$$N_f = \left(\frac{M\mu}{p_0' k_0'}\right)^{1/2}, \quad N_i = \left(\frac{M\mu}{p_0 k_0}\right)^{1/2}.$$

We use the metric, spinor normalizations, and Dirac matrices of Bjorken and Drell.[14]

For spin-$\frac{1}{2}$–spin-$\frac{1}{2}$ elastic scattering, the T-matrix element can be written

$$T_{fi} = \bar{u}_e(k')\bar{u}_p(p')\mathfrak{M}u_e(k)u_p(p), \tag{4}$$

where $\mathfrak{M}$ is a matrix in the combined spin-space of the electron and proton, and $u_p(p)$, etc., are the usual Dirac spinors. With the assumption of parity- and time-reversal invariance, $\mathfrak{M}$ can be written in terms of six scalar amplitudes.[15] The transverse polarization of the recoil proton is given by

$$P = \frac{\mathrm{Tr}[\gamma_5 \boldsymbol{s}\Lambda(p')\Lambda(k')\mathfrak{M}\Lambda(p)\Lambda(k)\bar{\mathfrak{M}}]}{\mathrm{Tr}[\Lambda(p')\Lambda(k')\mathfrak{M}\Lambda(p)\Lambda(k)\bar{\mathfrak{M}}]}, \tag{5}$$

where $\gamma_5\boldsymbol{s}$ acts only in the proton spin-space. $\Lambda(p)$, etc., are the positive-energy projection operators $\Lambda(p) = (\boldsymbol{p}+M)/2M$. Unitarity of the S matrix gives the following condition on $\mathfrak{M}$:

$$\begin{aligned}\Lambda(k')\Lambda(p')[\mathfrak{M}-\bar{\mathfrak{M}}]\Lambda(k)\Lambda(p)&\\ = (+i)\sum_n (2\pi)^4\delta^4(p+k-p_n-k_n)N_n{}^2&\\ \times\Lambda(k')\Lambda(p')\mathfrak{M}\Lambda(k_n)\Lambda(p_n)\bar{\mathfrak{M}}\Lambda(k)\Lambda(p),&\end{aligned} \tag{6}$$

where only the elastic intermediate state has been included in the unitarity sum, and four-vectors k_n and p_n of the intermediate electron and proton, respectively, have been defined; see Fig. 3.

For the two-particle intermediate state,

$$\sum_n \to \int \frac{d^3k_n}{(2\pi)^3}\frac{d^3p_n}{(2\pi)^3}.$$

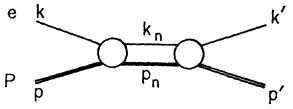

FIG. 3. Elastic intermediate state: notation.

On making an expansion of $\mathfrak{M}$ in powers of α,

$$\mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2 + \cdots, \tag{7}$$

Eq. (6) gives immediately that $\mathfrak{M}_1 = \bar{\mathfrak{M}}_1$. It is then easy to prove (essentially using the fact that $\mathfrak{M}_R = \mathfrak{M}$ where $\mathfrak{M}_R$ has the γ matrices written in reverse order), that the lowest-order contribution to the polarization results from an interference between $\mathfrak{M}_1$ and $\mathfrak{M}_2$:

$$\begin{aligned}I_0 P &= \mathrm{Tr}[\gamma_5\boldsymbol{s}\Lambda(p')\Lambda(k')(\mathfrak{M}_2-\bar{\mathfrak{M}}_2)\Lambda(p)\Lambda(k)\mathfrak{M}_1]\\ &= (+i)\int\frac{d^3k_n}{(2\pi)^3}\frac{d^3p_n}{(2\pi)^3}(2\pi)^4\delta(Q-p_n-k_n)N_n{}^2\\ &\quad\times\mathrm{Tr}[\gamma_5\boldsymbol{s}\Lambda(p')\Lambda(k')\mathfrak{M}_1\\ &\qquad\times\Lambda(p_n)\Lambda(k_n)\mathfrak{M}_1\Lambda(p)\Lambda(k)\mathfrak{M}_1],\end{aligned} \tag{8}$$

where

$$I_0 = \mathrm{Tr}[\Lambda(p')\Lambda(k')\mathfrak{M}_1\Lambda(p)\Lambda(k)\mathfrak{M}_1]$$

and we have used the unitarity relation for $(\mathfrak{M}_2-\bar{\mathfrak{M}}_2)$. This is the basic expression for our polarization calculations. $\mathfrak{M}_1$ is just the one-photon-exchange amplitude for e-p scattering and is given by the Feynman rules[16]

$$\mathfrak{M}_1(\Delta^2) = \gamma_{(e)}{}^\mu\frac{ee_p}{\Delta^2}\Gamma_\mu{}^{(p)}(\Delta^2). \tag{9}$$

The standard form for the proton current $\Gamma_\mu(\Delta^2)$ is[17]

$$\begin{aligned}\Gamma_\mu(\Delta^2) &= G_M(\Delta^2)\gamma_\mu - \frac{[G_M(\Delta^2)-G_E(\Delta^2)]}{2M(1+\tau)}P_\mu\\ &\equiv A(\Delta^2)\gamma_\mu + B(\Delta^2)P_\mu,\end{aligned} \tag{10}$$

where

$$\tau = -\Delta^2/4M^2.$$

Thus the traces now factorize into traces in the electron and proton spin-spaces:

$$\begin{aligned}&\mathrm{Tr}[\gamma_5\boldsymbol{s}\Lambda(p')\Lambda(k')\mathfrak{M}_1(\Delta_2{}^2)\\ &\qquad\times\Lambda(p_n)\Lambda(k_n)\mathfrak{M}_1(\Delta_1{}^2)\Lambda(k)\Lambda(p)\mathfrak{M}_1(\Delta^2)]\\ &\qquad= \frac{(ee_p)^3}{\Delta^2\Delta_1{}^2\Delta_2{}^2}\cdot\frac{1}{(2M)^3}\frac{1}{(2\mu)^3}T_e{}^{\mu\nu\rho}T_{p\mu\nu\rho},\end{aligned} \tag{11}$$

where

$$\begin{aligned}T_e{}^{\mu\nu\rho} &= \mathrm{Tr}[(\boldsymbol{k}'+\mu)\gamma^\mu(\boldsymbol{k}_n+\mu)\gamma^\nu(\boldsymbol{k}+\mu)\gamma^\rho],\\ T_{p\mu\nu\rho} &= \mathrm{Tr}[\gamma_5\boldsymbol{s}(\boldsymbol{p}'+M)\Gamma_\mu(\Delta_2{}^2)\\ &\qquad\times(\boldsymbol{p}_n+M)\Gamma_\nu(\Delta_1{}^2)(\boldsymbol{p}+M)\Gamma_\rho(\Delta^2)],\end{aligned}$$

[13] T. Powell *et al.*, Phys. Rev. Letters **24**, 753 (1970).

[14] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[15] The covariant M-function approach for fermion-fermion scattering is treated in all generality by B. H. Kellett, Nuovo Cimento **56A**, 1003 (1968). References to earlier work are given there.

[16] The Feynman rules are given for S-matrix elements in Ref. 14: The T-matrix element is extracted according to the definition in the text.

[17] Form factors F_1 and F_2 and their relation to G_E and G_M are defined, for example, in Ref. 14, p. 245.

and we have defined

$$\Delta_2=k'-k_n, \quad P_2=p'+p_n,$$
$$\Delta_1=k-k_n, \quad P_1=p+p_n.$$

III. CALCULATIONS

A. Trace Calculations

The traces were evaluated in covariant form before attempting the integration. Using the shorthand notation $A=A(\Delta^2)$, $A_1=A(\Delta_1^2)$, and so on, there are eight separate contributions to the proton trace arising from the eight combinations AA_1A_2, BA_1A_2, etc. The actual trace calculation is quite straightforward although very long and tedious. To simplify the calculations, terms proportional to the square of the electron mass μ^2 have been neglected. The explicit results are given in the Appendix, together with some useful trace identities.

In general, the A and B terms of the proton current have a different Δ^2 dependence, and all eight terms must be integrated separately. In Ref. 10, A and B were taken to have the same Δ^2 dependence: The three "A^2B" terms, and the three "AB^2" terms could then be added together before integration. The reported experimental behavior of G_E and G_M[18] does not suggest that A and B have the same Δ^2 dependence.

As a check on our calculations, the result of Ref. 10 was verified by adding the three "A^2B" terms and the three "AB^2" terms.[19] This result is a special case of our calculation[20] which applies for any Δ^2 dependence of G_E and G_M.

The denominator trace is just that involved in the Rosenbluth cross section.

B. Integrations and Infrared Divergence

The integrations are most easily performed in the c.m. system (see Fig. 4). The $\mathbf{k}$, $\mathbf{k}'$ plane is the scattering plane and, in general, $\mathbf{k}_n$ is not coplanar with $\mathbf{k}$ and $\mathbf{k}'$; furthermore,

$$k+p=k_n+p_n=k'+p'$$

and $|\mathbf{k}|=|\mathbf{k}'|=|\mathbf{k}_n|=k$ for elastic scattering; also,

$$\Delta^2=-2k^2(1-z), \quad \text{where} \quad \mathbf{k}\cdot\mathbf{k}'=k^2z=k^2\cos\theta. \tag{12a}$$

Similarly,

$$\Delta_1^2=-2k^2(1-z_1) \quad \text{and} \quad \Delta_2^2=-2k^2(1-z_2), \tag{12b}$$

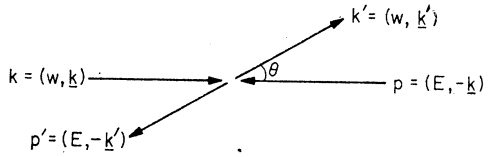

FIG. 4. Notation for the c.m. system.

where

$$z_2=zz_1+(1-z^2)^{1/2}(1-z_1^2)^{1/2}\cos\phi \tag{12c}$$

and

$$\mathbf{k}_n=(k,\theta_1,\phi)\equiv(k,\Omega_n),$$
$$z_1=\cos\theta_1, \quad z_2=\cos\theta_2.$$

With the aid of the δ function, the intermediate-state integration reduces to an angular integration in the c.m. system:

$$\sum_n (2\pi)^4\delta^4(Q-p_n-k_n)N_n{}^2\rightarrow\frac{kM\mu}{(2\pi)^2(E+\omega)}\int d\Omega_n.$$

We require integrals of the form

$$\int d\Omega_n\frac{F(\Delta_1^2,\Delta_2^2)}{\Delta_1^2\Delta_2^2}, \tag{13}$$

where F is some known function of Δ_1^2 and Δ_2^2. All the required nontrivial integrals can be obtained from the basic unitarity integral[21]

$$I(z)=\int\frac{d\Omega_n}{(T_1-z_1)(T_2-z_2)}, \quad T_1, T_2>1. \tag{14}$$

In our case we require limits such as $T_1=T_2=1$, where the integral diverges. As usual, a small photon mass λ must be introduced into the photon propagators; the limit $\lambda\rightarrow 0$ then gives rise to infrared divergent terms. However, it can be shown that there is no infrared divergence in the polarization. This is easy to show if we assume that A and B are constant; the traces can then be added and reduced to the result of Ref. 10. This result can be written in the form

$$T_eT_p=(skk'k_n)H+\Delta_1^2\Delta_2^2(skk'p)J, \tag{15}$$

where H and J are polynomials in Δ_1^2, Δ_2^2, $k\cdot p$, etc. The symbol $(abcd)$ is used here as a shorthand for $\mathrm{Tr}(\gamma_5\boldsymbol{abcd})$. In the unitarity integral (13), the J term clearly gives no divergence. For the H term, using the identity (A3) of the Appendix, we obtain a factor

$$[\Delta_1^2+\Delta_2^2-\Delta^2]/\Delta_1^2\Delta_2^2$$

in the integral. Now, for example, as $\Delta_1^2\rightarrow 0$, $\Delta_2^2\rightarrow\Delta^2$ and the divergence is cancelled. This result can of course be verified by detailed integration using the photon mass λ. Thus there is no singularity from the H term also,

[18] J. G. Rutherglen, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

[19] Note that the symmetry of the integral has been used to simplify one term in the expression quoted in Ref. 10 as being the result of the trace calculation.

[20] The term proportional to A_1A_2A is the trace required for the polarization of two Dirac particles with no anomalous magnetic moments. This term was also calculated keeping the electron mass terms, to compare with the result of Ref. 9. The expression obtained for the polarization in this case is identical with the result of Barut and Fronsdal except for the over-all sign. We believe the sign given here to be correct.

[21] See, for example, Appendix E of H. Burkhardt, *Dispersion Relation Dynamics* (North-Holland, Amsterdam, 1969).

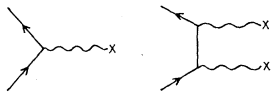

FIG. 5. One- and two-photon exchange diagrams for an electron scattering in a potential.

and the total result is finite.[22] For our case, where an explicit (different) Δ^2 dependence is taken for A and B (see Sec. III C), the trace cannot be written in the form of Eq. (15). However, if we consider the terms not of the form of Eq. (15) and evaluate the integrals using a finite photon mass λ in the photon propagators, in the limit $\lambda \to 0$ the divergent terms cancel explicitly.

The explanation[23] for this cancellation of the infrared divergence is most easily illustrated by considering the analogous problem of the polarization of an electron scattered by a potential. Again, the lowest-order polarization arises from an interference of the first Born amplitude with the two-photon-exchange amplitude (see Fig. 5). Dalitz[24] has shown that the infrared divergence in the two-photon graph can be isolated into a term proportional to the first Born amplitude. To this order in α, the divergence acts like a phase factor for the first Born amplitude, and it is precisely because of this that there is no divergence in the polarization. In the two-component spinor formalism (totally relativistic, of course),

$$T_{fi} \sim \chi_f^\dagger M \chi_i ,$$

and we make an expansion of M in powers of α,

$$M = \alpha M^{(1)} + \alpha^2 M^{(2)} + \cdots .$$

The infrared divergent term in $M^{(2)}$ is purely imaginary in this case and proportional to $M^{(1)}$. Thus we may write

$$\alpha^2 M^{(2)} = \alpha^2 M_F{}^{(2)} + i\alpha^2 \phi(\lambda^2) M^{(1)} ,$$

where $M_F{}^{(2)}$ is finite and $\phi(\lambda^2)$ is a scalar function of λ^2 which diverges as $\lambda \to 0$. Therefore we have

$$M = \alpha M^{(1)}[1 + i\alpha\phi(\lambda^2)] + \alpha^2 M_F{}^{(2)} .$$

Now M can be written

$$M = F + i\boldsymbol{\sigma}\cdot\mathbf{n}G ,$$

where n is the normal to the scattering plane. Consequently,

$$M^{(1)} = f^{(1)} + i\boldsymbol{\sigma}\cdot\mathbf{n}g^{(1)} ,$$
$$M_F{}^{(2)} = f^{(2)} + i\boldsymbol{\sigma}\cdot\mathbf{n}g^{(2)} .$$

The polarization is sensitive to the relative phase of F and G and since the divergent phase factor is the same for both $f^{(1)}$ and $g^{(1)}$, there is no divergence in the polarization to order α^3:

$$P \sim \alpha^3 \operatorname{Im}[f^{(1)} g^{(2)*} + g^{(1)} f^{(2)}] .$$

For the nonstatic case, Tsai[25] has shown that the infrared divergent part of the two-photon-exchange graph is still proportional to the first Born term. Consequently, the argument will be the same in this case. Cahn and Tsai[26] have recently given a similar explanation for the cancellation of the divergence in inelastic e-p scattering. A different explanation, which we believe is inadequate,[27] has been offered by Barut and Fronsdal.[9]

C. Results

The numerical calculations were carried out using the usual experimental parametrization of the two form factors

$$G_M(\Delta^2) = \mu_p G_E(\Delta^2) = \mu_p G_D(\Delta^2)$$

with

$$G_D(\Delta^2) = \frac{1}{(1-\Delta^2/\beta)^2} .$$

This is the so-called "dipole fit" and "scaling law" for G_E and G_M which is believed to be a good representation of the experimental data.[18] Experimentally, $\beta = 0.71$ $(\mathrm{GeV}/c)^2$.

With this form, after using partial fractions, the integrals can be performed analytically and finite expressions obtained after explicit cancellation of the infrared divergent terms. The explicit expressions are too long to be given here: they were programmed and the polarizations calculated using a KDF9 computer.

In the figures, the full line curves represent the polarization calculated using the experimental form factors, i.e., $\beta = 0.71$. The broken-line curves represent the polarization obtained for $G_D(\Delta^2) = 1$ $(\beta = \infty)$. This can be considered as some sort of "pointlike" limit for the proton, although such a limit is rather arbitrary. We note here that if we take as our pointlike form factors the definitions $F_1(\Delta^2) = F_2(\Delta^2) = 1$, then we obtain the zero in the polarization noted in Ref. 10. Their approximation is equivalent to parametrizing the form factors F_1 and F_2 by a dipole formula

$$F_1(\Delta^2) = F_2(\Delta^2) = G_D(\Delta^2) .$$

The graphs, Figs. 6 and 7, show, respectively, the behavior of this elastic effect with respect to the c.m. scattering angle for various electron lab energies, and

[22] A similar argument has been applied for the inelastic intermediate states where, in the limit of zero electron mass, current conservation restricts the trace to be of the form of Eq. (15). This is shown in the paper by Guerin and Piketty (Ref. 5), but there is a misprint in Eq. (12) of their paper where a term like the J term in Eq. (15) of our paper has been omitted. I thank Dr. Piketty for a private communication concerning these points, and am informed that the term omitted in Eq. (12) was not omitted in their calculations.

[23] I thank Professor R. H. Dalitz for a helpful discussion of his paper (Ref. 24).

[24] R. H. Dalitz, Proc. Roy. Soc. (London) A206, 509 (1951).

[25] Y. S. Tsai, Phys. Rev. 122, 1898 (1961).

[26] R. N. Cahn and Y. S. Tsai, Phys. Rev. D 2, 870 (1970).

[27] We are concerned here with the imaginary part of the infrared divergence of the two-photon-exchange amplitude. This is proportional to the one-photon-exchange amplitude, and is merely the second term in the expansion of the Coulomb phase factor, which is well known in the nonrelativistic limit and arises because of the infinite range of the Coulomb field. The soft-bremsstrahlung diagrams discussed in Ref. 9 are irrelevant for this type of divergence; the cancellation of this divergence in the lowest-order polarization is because the phase factor is spin independent.

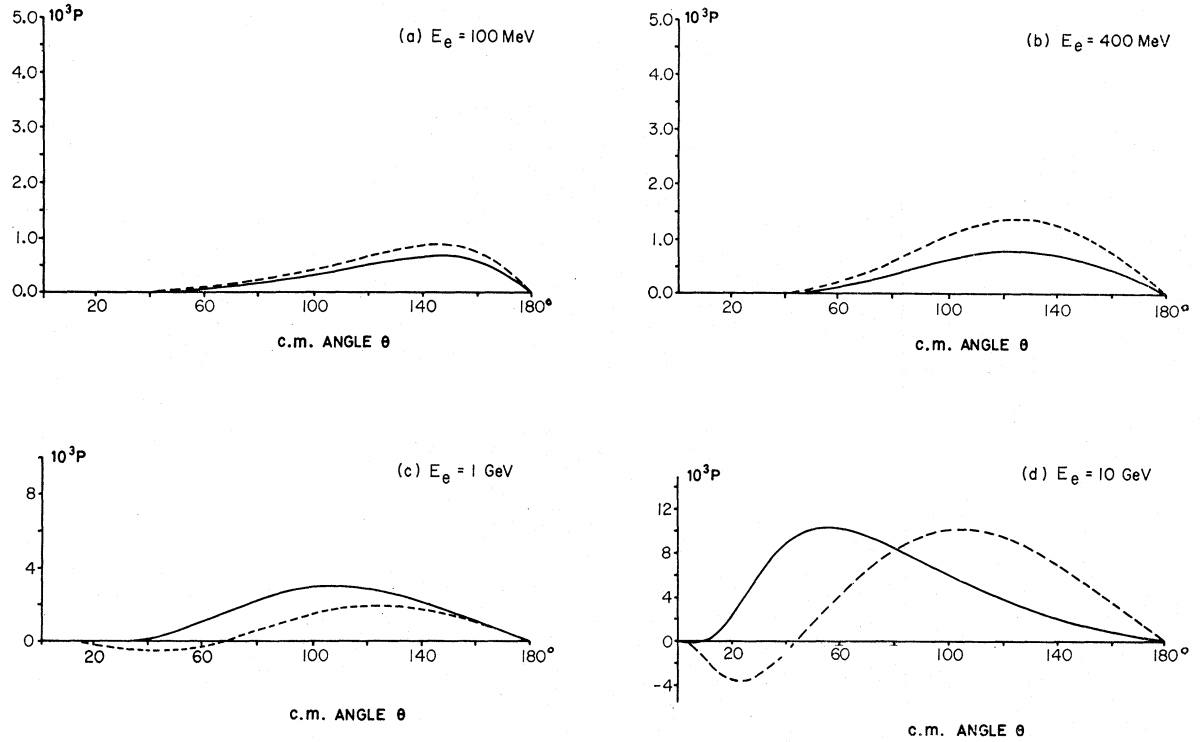

FIG. 6. $10^3\times$polarization vs c.m. scattering angle θ for various electron lab energies (E_e). Full curves: $\beta=0.71$. Broken curves: $\beta=\infty$.

with respect to electron lab energy for a fixed scattering angle.

IV. CONCLUSIONS

It is to be emphasized that these calculations are expected to correspond closely to the physical situation up to electron energies where pion production becomes important. Using the dipole form factors suggested by experiment, the maximum value of the polarization is found to be $\sim 0.08\%$ for electron energies below 400 MeV. We note that at 105 MeV, the maximum value is about 0.07% compared with 0.06% in Ref. 10 and 0.13% quoted in Ref. 5. The effect of the anomalous magnetic moment can be seen by noting that at 100 MeV, a "Dirac" proton with a dipole form factor gives a maximum polarization of $\sim 0.02\%$ compared with $\sim 0.07\%$ as calculated above.

The calculated maximum elastic effect rises to about 1% at 10 GeV and remains approximately constant for increasing energies. This value of 1% is to be compared with a maximum value of $\sim 0.6\%$ obtained in Ref. 10 (which they state is not reliable). At 10 GeV, there will, of course, be many other intermediate states contributing in the unitarity sum for the polarization, besides the elastic state. Guerin and Piketty[5] have found that the contributions from N^* resonances could be of opposite sign relative to the elastic contribution. If this is indeed a general feature, there could be considerable cancellations between the contributions from the various intermediate states, and consequently the elastic contribution may represent a reasonable order of magnitude estimate even at high energies.

A polarization of the order of 1% is probably too small to be measured with present experimental errors. To date, experimental data[18,13,28] exists for electron lab energies up to 18 GeV and for momentum transfers up to about 2 $(\mathrm{GeV}/c)^2$. The measurements appear to be consistent with $P=0$.[29] It is to be noted, however, that

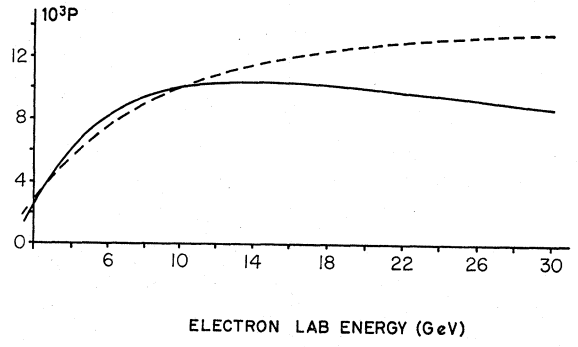

FIG. 7. $10^3\times$polarization vs electron lab energy in GeV, for fixed c.m. scattering angle θ. Full curve ($\beta=0.71$): $\theta=50°$. Broken curve ($\beta=\infty$): $\theta=100°$.

[28] H. C. Kirkmann *et al.*, Phys. Letters **32B**, 519 (1970).

[29] It may be remarked that since there are in principle six independent amplitudes for electron-proton scattering, from a strictly logical point of view, $P=0$ does not preclude the possibility of sizable two-photon-exchange amplitudes since cancellation may occur. The question of a complete set of experiments to determine the two-photon amplitudes uniquely is discussed in Ref. 32 (below).

the cross sections for inelastic e-p scattering in the so-called deep inelastic region[30] appear to show that resonance contributions are not the most important[31]: thus the possibility exists that in some kinematic region the polarization may be much larger than the elastic effect.

Note added in proof. Two-photon-exchange effects in elastic electron-proton scattering have recently been discussed by U. Günther and R. Rodenberg [Aachen Tech. Hochsch. report, 1970 (unpublished)] and by G. Leibbrandt [Nuovo Cimento **69A**, 153 (1970)].

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APPENDIX

In this appendix, we summarize the results of the trace calculation together with some useful identities. More details of these calculations and of the integrals may be found in Ref. 32. As a shorthand notation, we use

$$A(\Delta_1^2)=A_1, \quad A(\Delta^2)=A, \quad \text{etc.},$$

and

$$\mathrm{Tr}[\gamma_5 abcd]=(abcd).$$

We require to evaluate the trace product $T_eT_p=T_e{}^{\mu\nu\rho}T_{p\mu\nu\rho}$, where these traces are defined in Sec. II. The trace products T_{I}–T_{VIII} are defined by the expansion

$$\begin{aligned}T_eT_p=&AA_1A_2T_{\mathrm{I}}+A_2A_1BT_{\mathrm{II}}+A_2B_1AT_{\mathrm{III}}\\&+B_2A_1AT_{\mathrm{IV}}+A_2B_1BT_{\mathrm{V}}+B_2A_1BT_{\mathrm{VI}}\\&+B_2B_1AT_{\mathrm{VII}}+B_2B_1BT_{\mathrm{VIII}}.\end{aligned}\tag{A1}$$

In the calculation, the following trace identities are useful (we have made the approximation $k^2=k'^2=0$, $K^2=-\Delta^2$):

$$K^2(skpk_n)=\Delta_1^2(skk'p)+(2k\cdot p)(skk'k_n),\tag{A2a}$$

$$K^2(spk'k_n)=\Delta_2^2(skk'p)+(2k\cdot p+\Delta^2)(skk'k_n),\tag{A2b}$$

$$\begin{aligned}2M^2(skk'k_n)=&(2k_n\cdot p)(skk'p)+(2k\cdot p)(spk'k_n)\\&+(2k\cdot p+\Delta^2)(skpk_n),\end{aligned}\tag{A2c}$$

and finally

$$(skk'k_n)=\frac{\Delta^2(Q\cdot k)(\Delta_1^2+\Delta_2^2-\Delta^2)}{4N^2}(skk'p)\tag{A3}$$

with

$$4N^2=\Delta^2[K^2Q^2-4(Q\cdot k)^2].$$

For ease of integration, it is convenient to write the results in terms of $(skk'k_n)$ and $(skk'p)$—Eq. (A3) gives us some freedom in the actual form of these results.

The results of the trace calculations may be written in the following form:

$$\Delta^2T_N=a_N(skk'k_n)+b_N(skk'p),$$

$$T_{\mathrm{I}}\quad\begin{cases}a_{\mathrm{I}}=8M[(2k\cdot p)(\Delta_1^2+\Delta_2^2)-\Delta^2\Delta_1^2],\\ b_{\mathrm{I}}=16M\Delta_1^2\Delta_2^2,\end{cases}$$

$$T_{\mathrm{II}}\quad\begin{cases}a_{\mathrm{II}}=2\{(\Delta_1^2+\Delta_2^2)[4M^2(4k\cdot p-\Delta^2)-\Delta^2(8k\cdot p)]-(4k\cdot p+\Delta^2)4\Delta^2Q^2\},\\ b_{\mathrm{II}}=2[4\Delta_1^2\Delta_2^2(4M^2-\Delta^2)+\Delta^2(8k\cdot p)(4k\cdot p+\Delta^2)],\end{cases}$$

$$T_{\mathrm{III}}\quad\begin{cases}a_{\mathrm{III}}=2(2k\cdot p)[4Q^2\Delta_1^2+4M^2\Delta_2^2-4\Delta^2Q^2-(8k\cdot p)(4k\cdot p+\Delta^2)],\\ b_{\mathrm{III}}=2[\Delta_1^2\Delta_2^2(8M^2+8k\cdot p)-\Delta_1^2(8k\cdot p)(4k\cdot p+\Delta^2)],\end{cases}$$

$$T_{\mathrm{IV}}\quad\begin{cases}a_{\mathrm{IV}}=2\{(4k\cdot p+\Delta^2)[4\Delta^2Q^2+(4k\cdot p)^2]+4\Delta^2Q^2\cdot(2k\cdot p)+\Delta_1^2\cdot4M^2(2k\cdot p-\Delta^2)+\Delta_2^2\cdot4Q^2(2k\cdot p+\Delta^2)\},\\ b_{\mathrm{IV}}=2[\Delta_1^2\Delta_2^2(8k\cdot p+8M^2+4\Delta^2)+\Delta_1^2(8k\cdot p)(4k\cdot p+\Delta^2)-\Delta^2(8k\cdot p)(4k\cdot p+\Delta^2)],\end{cases}$$

$$T_{\mathrm{V}}\quad\begin{cases}a_{\mathrm{V}}=M\{2(4k\cdot p+\Delta^2)[8k\cdot p(4k\cdot p+\Delta^2)+4M^2\Delta^2]+\Delta_1^2\cdot(4k\cdot p+\Delta^2)\cdot8Q^2+\Delta_2^2[8k\cdot p(4M^2-2\Delta^2)-8M^2\Delta^2]\},\\ b_{\mathrm{V}}=M[4\Delta_1^2\Delta_2^2(8k\cdot p+8M^2)+4\Delta_2^2(8k\cdot p)(4k\cdot p+\Delta^2)],\end{cases}$$

$$T_{\mathrm{VI}}\quad\begin{cases}a_{\mathrm{VI}}=M\{2(4k\cdot p+\Delta^2)[8k\cdot p(4k\cdot p+\Delta^2)+4M^2\Delta^2]+\Delta_1^2[8k\cdot p(4M^2-2\Delta^2)-8M^2\Delta^2]+\Delta_2^2(4k\cdot p+\Delta^2)\cdot8Q^2\},\\ b_{\mathrm{VI}}=M[4\Delta_1^2\Delta_2^2(8k\cdot p+8M^2)+4\Delta_1^2(8k\cdot p)(4k\cdot p+\Delta^2)],\end{cases}$$

$$T_{\mathrm{VII}}\quad\begin{cases}a_{\mathrm{VII}}=M\{\Delta^2\cdot8Q^2\cdot[(8k\cdot p+\Delta^2)+(\Delta_1^2+\Delta_2^2)]\},\\ b_{\mathrm{VII}}=M\{-2\Delta^2\cdot(8k\cdot p)[(8k\cdot p+\Delta^2)+(\Delta_1^2+\Delta_2^2)]\},\end{cases}$$

$$T_{\mathrm{VIII}}\quad\begin{cases}a_{\mathrm{VIII}}=\Delta^2(4k\cdot p+4M^2)[(2(4k\cdot p)^2+4Q^2\Delta^2)+4Q^2(\Delta_1^2+\Delta_2^2)],\\ b_{\mathrm{VIII}}=0.\end{cases}$$

[30] F. J. Gilman, in Ref. 18.

[31] A recent paper by E. D. Bloom and F. J. Gilman [Phys. Rev. Letters **25**, 1140 (1970)] appears to cast some doubt on this conclusion.

[32] A. J. G. Hey, D. Phil. thesis, Oxford, 1970 (unpublished).