Using Eqs. (15) and (16) and the experimental data, we obtain

$$g_{\omega\rho\pi} = 1.42 \times 10^{-2}\ \mathrm{MeV}^{-1},$$
$$h_{\omega\rho\pi} = 0.24 \times 10^{-2}\ \mathrm{MeV}^{-1}.$$

The decay width $\Gamma(\omega \to 3\pi)$ now depends on two parameters δ and ξ. Table I shows the value of $\Gamma(\omega \to 3\pi)$ evaluated for the various choice of δ. Entries in the first row are obtained by putting $\xi = 0$, while in the second row we have given the result of Paper I, obtained with the neglect of the off-mass-shell effects in the $\omega\rho\pi$ system (again obtained by putting $\xi = 0$). Next we determine ξ by fitting the $\omega \to 3\pi$ amplitude with the decay width $\Gamma(\omega \to 3\pi) = 10.9$ MeV and use this value of ξ for predicting the value of $f_{\gamma-3\pi}$, the form factor which enters in the process $\gamma + \pi \to \pi + \pi$. Values of $f_{\gamma-3\pi}$ for the various choice of δ are shown in Table II.

We see that our results are an improvement over those of West and Brown, but are still smaller than the experimental ones if we neglect the contact terms. A comparison of the entries in the first two rows shows that off-mass-shell effects tend to decrease the $\omega \to 3\pi$ width, but are not as startling as predicted by West and Brown. The reason may be traced back to the fact that we have used the improved data and, consequently, our coupling constants $g_{\omega\rho\pi}$ and $h_{\omega\rho\pi}$ differ quite significantly from the ones used by West and Brown.

TABLE II. Form factor for $\gamma + \pi \to \pi + \pi$.

δ	0	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-1	Expt
$f_{\gamma-3\pi} \times 10^{-2}$	24.3	23.8	23.3	22.8	22.1	4 ± 15

We observe that it is not possible to determine the contact term from current algebra alone. Pending such a determination, the evaluation of $\Gamma(\omega \to 3\pi)$ is an open question. However, it seems that the pole term dominates the decay, and the contact term may not be as important as predicted by West and Brown. Finally, we remark that there is considerably uncertainty in the theory because of the variation in the ρ width in different experiments. With more accurate data on the ρ width, the contact terms could be determined to a higher degree of accuracy, and we could have a definite prediction for $f_{\gamma-3\pi}$.

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Regge Model for Inelastic Lepton-Nucleon Scattering*

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Lepton-nucleon scattering into hadrons is analyzed according to the simple Regge-pole model. Assuming $SU(3)$ for the factorized residues, we obtain from a strong-interaction analysis of tensor meson-baryon couplings $\sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma n}(\nu) / \sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma p}(\infty) = (0.26 \pm 0.05)$, to be compared with 0.31 obtained from the total photon-nucleon photoproduction data. For the inelastic structure functions, assuming all Regge poles couple in the deep-inelastic region, we find ${}^{\gamma}F_2{}^p(\rho) - {}^{\gamma}F_2{}^n(\rho) = +0.15\rho^{-1/2}$, $\rho = -\nu/q^2 > 4$. In the analogous neutrino process we establish from these assumptions ${}^{\nu}F_2{}^p(\rho) + {}^{\nu}F_2{}^n(\rho) \simeq 0.66 + (3.5 \pm 1.7)\rho^{-1/2}$, $\rho > 4$. Analyzing the Adler and Gross–Llewellyn Smith sum rules, we obtain the estimates ${}^{\bar{\nu}}F_2{}^p(\rho) - {}^{\nu}F_2{}^p(\rho) \simeq 1.5\rho^{-1/2}$, ${}^{\nu}F_3{}^p(\rho) + {}^{\nu}F_3{}^n(\rho) \simeq -8.9\rho^{1/2}$, $\rho > 4$. We also conclude that if scaling behavior of the neutrino process occurs for $-q^2 > 1\ \mathrm{GeV}^2$, then these sum rules are saturated to 90% only for energies $\nu > 200$ GeV for $-q^2 > 1\ \mathrm{GeV}^2$.

I. INTRODUCTION

IN this paper we analyze high-energy lepton-nucleon scattering into hadrons according to the simple Regge-pole model for high-energy forward scattering. In particular, we examine the photoproduction process for the total photon-nucleon cross sections and the highly inelastic region for photon- and neutrino-induced reactions. We assume that in the highly inelastic region that the inelastic form factors have the scaling behavior conjectured by Bjorken[1] and moreover that all Regge poles with the appropriate quantum numbers couple in this kinematic realm.[2] This later assumption is in accord with the observation that in the deep-inelastic region for electron-nucleon scattering there is a substantial nondiffractive component,[3] and that the observed structure function can be accounted for by trajectories other than the Pomeranchukon coupling.[2]

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TABLE I. Quantum numbers for Regge contributions to amplitudes.

Amplitude	$(-1)^J P$	$(-1)^J C$	I^G	Trajectories
${}^{\gamma}W_{1,2}{}^{p}+{}^{\gamma}W_{1,2}{}^{n}$	+	+	0^+	P, f_0, f'
${}^{\gamma}W_{1,2}{}^{p}-{}^{\gamma}W_{1,2}{}^{n}$	+	+	1^-	$A_2{}^0$
${}^{\bar{\nu}}W_{1,2}{}^{p}+{}^{\nu}W_{1,2}{}^{p}$	+	+	0^+	P, f_0, f'
${}^{\bar{\nu}}W_{1,2}{}^{p}-{}^{\nu}W_{1,2}{}^{p}$	+	+	1^+	ρ
${}^{\bar{\nu}}W_3{}^{p}+{}^{\nu}W_3{}^{p}$	+	+	0^-	ϕ, ω
${}^{\bar{\nu}}W_3{}^{p}-{}^{\nu}W_3{}^{p}$	−	−	1^-	A_1

First we examine the total cross-section data for photons on protons and neutrons[4] which indicate

$$r=\frac{\sigma_T{}^{\gamma p}(\nu)-\sigma_T{}^{\gamma n}(\nu)}{\sigma_T{}^{\gamma p}(\nu)-\sigma_T{}^{\gamma p}(\infty)}\simeq 0.31\,,$$

$$\nu=\text{photon energy/GeV}>4\,.$$

Assuming that the next-to-leading Regge poles (with the Pomeranchukon the leading trajectory) are the neutral members of the 2^+ tensor nonet $f_0(1250)$, $f'(1525)$, $A_2(1310)$, and assuming $SU(3)$ symmetry for the factorized Regge residues, we can parametrize r in terms of the f/d ratio of the tensor meson-nucleon couplings and the ratio γ_N/η_N of the singlet octet coupling to nucleons. From an analysis of the strong-interaction meson-baryon forward scattering,[5] one can obtain these ratios and one finds $r=0.26\pm0.05$ in agreement with the experimentally observed ratio. This suggests that $SU(3)$ symmetry for the factorized residues is having the same dramatic success for forward weak amplitudes that it had for the purely strong amplitudes.[6]

We also discuss the implications of the Regge model for the deep-inelastic region $\nu\to\infty$, $-q^2\to\infty$, $\rho=-\nu/q^2$ fixed, where q_μ=momentum transfer of the lepton. For the photon-induced process, we fit the reported data,[7] with R=ratio of longitudinal to transverse cross sections=0, for the inelastic structure function,

$${}^{\gamma}F_2{}^{p}(\rho)=0.11+0.48\rho^{-1/2}, \quad \rho>4$$

where the constant term is due to Pomeranchukon exchange and the rate of falloff is accounted for by the exchange of 2^+ nonet trajectories. Assuming $SU(3)$ for the residues, we obtain for the difference

$${}^{\gamma}F_2{}^{p}(\rho)-{}^{\gamma}F_2{}^{n}(\rho)=0.15\rho^{-1/2}, \quad \rho>4$$

consistent with the rough data reported.[8]

Deep-inelastic neutrino scattering is analyzed in the usual $V-A$ weak-interaction theory with the neglect of the final lepton mass and strangeness-changing interactions. Using the data for inelastic electron scattering and assuming $SU(3)$ for the couplings of the trajectories and the equality of vector and axial-vector transitions, we predict

$${}^{\nu}F_2{}^{\nu}(\rho)+{}^{\nu}F_2{}^{n}(\rho)\simeq 0.66+(3.5\pm1.7)\rho^{-1/2}, \quad \rho>4\,.$$

Finally, we examine the saturation of the Adler sum rule[9] in the deep inelastic region and the Gross–Llewellyn Smith (GLS) sum rule.[10] Assuming the validity of these sum rules, we obtain the estimates

$${}^{\bar{\nu}}F_2{}^{p}(\rho)-{}^{\nu}F_2{}^{p}(\rho)\simeq 1.5\rho^{-1/2},$$

$${}^{\nu}F_2{}^{p}(\rho)+{}^{\nu}F_3{}^{n}(\rho)\simeq -8.9\rho^{1/2}, \quad \rho>4\,.$$

If these estimates are valid and if the structure functions scale for $-q^2\gtrsim 1$ GeV^2 as indicated by the electron scattering experiments, then for $-q^2\gtrsim 1(\text{GeV})^2$, it is necessary to go to energies $\gtrsim 200$ GeV to obtain 90% saturation of the sum rules. This is to be contrasted with the estimate of Adler and Gilman,[11] who showed that for $-q^2\simeq 0.1$ GeV^2 the Adler sum rule was effectively saturated at 5 GeV.

II. PHOTOPRODUCTION AND ELECTROPRODUCTION

First we develop the Regge-pole-model analysis of photoproduction and electroproduction. The differential cross sections for electrons scattering on an unpolarized nucleon target into any hadronic state with one-photon exchange is specified in terms of two inelastic form factors:

$$\frac{d\sigma}{d\Omega dE'}=\frac{\alpha^2}{4E^2\sin^4(\frac{1}{2}\theta)}\times[W_1(q^2,\nu)\cos^2(\tfrac{1}{2}\theta)+2W_1(q^2,\nu)\sin^2(\tfrac{1}{2}\theta)],$$

where E=initial electron energy, E'=final electron energy, θ=scattering angle, $q^2=-4EE'\sin^2(\frac{1}{2}\theta)$, $\nu=q\cdot p/M=E-E'$, p_μ=nucleon momentum, $\alpha=1/137$. By the optical theorem, $W_i(q^2,\nu)$ are the absorptive parts of the forward virtual Compton amplitudes and can be used to define the total cross sections for transversely and longitudinally polarized photon incident on a hadron target. Defining $W_T(q^2,\nu)=W_1(q^2,\nu)$ and $W_L(q^2,\nu)=(1-\nu^2/q^2)W_2(q^2,\nu)-W_1(q^2,\nu)$, one has

$$4\pi^2\alpha W_T(q^2,\nu)=(\nu-|q^2|/2M)\sigma_T(q^2,\nu)\,,$$

$$4\pi^2\alpha W_L(q^2,\nu)=(\nu-|q^2|/2M)\sigma_L(q^2,\nu)\,.$$

At $q^2=0$, the photoproduction limit $\sigma_L(0,\nu)=0$ and $\lim_{q^2\to0}(-\nu^2/q^2)W_2(q^2,\nu)=W_1(0,\nu)=(\nu/4\pi^2\alpha)\sigma_T$ determines the total photon-nucleon cross section $\sigma_T(\nu)$.

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The Regge model for total photoabsorption has $\sigma_T(\nu) = \sum_i b_i \nu^{\alpha_i - 1}$, $\nu \to \infty$ where we will set the mass scale by normalizing the nucleon mass $M=1$, and the α_i are the $t=0$ interecpts of the leading trajectories. From Table I, which gives the quantum numbers of the contributing trajectories, we include the Pomeranchukon at $\alpha_P=1$ and the neutral 2^+ nonet numbers f_0, f', and $A_2{}^0$ with $\alpha_i \simeq \frac{1}{2}$ so that in the transresonance region observed at $\nu > 4$ GeV,

$$\sigma_T{}^{\gamma p}(\nu) = b_P \nu^{\alpha_P - 1} + b_{f_0} \nu^{\alpha_{f0}-1} + b_{f'} \nu^{\alpha_{f'}-1} + b_{A_2} \nu^{\alpha_{A2}-1}$$

and

$$\sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma n}(\nu) = 2 b_{A_2} \nu^{\alpha_{A2}-1}. \tag{1}$$

The experimental data[4] can be parametrized with a Regge fit according to

$$\sigma_T{}^{\gamma p}(\nu) = 94 + 79\nu^{-1/2}\ \mu b,$$
$$\sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma n}(\nu) = 24.6\nu^{-1/2}\ \mu\text{b}, \quad \nu > 4 \text{ GeV}$$

so that

$$r(\text{expt}) = \frac{\sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma n}(\nu)}{\sigma_T{}^{\gamma p}(\nu) - \sigma_T{}^{\gamma p}(\infty)} = 0.31.$$

In $SU(3)$ symmetry, assuming the nonet to be degenerate, we have $\alpha_{f_0} = \alpha_{f'} = \alpha_{A_2}$, so from (1)

$$r^{-1} = \frac{1}{2}\left(1 + \frac{b_{f_0}}{b_{A2}} + \frac{b_{f'}}{b_{A2}}\right). \tag{2}$$

If we further assume factorization of the Regge residues $b_{f'} = \gamma_{f'p}\gamma_{f'2\gamma}$, $b_{f_0} = \gamma_{f_0 p}\gamma_{f_0 2\gamma}$, $b_{A_2} = \gamma_{A_2 p}\gamma_{A_2 2\gamma}$, and use the strong-interaction analysis[5] of the tensor-nucleon couplings which specify the γ_{T_p} (here $f' = T_8$, the octet member, and $f_0 = T_1$, the $SU(3)$ singlet), then

$$\gamma_{f'p} = (1/\sqrt{3})(3f-d)\gamma_N, \quad \gamma_{A_2 p} = (f+d)\gamma_N,$$
$$\gamma_{f_0 p} = \sqrt{2}\eta_N, \quad f+d=1,$$

with $f/d = -1.90 \pm 0.1$, $\gamma_N = 1.18 \pm 0.09$, and $\eta_N = 2.17 \pm 0.8$.[5] For the tensor-meson two-photon reduced residue, we have $\gamma_{\alpha 2\gamma} = \gamma d_{\alpha QQ}$ with $Q = 3 + 8/\sqrt{3}$. These assumptions imply

$$\frac{b_{f'}}{b_{A_2}} = \frac{1}{3}\frac{3f-d}{f+d}, \quad \frac{b_{f_0}}{b_{A_2}} = \frac{4\eta_N}{\sqrt{3}(f+d)\gamma_N}.$$

For the ratio r, we obtain from (2) and the ratios f/d and γ_N/η_N,

$$r = 0.26 \pm 0.05,$$

in agreement with the experimentally observed ratio 0.31. This analysis suggests the applicability of $SU(3)$ for the factorized residues.

The structures functions are observed to have the Bjorken scaling behavior[1] for $-q^2 > 1$ GeV.[2] The scaling hypothesis is that for $\nu \to \infty$, $-q^2 \to \infty$, $\rho = -\nu/q^2$ fixed

$$\nu W_2(q^2,\nu) \to {}^{\gamma}F_2(\rho),$$
$$W_1(q^2,\nu) \to {}^{\gamma}F_1(\rho).$$

Here we discuss only the structure function $F_2(\rho)$ since the experimental data indicate for the ratio

$$R = F_L(\rho)/F_T(\rho) = [\rho F_2(\rho) - F_1(\rho)]/F_1(\rho) \simeq 0.18 \pm 0.18,$$

so that $\rho F_2(\rho) \simeq F_1(\rho)$. If we assume that all Regge poles with allowable quantum numbers couple, we expect that as $\rho \to \infty$,

$$\begin{aligned} {}^{\gamma}F_2{}^p(\rho) &= a_P \rho^{\alpha_P - 1} + a_{f_0}\rho^{\alpha_{f0}-1} + a_{f'}\rho^{\alpha_{f'}-1} + a_{A_2}\rho^{\alpha_{A2}-1}, \\ {}^{\gamma}F_2{}^p(\rho) - {}^{\gamma}F_2{}^n(\rho) &= 2a_{A_2}\rho^{\alpha_{A2}-1}, \quad \rho > 4. \end{aligned} \tag{3}$$

Since the form factors apparently scale for $-q^2 > 1$ GeV^2 and the photoproduction data indicate that the Regge region is $\nu > 4$ GeV, we expect (3) to hold for $\rho = -\nu/q^2 > 4$.

With $\alpha_P = 1$, $\alpha_{f'} = \alpha_{f_0} = \alpha_{A_2} \simeq \frac{1}{2}$, we have from (3)

$${}^{\gamma}F_2{}^p(\rho) = a_P + (a_{f_0} + a_{f'} + a_{A_2})\rho^{-1/2}.$$

The recent electroproduction data[8] can be fitted with $a_P = 0.275$, $a_{f_2} + a_{f'} + a_{A_2} = 0.13$. However, it should be remarked that the precise values obtained are dependent on the ratio $R = {}^{\gamma}F_L{}^p(\rho)/{}^{\gamma}F_T{}^p(\rho)$, for which, for the above values, we have used $R = 0.18$. For $R=0$ we have instead[7] $a_P = 0.11$, $a_{f_0} + a_{f'} + a_{A_2} = 0.48$. Assuming $SU(3)$ and following the analysis for photoproduction, with the only change being that the reduced residue for the tensor-meson two-photon coupling is $\gamma_{\alpha 2\gamma} = \gamma' d_{\alpha QQ}$, which does not change the ratios of the couplings, we have

$$\frac{{}^{\gamma}F_2{}^p(\rho) - {}^{\gamma}F_2{}^n(\rho)}{{}^{\gamma}F_2{}^p(\rho) - {}^{\gamma}F_2{}^p(\infty)} = r = 0.31.$$

Using $a_{f_0} + a_{f'} + a_{A_2} = 0.13$ ($R = 0.18$), we have $2a_{A_2} = 0.04$, and with $a_{f_0} + a_{f'} + a_{A_2} = 0.48$ ($R=0$), we have $2a_{A_2} = 0.15$, so

$$\begin{aligned} {}^{\gamma}F_2{}^p(\rho) - {}^{\gamma}F_2{}^n(\rho) &= 0.15\rho^{-1/2} \quad (R=0) \\ &= 0.04\rho^{-1/2} \quad (R=0.18), \\ &\rho > 4. \end{aligned}$$

This result is in agreement with the rough data reported.[8]

III. HIGHLY INELASTIC NEUTRINO-NUCLEON SCATTERING

The differential cross section for neutrino and antineutrino incident on a target nucleon, ngelecting the final lepton mass, is specified in the $V-A$ theory to lowest order in G= Fermi constant by

$$\frac{\pi}{MEE'}\frac{d^2\sigma^{\nu,\bar\nu}}{d\Omega dE'} = \frac{E'G^2}{2M^2E\pi}\left[{}^{\nu,\bar\nu}W_2(q^2,\nu)\cos^2(\tfrac{1}{2}\theta) + 2\,{}^{\nu,\bar\nu}W_1(q^2,\nu)\sin^2(\tfrac{1}{2}\theta) \pm \frac{E+E'}{M}\,{}^{\nu,\bar\nu}W_3(q^2,\nu)\sin^2(\tfrac{1}{2}\theta)\right].$$

If we neglect strangeness-changing interactions, approximating the Cabibbo angle $\sin^2\theta_C \simeq 0$, we obtain by isospin rotation

$$\begin{aligned} {}^{\nu}W_i^{p}(q^2,\nu) &= {}^{\bar\nu}W_i^{n}(q^2,\nu)\,, \\ {}^{\bar\nu}W_i^{p}(q^2,\nu) &= {}^{\nu}W_i^{n}(q^2,\nu)\,. \end{aligned} \tag{4}$$

This approximation, $\sin^2\theta_C \simeq 0$, can be tested experimentally by scattering on deuterium targets. Assuming small additivity corrections at high energy, we expect on this approximation

$${}^{\nu}W_i^{d}(q^2,\nu) = {}^{\bar\nu}W_i^{d}(q^2,\nu)$$

as $\nu \to \infty$.

Bjorken has conjectured[1] the scaling behavior for the inelastic form factors as $-q^2 \to \infty$, $\nu \to \infty$, $\rho = -\nu/q^2$ fixed:

$$\begin{aligned} {}^{\nu}W_1(q^2,\nu) &\to {}^{\nu}F_1(\rho)\,, \\ \nu\, {}^{\nu}W_2(q^2,\nu) &\to {}^{\nu}F_2(\rho)\,, \\ \nu\, {}^{\nu}W_3(q^2,\nu) &\to {}^{\nu}F_3(\rho)\,. \end{aligned}$$

From the standard Regge theory we expect for these amplitudes as $\nu \to \infty$, q^2 fixed: ${}^{\nu}W_1(q^2,\nu) \to \nu^{\alpha}$, ${}^{\nu}W_2(q^2,\nu) \to \nu^{\alpha-2}$, and ${}^{\nu}W_3(q^2,\nu) \to \nu^{\alpha-1}$. If we assume that all the trajectories with the allowed quantum numbers couple in the scaling limit, we then obtain from Table I and Eq. (4)

$$\begin{aligned} {}^{\nu}F_1^{p}(\rho) + {}^{\nu}F_1^{n}(\rho) = 2(c_P{}^V + c_P{}^A)\rho^{\alpha_P} &+ 2(c_{f_0}{}^V + c_{f_0}{}^A)\rho^{\alpha_{f0}} \\ &+ 2(c_{f'}{}^V + c_{f'}{}^A)\rho^{\alpha_{f'}}, \end{aligned} \tag{5a}$$

$${}^{\nu}F_1^{p}(\rho) - {}^{\bar\nu}F_1^{p}(\rho) = c_\rho \rho^{\alpha_\rho}\,, \tag{5b}$$

$$\begin{aligned} {}^{\nu}F_2^{p}(\rho) + {}^{\nu}F_2^{n}(\rho) = 2(a_P{}^V + a_P{}^A)\rho^{\alpha_P - 1} & \\ + 2(a_{f_0}{}^V + a_{f_0}{}^A)\rho^{\alpha_{f0} - 1} & \\ + 2(a_{f'}{}^V + a_{f'}{}^A)\rho^{\alpha_{f'} - 1}, & \end{aligned} \tag{5c}$$

$${}^{\bar\nu}F_2^{p}(\rho) - {}^{\nu}F_2^{p}(\rho) = a_\rho \rho^{\alpha_\rho - 1}\,, \tag{5d}$$

$${}^{\nu}F_3^{p}(\rho) + {}^{\nu}F_3^{n}(\rho) = a_\omega \rho^{\alpha_\omega} + a_\phi \rho^{\alpha_\phi}\,, \tag{5e}$$

$${}^{\bar\nu}F_3^{p}(\rho) - {}^{\nu}F_3^{p}(\rho) = a_{A_1} \rho^{\alpha_{A_1}}\,, \tag{5f}$$

$$\begin{aligned} {}^{\gamma}F_2^{p}(\rho) + {}^{\gamma}F_2^{n}(\rho) = (a_P{}^V + a_P{}^S)\rho^{\alpha_P - 1} &+ (a_{f_0}{}^V + a_{f_0}{}^S)\rho^{\alpha_{P0} - 1} \\ &+ (a_{f'}{}^V + a_{f'}{}^S)\rho^{\alpha_{f'} - 1}. \end{aligned} \tag{5g}$$

Here $c_i{}^{V,A}$ and $a_i{}^{V,A}$ refer to the vector and axial-vector current couplings, respectively, and $a_i{}^S$ refers to the coupling of the isoscalar photons. That the same couplings $a_i{}^V$ for the vector currents appear in a neutrino process and a photon process in (5c) and (5g) follows from an isotopic rotation. If the photon process is any indication of the behavior of the neutrino process, we expect the relations (5) to hold for $-q^2 > 1$ GeV^2 and $\rho > 4$. Here we have $\alpha_P = 1$ and all other trajectories $\alpha_i \simeq \frac{1}{2}$.

Assuming $SU(3)$ symmetry for the factorized residues and the equality of the vector and axial vector coupling which would follow from asymptotic $SU(2)\times SU(2)$ symmetry or the quark model,

$$a_i{}^V = a_i{}^A, \quad c_i{}^V = c_i{}^A, \tag{6}$$

we can give a prediction for ${}^{\nu}F_2^{p}(\rho) + {}^{\nu}F_2^{n}(\rho)$ based on the observed behavior of ${}^{\gamma}F_2^{p}(\rho) + {}^{\gamma}F_2^{n}(\rho)$. Our assumptions also require

$${}^{\nu}F_L{}^{(p+n)}(\rho) = \rho\, {}^{\nu}F_2{}^{(p+n)}(\rho) - {}^{\nu}F_1{}^{(p+n)}(\rho) \simeq 0$$

if the longitudinal amplitude is small for the photon process; so we will discuss only ${}^{\nu}F_2{}^{(p+n)}(\rho)$ with this expectation.

From $SU(3)$ symmetry and the assumption the Pomeranchukon and f_0 are $SU(3)$ singlets and the f' a member of an octet, we have a relation between the isovector and isoscalar amplitudes:

$$a_P{}^S = \tfrac{1}{3} a_P{}^V, \quad a_{f_0}{}^S = \tfrac{1}{3} a_{f_0}{}^V, \quad a_{f'}{}^S = -\tfrac{1}{3} a_{f'}{}^V. \tag{7}$$

Combining this result with (3),

$$2a_P = \tfrac{4}{3} a_P{}^V, \quad 2a_{f_0} = \tfrac{4}{3} a_{f_0}{}^V, \quad 2a_{f'} = \tfrac{2}{3} a_{f'}{}^V. \tag{8}$$

In order to obtain a_{f_0} and $a_{f'}$ from the photon-proton data which measure the combination $a = a_{f_0} + a_{f'} + a_{A_2}$, we use factorization and $SU(3)$ symmetry to obtain

$$\begin{aligned} 2a_{f'} &= a(3f-d)(3f+d+2\sqrt{3}\eta_N/\gamma_N)^{-1}, \\ a_{f_0} &= a[1+(3f+d)\gamma_N/2\sqrt{3}\eta_N]^{-1}. \end{aligned} \tag{9}$$

With $\alpha_P = 1$ and $\alpha_{f'} = \alpha_{f_0} = \frac{1}{2}$, we write ${}^{\nu}F_2^{p}(\rho) + {}^{\nu}F_2^{n}(\rho) = A + B\rho^{-1/2}$; and from (6), (8), and (9), we obtain

$$\begin{aligned} A &= 2(a_P{}^V + a_P{}^A) = 6a_P\,, \\ B &= 2(a_{f_0}{}^V + a_{f_0}{}^A) + 2(a_{f'}{}^V + a_{f'}{}^A) \\ &= 6a\{(3f-d)(3f+d+2\sqrt{3}\eta_N/\gamma_N)^{-1} \\ &\quad + [1+(3f+d)\gamma_N/2\sqrt{3}\gamma_N]^{-1}\} = 6a(1.2\pm0.6)\,, \end{aligned}$$

with $f/d = -1.90\pm0.1$, $\gamma_N = 1.18\pm0.09$, $\eta_N = 2.17\pm0.8$.[5] For $R = 0.18$, we have $a_P = 0.275$, $a = 0.13$, so $A = 1.65$, $B = 0.94\pm0.47$; for $R = 0$, we have $a_P = 0.11$, $a = 0.48$ and $A = 0.66$, $B = 3.5\pm1.7$, so

$$\begin{aligned} {}^{\nu}F_2^{p}(\rho) + {}^{\nu}F_2^{n}(\rho) &\simeq 0.66 + (3.5\pm1.7)\rho^{-1/2}, \\ (R=0,\ \ \rho>4)&. \end{aligned} \tag{10}$$

This result for the numerical coefficients is dependent on the analysis of the proton-photon data with $R = 0$ or 0.18; however, the qualitative feature that the Pomeranchukon limit is reached from above is only dependent on the observation that apparently this limit is reached from above in the electron-scattering experiment.

This result can be tested by neutrino scattering on nuclei with equal numbers of protons and neutrons (assuming simple additivity of the scattering amplitude). Statistical errors are still too large on the present data to reach a definite conclusion but (10) is completely consistent with these rough data.[12]

IV. SUM RULES

We can also examine the implications of the Adler sum rule[9] and the GLS sum rule.[10] The Adler sum rule

[12] I. Budagov *et al.*, Phys. Letters **30B**, 364 (1969).

can be derived on the assumption of the time-time component current-algebra commutator and the absence of subtractions in certain current-nucleon scattering amplitudes. This rule reads

$$\int_0^\infty d\nu[{}^{\bar\nu}W_2{}^p(q^2,\nu)-{}^{\nu}W_2{}^p(q^2,\nu)]=2\cos^2\theta_C\simeq 2.$$

For $q^2\simeq -0.1$ GeV^2, Adler and Gilman[11] concluded that for $\nu_0=5$ GeV this rule would be 90% saturated. We will examine here the saturation of the sum rule in the scaling region which may be expected to set in around $-q^2\sim 1$ GeV^2. Assuming scaling behavior and the uniform convergence of the integral as $-q^2\to\infty$, we have

$$\int_{1/2}^\infty \frac{d\rho}{\rho}[{}^{\bar\nu}F_2{}^p(\rho)-{}^{\nu}F_2{}^p(\rho)]\simeq 2. \tag{11}$$

Using (5d) and $\alpha_\rho=\frac{1}{2}$, we have for $\rho>4$, ${}^{\bar\nu}F_2{}^p(\rho)-{}^{\nu}F_2{}^p(\rho)=a_\rho\rho^{-1/2}$. Since these structure functions vanish at threshold $\rho=\frac{1}{2}$ and at $\rho=4$ are given by $a_\rho/2$, we will use a linear form ${}^{\nu}F_2{}^p(\rho)-{}^{\nu}F_2{}^p(\rho)=(\rho-\frac{1}{2})(a_\rho/7)$, $\frac{1}{2}<\rho<4$ to estimate the integral. From the sum rule (11), we then obtain the estimate $a_\rho\simeq 1.5$. It should be remarked that 75% of the contribution to the integral is for $\rho>4$ so that our estimate does not depend too critically on the extrapolation for $\frac{1}{2}<\rho<4$. Hence we predict

$${}^{\bar\nu}F_2{}^p(\rho)-{}^{\nu}F_2{}^p(\rho)\simeq 1.5\rho^{-1/2},\quad \rho>4.$$

We may also conclude on the basis of this estimate that the sum rule is saturated to 90% for $\rho_0=a_\rho{}^2\times 10^2=225$. If scaling sets in at $-q^2\simeq 1$ GeV^2, then one must go to energies $\nu_0=-q^2\rho_0=\rho_0$ (GeV)$\sim$225 GeV to achieve 90% saturation. The scaling feature of the structure functions makes it difficult even at National Accelerator Laboratory energies to test this rule.[13] The question of saturation for $-q^2<1$ GeV^2 is difficult to answer, depending in detail on how the resonance, which give the dominant contribution for small $-q^2$, behave in this region.

The GLS sum rule is based on quark-model current commutators and the existence of the first Bjorken-Johnson-Low[14] limit. For nucleons with $\theta_C=0$ it reads

$$\int_{1/2}^\infty \frac{d\rho}{\rho^2}[{}^{\nu}F_3{}^p(\rho)+{}^{\bar\nu}F_3{}^p(\rho)]=-12. \tag{12}$$

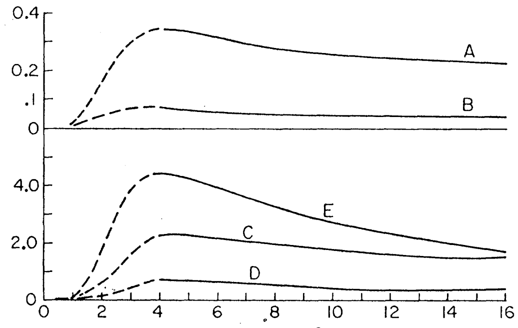

FIG. 1. Inelastic structure functions in the Regge model. The solid line for $\rho>4$ is the Regge region and the dashed lines represent extrapolations to $\rho=\frac{1}{2}$. *A*: ${}^{\gamma}F_2{}^p=0.11+0.48\rho^{-1/2}$ (input); *B*: ${}^{\gamma}F_2{}^p-{}^{\gamma}F_2{}^n=0.15\rho^{-1/2}$; *C*: ${}^{\nu}F_2{}^p+{}^{\nu}F_2{}^n=0.66+3.5\rho^{-1/2}$; *D*: ${}^{\bar\nu}F_2{}^p-{}^{\nu}F_2{}^p=1.5\rho^{-1/2}$; *E*: $({}^{\nu}F_3{}^p+{}^{\nu}F_3{}^n)/(-\rho)=8.9\rho^{-1/2}$.

From (5e) with $\alpha_\omega=\alpha_\phi=\frac{1}{2}$, we have, for $\rho>4$, ${}^{\nu}F_3{}^p(\rho)+{}^{\bar\nu}F_3{}^p(\rho)=(a_\omega+a_\phi)\rho^{1/2}$. Using a linear form for $\frac{1}{2}<\rho<4$, we obtain, using (12), $a_\omega+a_\phi\simeq -8.9$, so

$${}^{\nu}F_3{}^p(\rho)+{}^{\bar\nu}F_3{}^p(\rho)\simeq -8.9\rho^{-1/2},\quad \rho>4.$$

In order to test this sum rule, we would conclude, as in the case of the Adler sum rule, that one requires an energy $\nu_0>220$ GeV for $-q^2>1$ GeV^2.

We may also examine the Bjorken backward-scattering sum rule[15] which, when combined with Adler's sum rule, implies

$$\int_{1/2}^\infty \frac{d\rho}{\rho^2}\,{}^{\nu}F_L{}^{(p-n)}(\rho)=0. \tag{13}$$

Since our other assumptions imply ${}^{\nu}F_L{}^{(p+n)}(\rho)=0$ if ${}^{\gamma}F_L{}^{(p+n)}(\rho)=0$, we conclude from this and (12) and the positivity of ${}^{\nu}F_L{}^{(p,n)}(\rho)$ that ${}^{\nu}F_L{}^{(p,n)}(\rho)=0$, so

$$\rho\,{}^{\nu}F_2{}^{(p,n)}(\rho)={}^{\nu}F_1{}^{(p,n)}(\rho).$$

In conclusion, we see that the Regge model, when combined with $SU(3)$ symmetry and sum rules, gives estimates for the structure functions which are subject to experimental tests. Our estimates for the various combination of the structure functions are plotted in Fig. 1. The primary uncertainty is in the measured rate of falloff for the electron-proton scattering amplitude depending on R, and greater experimental resolution in this parameter is desired.

[13] I would like to thank Dr. E. A. Paschos for discussions on the saturation of the Adler sum rule. See also E. A. Paschos, NAL Summer Report No. SS-175-2022, 1970 (unpublished).

[14] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966). K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. **37–38**, 74 (1966).

[15] J. D. Bjorken, Phys. Rev. **163**, 1767 (1967).