

Direct-Channel Resonance Model of Deep-Inelastic Electron Scattering. II.* Spin Dependence of the Cross Section

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The spin dependence of the cross section of deep-inelastic electron (muon) scattering is investigated in the framework of the direct-channel resonance model. The structure functions entering the spin-dependent part of the cross section obey the following scaling laws in the Bjorken limit: νW_2 is a function of ω , and $\nu W_4 = W_3$. The polarization asymmetry is calculated in the framework of a quark model with orbital excitations; sizable asymmetries are predicted. The spin-dependent part of the cross section is found to be strongly model dependent; therefore, polarized-beam-polarized-target experiments should be able to distinguish between various models.

I. INTRODUCTION

INELASTIC scattering experiments with charged, polarized lepton beams on polarized nucleon targets have become feasible lately.^{1,2} From a theoretical point of view, this means that by measuring the cross section for inelastic electron (or muon) scattering at various incident lepton energies and scattering angles and at two different relative orientations of the target and beam spin, the amplitude of virtual forward Compton scattering can be—in principle—completely determined. There exist already several works in which the spin dependence of the Compton amplitude is investigated either by using general principles (locality, etc.) or specific models.³⁻⁵ In Paper I of this work⁶ we have demonstrated that at least a substantial part of the experimental data on the spin-averaged cross section can be explained in terms of nucleon resonance contributions. The model constructed in I described the inelastic electron scattering data on unpolarized proton targets in terms of one free parameter, and predicted—without free parameters—the difference between the structure functions of the proton and neutron. Good agreement was obtained between the experimental data and the predictions of the theory in the region of Bjorken's scaling variable $\omega = 2m\nu/q^2$ [or rather the modified variable, $\omega' = (2m\nu + m^2)/q^2$], where scale invariance is well established experimentally ($\omega' \lesssim 9$).

The purpose of the present work is to investigate the scattering of polarized electron (or muon) beams on polarized targets in the framework of the resonance model described in I.

We find—as may be expected—that the spin-depen-

dent part of the virtual Compton scattering amplitude is quite sensitive to the details of a resonance model. In particular, the behavior of the respective structure functions and the polarization asymmetry depend quite sensitively on the assumptions one makes about the transition form factors. This is in sharp contrast with the spin-averaged cross section, where almost any reasonable model is able to reproduce the observed scaling behavior of the structure functions and more or less their shape as well. We believe that this is not a specific feature of the resonance model, but rather reflects a general physical property of the virtual Compton amplitude; this further underlines the importance of performing polarized-beam-polarized-target experiments in order to gain insight into the structure of hadrons.

Our basic assumptions are the same as in I; we list them here only briefly.

(A) The nucleon spectrum is “oscillatorlike”; the masses squared and total widths of the states satisfy the approximate semiempirical formulas

$$s_n = 1 + n, \quad (1.1)$$

$$s_n^{1/2} \Gamma_n = 0.13(s_n - 1). \quad (1.2)$$

Even and odd values of n correspond to $I = \frac{3}{2}$ and $I = \frac{1}{2}$ resonances, respectively. Each level characterized by a fixed value of n contains several states with spins ranging from $\frac{1}{2}$ or $\frac{3}{2}$ to $n + \frac{1}{2}$ and of both normalities.

(B) The transition form factors $G(q^2)$ are—essentially—universal functions of the variable $x = q^2/s_n$. We choose a “dipole form,” viz., $G(x) \propto (1 + r^2 x)^{-2}$, where $r^2 \approx 1.41$ from a fit to elastic electron-proton scattering.

(C) At high resonance masses, each decay channel has the same relative weight; hence, the relative weight—as defined in I—is inversely proportional to the number of channels, i.e., roughly to $s_n^{-1/2}$.

Clearly, none of these assumptions is satisfied exactly in nature. Nevertheless, it is hoped that in those kinematic regions where many resonances give big contributions to the virtual Compton amplitude, the finer details of the individual resonance terms are washed out and thus a model based on these assump-

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¹ V. Hughes, M. Lubell, M. Posner, and W. Raith, in Proceedings of the Sixth International Conference on High-Energy Accelerators, Cambridge, Mass., 1967 (unpublished).

² J. R. Chen, T. Kick, D. Kreinick, and J. T. Sanderson, AGS Report, 1970 (unpublished).

³ L. Galfi, J. Kuti, and A. Patkos, Phys. Letters **31B**, 465 (1970).

⁴ J. D. Bjorken, Phys. Rev. D **1**, 1376 (1970).

⁵ L. Galfi, P. Gnadig, J. Kuti, F. Niedermayer, and A. Patkos, ITP-Budapest Report No. 281, 1970 (unpublished).

⁶ G. Domokos, S. Kovesi-Domokos, and E. Schonberg, preceding paper, Phys. Rev. D **3**, 1184 (1971).

tions is quite adequate. The basic advantage of these assumptions is their simplicity; this allows the summation over resonances to be carried out relatively easily.

Section II is devoted essentially to the kinematics of the virtual Compton amplitude, including the spin dependence. The appropriate structure functions are expressed through the contributions of the nucleon resonances. We discover the reason for the strong model dependence of the spin-dependent amplitude in that the structure functions contain the *difference* of two terms of comparable magnitude.

In Sec. III, the structure functions and the asymmetry in the electron scattering cross sections are estimated in the framework of a relativistic quark model with orbital excitations. Even though we do not know the exact form of the level density, we get a reasonable estimate by making use of the fact that at high energies the strength function must peak around the semiclassical value of the angular momentum. The resulting asymmetry is quite sizable; we calculate it both in the limit of an exactly symmetric quark model and also adding a small symmetry breaking which accounts for the small ($\approx 5\%$) electric quadrupole term in Δ photoproduction.

The results are discussed in Sec. IV. Throughout this paper we use consistently the same—essentially standard—notation as in I. The units are so chosen that the mass of the nucleon is equal to 1.

II. SPIN-DEPENDENT PART OF VIRTUAL COMPTON AMPLITUDE IN RESONANCE MODEL

Let $M_{\mu\nu}$ be the absorptive part of the spinor amplitude of forward virtual Compton scattering. We define the function $W_{\mu\nu}$ by

$$W_{\mu\nu} = \text{Tr}[M_{\mu\nu}P(\xi)], \quad (2.1)$$

where ξ_μ is the axial vector describing the polarization of the target, and $P(\xi)$ is the covariant density matrix. In the rest frame of the target, $\xi_0=0$, $P(\xi) \rightarrow \frac{1}{2}(1 + \boldsymbol{\sigma} \cdot \boldsymbol{\xi})$, the latter being the conventional, “nonrelativistic” form of the density matrix.

This is sufficient to define the density matrix completely, although its explicit expression depends on the normalization of the nucleon spinor. (However, $W_{\mu\nu}$ is, of course, independent of the normalization convention used.)

The tensor $W_{\mu\nu}$ can be decomposed into invariant amplitudes as follows⁷:

$$\begin{aligned} W_{\mu\nu} = & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1 + \left(\hat{p}_\mu - \frac{\hat{p} \cdot q}{q^2} q_\mu \right) \left(\hat{p}_\nu - \frac{\hat{p} \cdot q}{q^2} q_\nu \right) W_2 \\ & + 2i[\epsilon_{\mu\nu\sigma\tau} \hat{p}^\sigma \xi^\tau (\hat{p} \cdot q) + \hat{p}_\mu \epsilon_{\nu\sigma\tau\rho} \hat{p}^\sigma \xi^\tau q^\rho \\ & - \hat{p}_\nu \epsilon_{\mu\sigma\tau\rho} \hat{p}^\sigma \xi^\tau q^\rho] W_3 + 2i(\epsilon_{\mu\nu\sigma\tau} \hat{p}^\sigma \xi^\tau q^2 + q_\mu \epsilon_{\nu\sigma\tau\rho} \hat{p}^\sigma \xi^\tau q^\rho \\ & - q_\nu \epsilon_{\mu\sigma\tau\rho} \hat{p}^\sigma \xi^\tau q^\rho) W_4. \quad (2.2) \end{aligned}$$

⁷ S. D. Drell and J. D. Sullivan, Phys. Rev. **154**, 1477 (1967), and Ref. 3. The amplitude W_4 as defined in Ref. 3 differs from ours in a factor $(\hat{p} \cdot q)$.

Assuming a lepton beam of 100% longitudinal polarization, the differential cross section of the inelastic lepton scattering becomes

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \{ W_2 + 2 \tan^2(\frac{1}{2}\theta) W_1 + 4\xi \tan^2(\frac{1}{2}\theta) \times [-W_3(E+E' \cos\theta) + q^2 W_4] \}. \quad (2.3)$$

In the last equation, ξ stands for the magnitude of the target polarization, *together with its sign* relative to the polarization of the beam. It is assumed that the target is polarized parallel or antiparallel to the beam; the general formula for the cross section has been written out, e.g., in Ref. 8.

The theoretical polarization asymmetry A is the difference between the cross sections with target spin “up” and “down” divided by their sum, assuming $\xi=1$, from Eq. (2.3):

$$A = 4 \tan^2(\frac{1}{2}\theta) \frac{-W_3(E+E' \cos\theta) + W_4}{W_2 + 2 \tan^2(\frac{1}{2}\theta) W_1}. \quad (2.4)$$

Hence a measurement of the spin-averaged cross section and A under various kinematic conditions allows one to determine the “spin-dependent structure functions” W_3 and W_4 .

In order to determine the resonance contributions to the structure functions W_3 and W_4 , we proceed in the same way as in I. The matrix element of the current between a resonance of total momentum, spin, helicity, and normality equal to P, j, Λ, N , respectively, and a nucleon of momentum \hat{p} , helicity λ , is written as⁹

$$\begin{aligned} \langle P j \Lambda N | j_\mu(0) | \hat{p} \lambda \rangle = & \left[\frac{s_n}{(\mathbf{P}^2 + s_n)(\mathbf{p}^2 + 1)} \right]^{1/4} \\ & \times \bar{\psi}_\Lambda^{\alpha_1 \dots \alpha_{j-1/2}}(P) q_{\alpha_2} \dots q_{\alpha_{j-1/2}} \left\{ q_{\alpha_1} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) G_1 \right. \\ & \left. + 2S_{\alpha_1 \mu} G_2 + i(\sqrt{s_n}) q_{\alpha_1} \epsilon_{\mu\rho\sigma\tau} \gamma_5 \gamma^\rho \hat{p}^\sigma q^\tau (G_2 + G_3) \right\} \\ & \times \left(\frac{1 + \gamma_5}{2} + N \frac{1 - \gamma_5}{2} \right) u_\lambda(\hat{p}), \quad (2.5) \end{aligned}$$

where $N = \pm 1$ is the normality of the resonance. Here $\psi^{\alpha_1 \dots \alpha_{j-1/2}}(P)$ and $u_\lambda(\hat{p})$ are the usual plane-wave spinors of the resonance and the nucleon.

In the approximation described in I [state vectors approximated by those of a single particle, the δ function $\delta(P^2 + s_n)$ smoothed out to a Breit-Wigner factor], the contribution of a resonance to $W_{\mu\nu}$ can be written as

$$\begin{aligned} W_{\mu\nu}^{(jNn)} = & \sum_{\Lambda\Lambda'} P_{\Lambda\Lambda'}(\xi) \langle \hat{p} \lambda' | j_\mu(0) | \hat{p} + q, j\Lambda, N \rangle \\ & \times \langle \hat{p} + q j \Lambda N | j_\nu(0) | \hat{p} \lambda \rangle B_n(s), \quad (2.6) \end{aligned}$$

⁸ G. Domokos, S. Kovesi-Domokos, and E. Schonberg, JHU Report No. NYO-4076-11, 1970 (unpublished).

⁹ This form is equivalent to the one used by J. D. Bjorken and J. D. Walecka, Ann. Phys. (N. Y.) **38**, 35 (1966).

where

$$P_{\lambda\lambda'}(\xi) = \bar{u}_\lambda(p)P(\xi)u_{\lambda'}(p)$$

and

$$B_n(s) = \frac{1}{\pi} \frac{\Gamma_0 s_n}{(s_n - s)^2 + \Gamma_0^2 s_n^2}.$$

The total resonance contribution is obtained by multiplying (2.6) by the number of states, $\rho(j, n)$, and summing over j, N, n .

The invariant amplitudes W_3 and W_4 are projected out by contracting $W_{\mu\nu}$ with suitably chosen anti-symmetric tensors, e.g., those standing in front of W_3, W_4 in the decomposition (2.2). In this way, one obtains two invariant equations, from which W_3 and W_4 can be expressed.

It is convenient to evaluate (2.6) in the rest frame of the resonance, because the projection operator $\sum \psi^{\alpha_1 \dots \alpha_{j-1/2}} \bar{\psi}^{\beta_1 \dots \beta_{j-1/2}}$ simplifies there and a two-component formalism can be used. The details of this procedure have been described in I for the case of the spin-independent part of the amplitude. Here the only difference is that instead of summing over the helicities of the nucleon, one must insert the density matrix of the target. Using "covariant" spinor normalization $\bar{u}_\lambda(p)u_{\lambda'}(p) = \delta_{\lambda\lambda'}$, we have

$$P(\xi) = \frac{1}{4}(1 - i\xi_\mu \gamma_\mu \gamma_5). \quad (2.7)$$

Inserting (2.5) and (2.7) into (2.6) and noticing that the sums over nucleon helicities give just $2^{-1}(1 - i\hat{p})$, the projection operator for positive energies, we obtain after a straightforward but tedious calculation

$$\begin{aligned} W_3^{(jNn)} &= -\frac{1+E}{4} B_n(s) \frac{s_n}{q^{*3}} \left(\frac{q^{*2}}{2}\right)^{j+1/2} \frac{A_N^2 (2j+1)!!}{(2j)!!} \\ &\quad \times \left\{ NG_1 G_3 + \nu(\sqrt{s_n}) \left[G_3^2 - \frac{2j+3}{2j-1} G_2^2 \right] \right\}, \\ W_4^{(jNn)} &= \frac{1+E}{4} B_n(s) \frac{s_n}{q^{*2}} \left(\frac{q^{*2}}{2}\right)^{j+1/2} \frac{A_N^2 (2j+1)!!}{(2j)!!} \\ &\quad \times \left\{ -NG_1 G_3 \frac{\nu}{q^2} + (\sqrt{s_n}) \left[G_3^2 - \frac{2j+3}{2j-1} G_2^2 \right] \right\}. \end{aligned} \quad (2.8)$$

Here, as in I, A_N stands for the "normality factor":

$$A_N = \begin{cases} 1, & \text{if } N=1 \\ q^*/(1+E), & \text{if } N=-1. \end{cases}$$

Next, we would like to sum over j, N, n in order to obtain the full resonance contribution to the invariant functions. At this point, however, we encounter an infamous difficulty. In order to understand its nature, let us compare, e.g., the expressions of W_2 and W_3 , summed over the normalities. Using the results of I

and Eq. (2.8), we find

$$\begin{aligned} \sum_{N=\pm 1} W_2^{(jNn)} &= \frac{q^2 E}{q^{*2} s_n} B_n(s) \left[s_n^{5/2} \left(\frac{q^{*2}}{2}\right)^{j+1/2} \frac{(2j+1)!!}{(2j)!!} \right] \\ &\quad \times \left\{ \frac{G_1^2}{s_n} + \left[G_3^2 + \frac{2j+3}{2j-1} G_2^2 \right] \right\}, \end{aligned} \quad (2.9)$$

$$\begin{aligned} \sum_{N=\pm 1} W_3^{(jNn)} &= -\frac{E\nu}{2q^{*2} s_n} B_n(s) \left[s_n^{5/2} \left(\frac{q^{*2}}{2}\right)^{j+1/2} \frac{(2j+1)!!}{(2j)!!} \right] \\ &\quad \times \left\{ \frac{G_1 G_3}{E\nu \sqrt{s_n}} + \left[G_3^2 - \frac{2j+3}{2j-1} G_2^2 \right] \right\}. \end{aligned} \quad (2.10)$$

We could argue in I that in the Bjorken limit, when $s \rightarrow \infty$, $q^2 \rightarrow \infty$, and $s/q^2 = \omega' - 1 = O(1)$, the electric contribution to (2.9) (proportional to G_1^2) could be safely neglected, since it contained an extra factor $s_n^{-1} \sim s^{-1}$. Let us realize, however, that this argument depends crucially upon the fact that the expression in the square brackets, containing G_1^2 and G_2^2 , is positive definite and hence no compensations can occur. (For the same reason, almost any reasonable hypothesis about the transition form factors and the level density leads *essentially* to the same shape of W_2 .) Not so in Eq. (2.10) and the analogous expression for W_4 . There the form factors G_2 and G_3 contribute with opposite signs; their contributions may compensate each other. Therefore, even the relative orders of magnitude of the first and second terms in the curly bracket are hard to estimate without making rather detailed assumptions about the level density and the transition form factors. Also, various assumptions may lead to quite different shapes and scaling properties of W_3 and W_4 and hence the asymmetry A .

In the Sec. III we first investigate those properties of the spin-dependent structure functions which are essentially independent of the detailed assumptions made about the level density and form factors (but depend on the fact that we are using a resonance model), and then calculate the relevant quantities in a specific model.

III. CALCULATION OF SPIN-DEPENDENT STRUCTURE FUNCTIONS

A. General Properties

We introduce the relative weight of the photon-nucleon channel as in I:

$$F_n = \sum_j \rho(j, n) s_n^{5/2} \left(\frac{q^{*2}}{2}\right)^{j+1/2} \frac{(2j+1)!!}{(2j)!!} \approx \frac{f}{\sqrt{(n+1)}},$$

where $\rho(j, n)$ is the density of states. On summing over

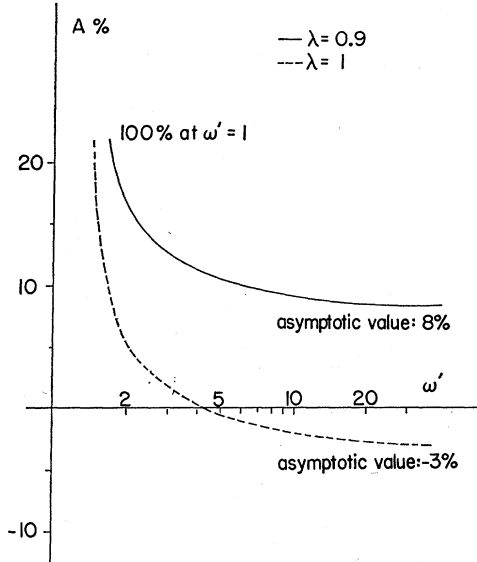


FIG. 1. Predicted asymmetry in polarized-beam-polarized-target experiments. Dashed curve: symmetric quark model. Full curve: quark model with symmetry breaking.

resonances, we obtain

$$W_3 = -\frac{E\nu}{2q^{*2}} \sum_n \frac{B_n(s)}{s_n} F_n \left\{ \frac{G_1 G_3}{E\nu\sqrt{s_n}} + G_3^2 - \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle \right\}, \quad (3.1)$$

$$W_4 = \frac{E}{2q^{*2}} \sum_n \frac{B_n(s)}{s_n} F_n \left\{ -\frac{\omega}{2} \frac{G_1 G_3}{E\sqrt{s_n}} + G_3^2 - \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle \right\}. \quad (3.2)$$

In the last two equations, we used the definition

$$\sum_j \rho(j, n) \left(\frac{q^{*2}}{2} \right)^{j+1/2} \frac{(2j+1)!!}{(2j)!!} \frac{2j+3}{2j-1} G_2^2 = F_n \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle.$$

(The form factors G_1 , G_3 have been assumed to be "universal," i.e., independent of j for any given n .)

Provided that the difference

$$G_3^2 - \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle$$

does not decrease faster than s_n^{-1} for large resonance masses, in the Bjorken limit the electric contribution to (3.1) and (3.2) can be neglected.¹⁰ Thus, in this limit, we

¹⁰ A very fast decrease of the difference between the averages of G_2^2 and G_3^2 can probably be excluded on physical grounds: It would lead to a pathologically smooth behavior of the current commutators on the light cone; cf. Ref. 5.

find the relation

$$\nu W_4 \sim W_3. \quad (3.3)$$

Further, using the scaling properties of W_1 and W_2 , established in I, we find for the expression for the theoretical asymmetry [Eq. (2.4)] in the same limit

$$A \sim \sum_n \frac{B_n(s)}{s_n} F_n \left\{ G_3^2 - \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle \right\} / \sum_n \frac{B_n(s)}{s_n} F_n \left\{ G_3^2 + \left\langle \frac{2j+3}{2j-1} G_2^2 \right\rangle \right\}. \quad (3.4)$$

It is easy to verify that this can be expressed through the virtual photoabsorption cross sections

$$A \sim (\sigma_{4\uparrow} - \sigma_{4\downarrow}) / (\sigma_{4\uparrow} + \sigma_{4\downarrow}),$$

a relation already derived by Bjorken.⁴ (The arrows indicate the relative orientation of the photon and nucleon spins.)

Relation (3.3) is characteristic of a resonance model. Other models (Regge models, parton models, etc.) lead to different types of relations between the spin-dependent structure functions; cf. Ref. 5.

B. Quark-Model Calculation

Assume that the nucleon spectrum can be described in terms of orbital excitations of relativistic quark states. (This assumption is not incompatible with the empirical properties of the spectrum, discussed in I; indeed it would lead to a natural explanation of the observed fact that $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states follow each other alternately.) In an orbital excitation model we expect that the ratio G_2/G_3 is—at least approximately— independent of the excitation and, hence, can be determined from the properties of the ground state. As is well known,¹¹ a relativistic, symmetric "quark" model gives $G_2=G_3$ for $I=\frac{3}{2}$, and $G_2=0$ for $I=\frac{1}{2}$ nonstrange states. We estimate the average of $(2j+3)/(2j-1)$ semiclassically. Let $j-\frac{1}{2}=l\sim q^*R$, where R is the average radius at which the photon is absorbed. Clearly, for high energies, the function

$$\rho(j, n) \frac{(2j+1)!!}{(2j)!!} \left(\frac{q^{*2}}{2} \right)^{j+1/2}$$

has to have a sharp peak around the semiclassical value of j . Therefore

$$\sum s_n^{5/2} \rho(j, n) \frac{(2j+1)!!}{(2j)!!} \left(\frac{q^{*2}}{2} \right)^{j+1/2} \frac{2j+3}{2j-1} \approx \frac{2g+3}{2g-1} F_n,$$

where $g=l+\frac{1}{2}$ is the semiclassical estimate of the dominant angular momentum.

¹¹ A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965). The equivalent statement in "static" models is that the Δ photoproduction amplitude is pure $M1$.

From the expression of the form factor we find $R \approx r/\sqrt{s_n} \approx r/\sqrt{s}$, since the factors $B_n(s)$ peak around $s_n \approx s$. Thus, in the limit $q^2 \rightarrow \infty$, we get $l \sim \frac{1}{2}r\omega(\omega-1)^{-1}$. (This estimate breaks down in a small neighborhood of $\omega=1$, roughly where the maximal spins of the intermediate states become smaller than l as estimated here.) For $I = \frac{3}{2}$ states, l has to be replaced by $l+1$, since the minimum spin is $\frac{3}{2}$. Converting the series in (3.1) into an integral, as described in I, and using the narrow-resonance approximation, gives the following estimate for the structure function $W_3^{(I)}$:

$$\nu W_3^{(1/2)} \sim \frac{1}{2}\mu_{1/2}^2 fF(\omega), \quad (3.5)$$

$$\nu W_3^{(3/2)} \sim -\mu_{3/2}^2 fF(\omega) \frac{\omega-1}{(1+\frac{1}{2}r)\omega-1}. \quad (3.6)$$

Here

$$F(\omega) = \frac{\omega(\omega-1)^3}{(\omega-1+\frac{1}{2}r)^4},$$

and the superscript I indicates the isospin of the intermediate state.

Using the known expressions for the spin-independent structure functions, the expression for the asymmetry A becomes

$$A(\omega) = \frac{\frac{1}{2}r\mu_{1/2}^2\omega + (\mu_{1/2}^2 - 2\mu_{3/2}^2)(\omega-1)}{\frac{1}{2}r(\mu_{1/2}^2 + 2\mu_{3/2}^2)\omega + (\mu_{1/2}^2 + 4\mu_{3/2}^2)(\omega-1)}. \quad (3.7)$$

This expression is not valid for $\omega-1 \ll 1$, since there our estimate of l breaks down. It can be shown that $A(1) = 1$. The curve $A(\omega)$, given by (3.7), is plotted for a proton target in Fig. 1 (dashed curve). The symmetric quark model predicts that the $\gamma N \rightarrow \Delta$ transition-matrix element is pure $M1$, whereas experimentally there seems to be an admixture of $E2$ of about 4-5% (see Ref. 12).

Making the assumption that $G_2 = \lambda G_3$, one can fix the parameter λ from the ratio of the $E2$ and $M1$ matrix elements. Indeed, the appropriate transition-matrix elements are given by^{9,13}

$$F(E2) = -(\sqrt{s}) \left(\frac{M_\Delta}{2(E+1)} \right)^{1/2} M_{\Delta q}^{*3}(G_2 - G_3),$$

$$F(M1) = - \left(\frac{M_\Delta}{2(E+1)} \right)^{1/2} M_{\Delta q}^{*3}(G_2 + G_3).$$

Assuming a 5% admixture of $E2$, one gets $\lambda \approx 0.9$. The asymmetry can be recalculated under this condition; the resulting curve is drawn in Fig. 1 as a full line.

¹² G. Morpurgo, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

¹³ L. Durand, III, P. de Celles, and R. Marr, *Phys. Rev.* **126**, 1882 (1962).

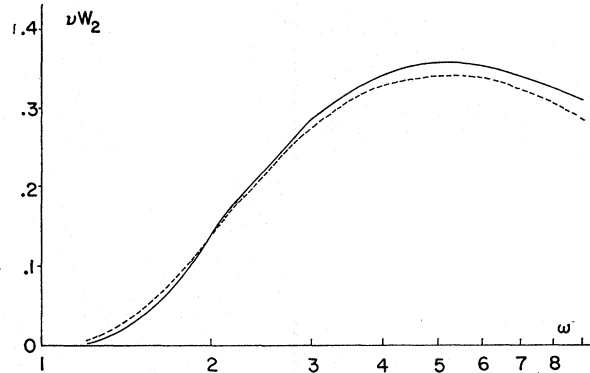


FIG. 2. Structure function νW_2 . Dashed curve: $G_2=0$. Full curve: symmetric quark model.

IV. DISCUSSION

The resonance model predicts a sizable polarization asymmetry in the inelastic scattering cross section of charged leptons. A particularly pleasant feature of the spin-dependent part of the cross section is that it is considerably more sensitive to model-dependent assumptions than the spin-averaged cross section. The predicted polarization asymmetry (taking the quadrupole term in the $\gamma N \rightarrow \Delta$ matrix element into account) levels off at about 8% for large values of ω . Such an asymmetry should be detectable already in the proposed AGS experiment of Chen *et al.*,² and even more in the next generation of experiments to be performed at SLAC and NAL. It is important to point out that these experiments should be able to distinguish between various models. Compared with other calculations, based on quark-algebra or various versions of the parton model,^{4,14} the asymmetries predicted by the resonance model in its present form are lower typically by a factor of 2, and also the dependence on ω is different.

In order to emphasize further the sensitivity of the asymmetry to various physical assumptions (in contrast with the spin-averaged cross section) we recalculated νW_2 under two extreme assumptions. The results are shown in Fig. 2. The dashed curve is calculated under the extreme—and unphysical—assumption that the form factor G_2 vanishes identically, while the full curve corresponds to the symmetric-quark-model value. (The free parameter in the relative weight has been adjusted to make both curves pass through the same value at $\omega=2$.) The difference between both curves is nowhere bigger than $\approx 5\%$, whereas the assumption $G_2 \equiv 0$ leads to an asymmetry which is 100% at every $\omega > 1$.

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¹⁴ J. Kuti (private communication).