Asymptotic Behavior of Weak Spectral Functions and Hadronic Decays of the W Boson*

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(Received 5 November 1970)

Models of current algebra are employed to study the asymptotic properties of the spectral functions of weak currents. Asympotically it is found that they all approach constant values closely related to each other. As an application, we use our results to estimate the hadronic decay of the intermediate vector boson. The branching ratio of the W decay into hadrons to the decay into leptons is of order unity, provided that the electron-positron annihilation total cross section is pointlike and predominantly isovector in character. Another contribution of this paper is a consequence of partial conservation of axial-vector current, which relates the decay rate of the W boson into a soft pion plus any number of bosons to the decay into all bosons through a known constant independent of the W mass.

I. INTRODUCTION

THE advent of high-energy electron-positron colliding beams[1] has stimulated considerable interest in the asymptotic properties of the spectral functions of the electromagnetic current.[2] The purpose of this paper is to study the closely related spectral functions of the weak hadronic currents and to point out their relevance for future experiments. Knowledge of the spectral functions corresponding to weak currents is very important, since it provides one of the few ways for investigating the contribution of the axial-vector current.

We first classify several relations among the asymptotic values of the spectral functions according to models of current algebra and the transformation properties of symmetry-breaking terms. As an application, we estimate the hadronic decay of the intermediate vector boson (IVB), under the assumption that its mass is considerably larger than the mass of the proton. We arrive at the conclusion that if the total cross section for electron-positron annihilation into hadrons is pointlike and predominantly isovector in character, then the branching ratio h/l (hadrons/leptons) for the decay of the intermediate vector boson is of order unity, a result that has been suggested in the past by Bjorken and subsequently by Gribov *et al.* Such a result is of experimental importance for the W searches which will take place at the new accelerators. In Sec. IV, we study the model dependence of our results by computing them in several other ways: (1) in perturbation theory by evaluating the vacuum polarization tensor, (2) in the parton model, and (3) in terms of recent developments on scale invariance.

Section V contains an amusing consequence of partial conservation of axial-vector current (PCAC). It states that the decay probability of the W boson into a soft pion plus bosons divided by the decay probability into all bosons is a number completely determined by PCAC.

Finally, it is needless to emphasize that the properties of the spectral functions obtained in this paper are completely independent of the existence of the IVB but their physical significance is immensely enhanced if such a particle exists.

II. ASYMPTOTIC PROPERTIES OF SPECTRAL FUNCTIONS

Using the Lehmann-Källén representation, we can write[3] the vacuum expectation value of the hadronic currents in terms of two spectral functions ρ_1, ρ_2:

$$M_{\mu\nu}=\sum_n \langle 0|J_\mu{}^+(0)|n\rangle\langle n|J_\nu{}^-(0)|0\rangle(2\pi)^4\delta^4(q-P_n)\epsilon(q_0)$$
$$=\int e^{-iq\cdot x}\langle 0|[J_\mu{}^+(x),J_\nu{}^-(0)]|0\rangle d^4x$$
$$=[q_\mu q_\nu\rho_2(q^2)-g_{\mu\nu}q^2\rho_1(q^2)]\epsilon(q_0)\,. \quad (2.1)$$

The weak current is believed to be of the form

$$J_\mu{}^+(x)=[V_\mu{}^1(x)+iV_\mu{}^2(x)-A_\mu{}^1(x)-iA_\mu{}^2(x)]\cos\theta_C$$
$$+[V_\mu{}^4(x)+iV_\mu{}^5(x)-A_\mu{}^4(x)-iA_\mu{}^5(x)]\sin\theta_C\,, \quad (2.2)$$

where θ_C is the Cabibbo angle. Each of the spectral functions in (2.1) can be written as the sum of four terms:

$$\rho_i(q^2)=[\rho_i{}^V(q^2)+\rho_i{}^A(q^2)]\cos^2\theta_C$$
$$+[\sigma_i{}^V(q^2)+\sigma_i{}^A(q^2)]\sin^2\theta_C\,, \quad (2.3)$$

* Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-4204.

[1] B. Bartoli, B. Coluzzi, F. Felicetti, G. Goggi, M. Marini, F. Massa, D. Scannicchio, V. Silvestrini, and F. Vanolio, Frascati Report No. LNF-70/37, 1970 (unpublished).

[2] For a review of e^+-e^- annihilation into hadrons see R. Gatto, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970). Relevant references since the conference are mentioned explicitly.

[3] S. Okubo, Nuovo Cimento **44A**, 1015 (1966); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); **19**, 470 (1967).

where $i=1,2$,

$$\epsilon(q_0)\left[q_\mu q_\nu\begin{pmatrix}\rho_2{}^V\\ \rho_2{}^A\end{pmatrix}-g_{\mu\nu}q^2\begin{pmatrix}\rho_1{}^V\\ \rho_1{}^A\end{pmatrix}\right]=\int e^{-iq\cdot x}\langle 0|\begin{pmatrix}[V_\mu{}^1(x)+iV_\mu{}^2(x),V_\nu{}^1(0)-iV_\nu{}^2(0)]\\ [A_\mu{}^1(x)+iA_\mu{}^2(x),A_\nu{}^1(0)-iA_\nu{}^2(0)]\end{pmatrix}\times|0\rangle d^4x, \quad (2.4)$$

and $\sigma_i{}^V$, $\sigma_i{}^A$ are similarly defined in terms of the strangeness-violating vector and axial-vector currents. The vacuum expectation values of the interference terms between vector and axial-vector currents vanish. The same is also true for the interference terms between strangeness-conserving and strangeness-violating currents.

From representations (2.1) and (2.4), we obtain positivity results by taking components of the currents parallel to and perpendicular to $\mathbf{q}$:

$$\rho_2{}^m(q^2)\geqslant\rho_1{}^m(q^2)>0 \quad\text{and}\quad \sigma_2{}^m(q^2)\geqslant\sigma_1{}^m(q^2)>0, \quad (2.5)$$

with $m=V$ or A and with the equality satisfied if, and only if, the currents are conserved. Considering the time-space component of (2.4) and integrating over q_0, we obtain

$$q_i\int_0^\infty \rho_2{}^V(q^2)dq^2=2\pi\int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}}\langle 0|[V_0{}^+(\mathbf{x},0),V_i{}^-(0,0)]|0\rangle, \quad (2.6)$$

which is just the well-known spectral representation of the Schwinger term. In this as well as the following equations of this section, we discuss only the strangeness-conserving vector contribution with the understanding that similar relations also hold for the other three spectral functions. A similar relation is obtained by multiplying (2.4) by q_0 and integrating over q_0:

$$\int_0^\infty [\eta_\mu\eta_\nu\rho_2(q^2)-g_{\mu\nu}\rho_1(q^2)]q^2dq^2=2\pi\int d^3x\langle 0|[[H,J_\mu{}^+(\mathbf{x},0)],J_\nu{}^-(0,0)]|0\rangle, \quad (2.7)$$

where $\eta_\mu=(1,0,0,0)$ and $\mathbf{q}$ has been set equal to zero. The equal-time commutators which appear in (2.6) and (2.7) are not fixed by the $SU(3)\times SU(3)$ current algebra. The double commutator in (2.7) depends on the transformation properties of the symmetry-breaking term in the Hamiltonian, while the Schwinger term in (2.6) depends on the explicit form of the currents. We shall assume some simple models in order to evaluate the equal-time commutators leading to relations between the asymptotic values of the spectral functions. Much of this discussion has considerable overlap with published work[3,4] and we include it in the interest of summarizing the asymptotic properties of the spectral function that are relevant to the hadronic decays of the $W^\pm$ bosons.

We now classify several relations between the asymptotic values of the spectral functions according to two models.

A. Violation of $SU(3)\times SU(3)$ by Term Belonging to $(3,\bar{3})+(\bar{3},3)$ Representation

Consider the time-time component of (2.4) for the axial-vector currents

$$\frac{-1}{2\pi}\int_0^\infty [\rho_2{}^A(q^2)-\rho_1{}^A(q^2)]q^2dq^2=\int d^3x\langle 0|[[H,A_0{}^+(\mathbf{x},0)],A_0{}^-(0,0)]|0\rangle \quad (2.8)$$

and calculate the commutator in the model where the symmetry-violating term[5–7] is given by

$$H=-\int[u_0(\mathbf{x},0)+cu_8(\mathbf{x},0)]d^3x. \quad (2.9)$$

Here u_i,v_i are the scalar and pseudoscalar nonets contained in the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$ symmetry group. In evaluating the commutators one only uses the commutation relations of the charges with the densities v_i,u_i:

$$\int d^3x[H,A_0{}^+(\mathbf{x},0)]=[H,Q_5{}^+(0)]=\frac{i(\sqrt{2}+c)}{\sqrt{3}}\int d^3x[v_1(\mathbf{x},0)+iv_2(\mathbf{x},0)] \quad (2.10)$$

and

$$\int d^3x\langle 0|[[H,A_0{}^+(\mathbf{x},0)],A_0{}^-(0,0)]|0\rangle=-\frac{i(\sqrt{2}+c)}{\sqrt{3}}\times\int d^3x\langle 0|[A_0{}^-(0,0),v_1(\mathbf{x},0)+iv_2(\mathbf{x},0)]|0\rangle=\tfrac{2}{3}(\sqrt{2}+c)(\langle 0|u_8|0\rangle+\sqrt{2}\langle 0|u_0|0\rangle). \quad (2.11)$$

Equation (2.8) together with (2.11) leads to the following sum rule:

$$\int_0^\infty [\rho_2{}^A(q^2)-\rho_1{}^A(q^2)]q^2dq^2=+\tfrac{2}{3}(\sqrt{2}+c)(\langle 0|u_8|0\rangle+\sqrt{2}\langle 0|u_0|0\rangle). \quad (2.12)$$

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[4] The prediction for the asymptotic behavior of e^+-e^- annihilation and the suggestion to extend it to charged currents was made by J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[5] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[6] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[7] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

The right-hand side vanishes for $c=-\sqrt{2}$, which corresponds to exact $SU(2)\times SU(2)$. In general, $c\neq-\sqrt{2}$ and the double commutator depends on the vacuum expectation values of u_0 and u_8.

Application of low-energy limits and partial conservation of axial-vector current (PCAC) indicate[7] that the vacuum expectation values of u_0 and u_8 are finite with

$$\langle 0|u_8|0\rangle\approx 0 \tag{2.13}$$

and

$$\langle 0|u_0|0\rangle\approx\langle m^2\rangle_{\rm av}f^2, \tag{2.14}$$

where $\langle m^2\rangle_{\rm av}$ is the average of the square of the mass of the pseudoscalar mesons and $f\approx f_\pi\approx 0.9m_\pi$, the pion decay constant. The convergence of the sum rule implies that, within $\ln q^2$ terms,

$$q^4[\rho_2{}^A(q^2)-\rho_1{}^A(q^2)]\xrightarrow[q^2\to\infty]{} 0. \tag{2.15}$$

We can obtain similar results for the spectral functions corresponding to the strangeness-violating current. Setting $\sigma=\sigma^V+\sigma^A$, we state the corresponding sum rule:

$$\int_0^\infty [\sigma_2(q^2)-\sigma_1(q^2)]q^2dq^2$$
$$=-\tfrac{2}{3}\sqrt{2}(\sqrt{2}+\tfrac{1}{2}c)\langle 0|u_0|0\rangle+(1/\sqrt{3})(c-\sqrt{2})\langle 0|u_3|0\rangle$$
$$+\tfrac{1}{3}(\sqrt{2}+5c)\langle 0|u_8|0\rangle. \tag{2.16}$$

The constant c occurring in (2.12) has been estimated by Gell-Mann, Oakes, and Renner.[7] What is still needed for saturating the sum rules are the couplings and masses of resonances with negative G parity and $J^P=0^-$ or 1^+. The convergence of the sum rules, together with the result $\rho_2(q^2)\to$ const as $q^2\to\infty$ (shown in Sec. II B) implies that the functions $\rho_i{}^V(q^2)$ and $\rho_2{}^A(q^2)$ are asymptotically equal. Similar results follow for the spectral functions σ^A and σ^V.

B. Quark Model

The quark model offers a possibility of evaluating the Schwinger terms occurring in (2.6). The currents are assumed to have a "split-point" definition

$$\begin{aligned}V_\mu{}^\alpha(x)&=\bar\psi(x+\tfrac{1}{2}\epsilon)\tfrac{1}{2}\lambda^\alpha\gamma_\mu\psi(x-\tfrac{1}{2}\epsilon),\\ A_\mu{}^\alpha(x)&=\bar\psi(x+\tfrac{1}{2}\epsilon)\tfrac{1}{2}\lambda^\alpha\gamma_5\gamma_\mu\psi(x-\tfrac{1}{2}\epsilon),\end{aligned} \tag{2.17}$$

where ϵ is a spacelike vector to be averaged over all directions and then taken in the limit $\epsilon\to 0$. The most singular term in the equal-time commutators in (2.6) is

$$\langle 0|[V_0{}^\alpha(\mathbf{x},0),V_i{}^\beta(0,0)]|0\rangle$$
$$=\langle 0|A_0{}^\alpha(\mathbf{x},0),A_i{}^\beta(0,0)|0\rangle\to\langle 0|\bar\psi(0)\gamma_i$$
$$\times\{\tfrac{1}{2}\lambda^\alpha,\tfrac{1}{2}\lambda^\beta\}\psi(0)|0\rangle\epsilon^j\frac{\partial\delta(x)}{\partial x_j}. \tag{2.18}$$

Evaluating the term on the right-hand side, one obtains that it diverges quadratically, as has been repeatedly discussed in literature.[8] This together with Eq. (2.6) implies that $\rho_2{}^V(q^2)\to$ const. Similarly, $\rho_2{}^A(q^2)$, $\sigma_2{}^V(q_2)$, $\sigma_2{}^A(q^2)$ tend to constants in the limit $q^2\to\infty$. Furthermore, for a cutoff Λ very large in comparison to M_W, we find

$$\int_0^{\Lambda^2}[\rho_2{}^V(q^2)-\rho_2{}^A(q^2)]dq^2$$
$$=\int_0^{\Lambda^2}[\sigma_2{}^V(q^2)-\sigma_2{}^A(q^2)]dq^2=0, \tag{2.19}$$

since the Schwinger terms for vector and axial-vector currents are equal. We conclude that to leading order in q^2 the spectral functions $\rho_2{}^V(q^2)$ and $\rho_2{}^A(q^2)$, as well as $\sigma_2(q^2)$ and $\sigma_1(q^2)$, become asymptotically equal.

Asymptotic equality of the spectral functions also follows from the first Weinberg sum rule,[9] since its convergence demands that the difference $\rho_2{}^V(q^2)-\rho_2{}^A(q^2)$ decreases faster than $1/q^2$ in the limit $q^2\to\infty$.

Combining the results, we have

$$\rho_1{}^V=\rho_2{}^V\approx\rho_2{}^A\approx\rho_1{}^A=\text{const}\quad\text{as}\quad q^2\to\infty, \tag{2.20}$$

where the first equality follows from conserved vector current (CVC).

III. HADRONIC DECAY OF INTERMEDIATE VECTOR BOSON

The most important application of the results in Sec. II is an estimate of the hadronic decay of the W boson, provided that such a particle exists. Great interest for its existence is indicated by the numerous proposals that have been submitted to the National Accelerator Laboratory.[10] Most of the proposals hope to observe the intermediate vector boson through its leptonic decays. It was not until recently that it was suggested[11] that the events from the hadronic decay of W boson may be clustered in a small region of phase space so that they peak prominently on top of the inelastic neutrino-nucleon background, which is more or less uniform in phase space. This feature can be used to search for the W boson through its hadronic decay mode. Furthermore, Lee[12] has recently presented an example where the W boson can be observed only through its hadronic decays. It seems from these developments that an estimate or a lower bound for the hadronic decay of W boson is of considerable interest.

[8] V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Phys. Letters **24B**, 554 (1967); see also R. Jackiw and G. Preparata, Phys. Rev. Letters **22**, 975; **22**, 1162(E) (1969).

[9] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[10] As of the summer 1970, searches for the intermediate vector boson with neutrino beams were contained in eleven of the proposals submitted to NAL. Additional searches were suggested using other beams.

[11] D. Cline, A. K. Mann, and C. Rubbia, Phys. Rev. Letters **25**, 1309 (1970).

[12] T. D. Lee, Phys. Rev. Letters **25**, 1144 (1970).

Using the results of Sec. II and the conserved-vector-current hypothesis, we can estimate the W decay in terms of the isovector part of the spectral function measured in electron-positron annihilation experiments. The decay rate of the W is

$$\Gamma=\frac{GM_W}{2\sqrt{2}}\epsilon_\mu\epsilon_\nu\sum_n \langle 0|J_\mu^-|n\rangle\langle n|J_\nu^+|0\rangle \times(2\pi)^4\delta^4(q-P_n)\,, \quad (3.1)$$

where G is the Fermi coupling constant $[GM_P{}^2\simeq 10^{-5}$ and $g^2=(G/\sqrt{2})M_W{}^2]$, M_W is the mass of the IVB, and ϵ_μ is the polarization of the IVB. Averaging over initial polarizations and using (2.1), we have

$$\Gamma=\frac{GM_W{}^3}{2\sqrt{2}}\rho_1(q^2)\,. \quad (3.2)$$

The spectral function $\rho_1(q^2)$ is the sum of the vector and the axial-vector-current contributions, both being positive definite. For $\Delta S=0$ transitions, we can use the CVC hypothesis to make an isospin rotation and obtain

$$\rho_1{}^V(q^2)=2\rho^{I=1,\mathrm{em}}(q^2)\,. \quad (3.3)$$

where $\rho^{I=1,\mathrm{em}}(q^2)$ is the isovector part of the electromagnetic spectral function. Good knowledge of the electromagnetic spectral function is to be obtained from electron-positron colliding-ring experiments where the total annihilation cross section measures $\rho(q^2)$:

$$\sigma_{\mathrm{tot}}{}^{e^+e^-}(q^2)=(8\pi^2\alpha^2/q^2)\rho(q^2)\,. \quad (3.4)$$

The point cross section[13] for this process corresponds to

$$\rho(q^2)=1/6\pi\,. \quad (3.5)$$

By assuming that the isoscalar contribution to $\rho(q^2)$ is negligible and using the positivity properties of the spectral functions, we obtain a lower bound:

$$\Gamma(W\to\text{hadrons})>\frac{GM_W{}^3}{2\sqrt{2}}\rho_1{}^V(q^2) \cong\frac{GM_W{}^3}{\sqrt{2}}\frac{q^2\sigma_{\mathrm{tot}}{}^{e^+e^-}}{8\pi^2\alpha^2}. \quad (3.6)$$

The lower bound together with a point cross section for the isovector part of electron-positron annihilation into hadrons will imply that the hadronic decay of IVB is also pointlike. This result depends only on CVC and it is independent of the mass of the IVB. In order to improve the inequality, we must further assume that the mass of the W is considerably larger than the proton mass, so that the asymptotic limits of the previous section hold. Including also the axial-vector-current contribution from (2.20),

$$\Gamma(W\to\text{hadrons})\simeq\frac{GM_W{}^3}{\sqrt{2}}\frac{q^2\sigma_{\mathrm{tot}}{}^{e^+e^-}}{4\pi^2\alpha^2}. \quad (3.7)$$

Presently there are no accurate measurements of the electron-positron total annihilation into hadrons. Results from the initial operation of the Frascati electron-positron ring[1] indicate the possibility of abundant production of hadrons in the range 1.6–2.0 BeV e^+e^- c.m. energy. However, it is not clear whether all the events can be attributed to electron-positron annihilation. Contributions from two-photon exchange may form a large background[14] even at these energies. Under the circumstances, a reliable estimate of the hadronic decay of the W through (3.7) must wait for a more accurate measurement of $\sigma_{\mathrm{tot}}{}^{e^+e^-}$ at higher energies and a better understanding of the background.

Using (3.7) we calculate the branching ratio for the decay of IVB into hadrons and leptons. It is well known that

$$\Gamma(W^-\to l^-\bar{\nu})=\frac{GM_W{}^3}{\sqrt{2}6\pi}\left(1-\frac{m_l{}^2}{M_W{}^2}\right)\left(1+\frac{1}{2}\frac{m_l{}^2}{M_W{}^2}\right). \quad (3.8)$$

Therefore

$$B=\frac{\Gamma(W^-\to\text{hadrons})}{\Gamma(W^-\to\mu^-\bar{\nu})+\Gamma(W^-\to e^-\bar{\nu})} \approx\pi\frac{q^2\sigma_{\mathrm{tot}}{}^{e^+e^-}}{8\pi^2\alpha^2}=6\pi\rho(q^2)\,. \quad (3.9)$$

As an estimate, we observe that $B\approx 1$ for $\rho(q^2)=1/6\pi$, which is the value for a point cross section.[13]

Previous estimates[15] for the hadronic decays of the W boson consisted of summing up several hadronic channels, each of them being rather small. In this manner one arrives at the conclusion that the purely leptonic decays are the dominant ones. The present calculation indicates that for M_W sufficient large, by summing over all hadronic channels we obtain a decay rate of W into hadrons comparable to the decay rate into leptons, provided that at high energies the isovector part electron-positron annihilation is pointlike.

[13] By point cross section we mean the cross section for $e^+e^-\to\mu^+\mu^-$.

[14] V. E. Balakin, V. M. Budnev, and I. F. Ginzburg, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **11**, 559 (1970) [Soviet Phys. JETP Letters **11**, 388 (1970)]; S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters **25**, 972 (1970).

[15] R. G. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions* (Wiley, New York, 1969). A previous estimate by Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **35**, 5 (1966), suggested that each one of the form factors occurring in the decay of the W into specific pairs of the pseudoscalar meson octet is close to unity. In this manner he arrived at a branching ratio considerably larger than unity. This seems to be an overestimate since each form factor may decrease very fast with M_W. What is suggested in this analysis is a pointlike spectral function only after a summation over all hadronic channels is performed. For comparison see also Ref. 4 and the first paper of Ref. 8.

IV. OTHER MODELS

In this section, we compare the previous results to those of other models. We note that the derivation of the fundamental equations (2.6) and (2.7) involves interchange of limiting procedures, whose justification is by no means trivial.[16] Such an interchange is justified, provided that the "$q_0 \to \infty$ limit for T^* products" is valid[17] to order $1/q_0^2$. However, it has been pointed out that the equal-time commutators in (2.6) and (2.7) computed from Feynman diagrams do not in general agree with the "naive" canonical commutation relations.[18] Thus it seems instructive to study the spectral functions in terms of the vacuum polarization tensor. To lowest order in perturbation theory,

$$\Pi_{\mu\nu}{}^{(V,A)} = \frac{i}{2\pi^2}\int_0^1 \left(\ln\left|1-\frac{q^2}{m^2}x(1-x)\right|\right)$$
$$\times[(q_\mu q_\nu - g_{\mu\nu}q^2)x(1-x)+\tfrac{1}{2}m^2 g_{\mu\nu}(1\mp 1)]dx. \quad (4.1)$$

The sign in the term proportional to $m^2 g_{\mu\nu}$ is $-$ $(+)$ when the current is vector (axial-vector). In the limit as $q^2\to\infty$ the leading term in the vacuum polarization tensor is proportional to $(q_\mu q_\nu - g_{\mu\nu}q^2)$, whidh gives the asymptotic equality of $\rho_1(q^2)$ and $\rho_2(q^2)$. Taking time-space components, we obtain the equality of vector and axial-vector contributions to $\rho_1(q^2)$. Asymptotically, the spectral functions diverge logarithmically in q^2. This is consistent with our previous results, since the asymptotic behaviors determined in Sec. II are not sensitive enough to account for $(\ln q^2)$ terms.

A model that has been rather successful from the practical point of view is the parton model. Cabibbo, Parisi, and Testa[19] have extended its applications to include electron-positron annihilation into hadrons. They conclude that the electromagnetic current in the problem can be replaced by a free current, provided that transverse momenta of the particles, appearing in intermediate states, have a definite cutoff. The same holds true for the hadronic decay of the W boson, provided its mass is considerably larger than the proton mass. We chose the free current to be the Cabibbo current:

$$J_\mu{}^+(x) = \bar{p}'(x)\gamma_\mu(1-\gamma_5)[n'(x)\cos\theta_C + \lambda'(x)\sin\theta_C], \quad (4.2)$$

where $p'(x)$, $n'(x)$, and $\lambda'(x)$ are the free quark fields. The decay rate into hadrons is obtained by assuming that the distribution of $p'\bar{p}'$, $n'\bar{n}'$, $\lambda'\bar{\lambda}'$ pairs in the final state is statistical,

$$\Gamma(W\to\text{hadrons})$$
$$= \frac{GM_W{}^3}{\sqrt{2}6\pi}\left(1-\frac{m_q{}^2}{M_W{}^2}\right)\left(1+\frac{1}{2}\frac{m_q{}^2}{M_W{}^2}\right). \quad (4.3)$$

Imposing the condition $(m_q/M_W)\ll 1$, as is required by the model, we obtain

$$\rho_1{}^{V,\Delta S=0}(q^2) = 1/6\pi. \quad (4.4)$$

This should be contrasted with the electromagnetic spectral function when it is also calculated in the quark-parton model:

$$\sigma_{\text{tot}}{}^{e^+e^-} = (8\pi^2\alpha^2/q^2)[\rho^{I=1}(q^2)+\rho^{I=0}(q^2)], \quad (4.5)$$

with

$$\rho^{I=1}(q^2) = \frac{1}{12\pi}, \quad \rho^{I=0}(q^2) = \frac{1}{36\pi}. \quad (4.6)$$

The specific values of the spectral functions depend on the explicit form of the current and the specific distribution of final states, as has already been stated. The branching ratio in the quark-parton model obtained from Eqs. (3.9) and (4.6) is evidently equal to $\frac{1}{2}$.

It is our hope that the electromagnetic spectral function will eventually be measured in colliding-beam experiments and its isovector-to-isoscalar ratio determined with the help of soft-pion theorems.[20]

The results in Sec. II are also consistent with the expectations of scale invariance according to Wilson.[21] In such a theory the $SU(3)\times SU(3)$ algebra demands that the local charge densities must have dimension 3. The spatial components of the local densities have the same dimension[22] provided that the symmetry term is the same one discussed in Sec. II. The two results imply that asymptotically the spectral functions approach constant values. Furthermore, it has been shown that within this framework, the assumptions in Sec. II guarantee the convergence of the first Weinberg sum rule. As it has already been remarked, the convergence of the sum rule implies that the contributions of the vector and the axial-vector currents are asymptotically equal.

Finally, our conclusions will be seriously modified, if the equal-time commutators are evaluated in the algebra

[16] For greater peace of mind, we may define instead of (2.6) a Fourier transform in the sense of generalized functions:

$$q_i\int e^{-iq_0\cdot\lambda}\rho_2(q^2)q_0dq_0 = \int dq_0 e^{-iq_0\cdot\lambda}\int d^4x\, e^{iq\cdot x}\langle 0|[V_0{}^+(x),V_i{}^-(0)]|0\rangle$$

and then evaluate them at the point $\lambda=0$.

[17] See Bjorken, Ref. 4; K. Johnson and F. Low, Progr. Theoret. Phys. (Kyoto) Suppl. **37–38**, 74 (1966).

[18] S. L. Adler and W. K. Tung, Phys. Rev. Letters **22**, 978 (1969); A. I. Vainshtein and B. L. Ioffe, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **6**, 917 (1967) [Soviet Phys. JETP Letters **6**, 341, (1967)]; R. Jackiw and G. Preparata, Phys. Rev. Letters **22**, 975 (1969); K. M. Bitar and N. N. Khuri, Phys. Rev. D (to be published).

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of fields. We do not discuss this model in any detail because it cannot account in a simple way for the ratio $R=\sigma_l/\sigma_t$ that has already been measured[23] in inelastic electron-proton scattering experiments. However, it is of interest to know that the predictions of the quark commutators or the parton model are consistent with the observed value of R.

V. A SOFT-PION THEOREM

Recently, Pais and Treiman[20] have shown, on the basis of PCAC and current-algebra notions, how studies of soft-pion production can provide information on the axial-vector current and the isovector part of the electromagnetic current. The same analysis can be applied here. We consider both vector and axial-vector spectral functions given by

$$\sum_n \langle 0|V_\mu{}^+|n\rangle\langle n|V_\nu{}^-|0\rangle(2\pi)^4\delta^4(q-P_n) = q_\mu q_\nu \rho_{B2}{}^V(q^2) - g_{\mu\nu}q^2\rho_{B1}{}^V(q^2)\,, \quad (5.1)$$

$$\sum_n \langle 0|A_\mu{}^+|n\rangle\langle n|A_\nu{}^-|0\rangle(2\pi)^4\delta^4(q-P_n) = q_\mu q_\nu \rho_{B2}{}^A(q^2) - g_{\mu\nu}q^2\rho_{B1}{}^A(q^2)\,, \quad (5.2)$$

where the summation over the states $|n\rangle$ extends over *bosons only*. Using PCAC and a low-energy theorem, we have in the soft-pion limit

$$\langle 0|V_\mu{}^+(0)|\pi^i n\rangle = -\frac{i}{f_\pi}\int d^4x\, e^{-iq\cdot x} \times\langle 0|\delta(x_0)[A_0{}^i(x),V_\mu{}^+(0)]|n\rangle\,, \quad (5.3)$$

$$\langle 0|A_\mu{}^+(0)|\pi^i n\rangle = -\frac{i}{f_\pi}\int d^4x\, e^{-iq\cdot x} \times\langle 0|\delta(x_0)[A_0{}^i(x),A_\mu{}^+(0)]|n\rangle\,. \quad (5.4)$$

We have excluded from the state $|n\rangle$ all $N\bar{N}$ pairs in order to avoid the pole terms arising from bremsstralung of a soft pion from the nucleons.

Let us consider $\Delta S=0$ transitions and denote the momentum of the soft pion in the final state by k. The partial decay width of W into all bosons, indicated by the "m" and a soft pion π^i is given by

$$\Gamma(W\to \text{“}m\text{”}+\pi^i) = \frac{1}{2M_W} g^2 \cos\theta_C \epsilon_\mu \epsilon_\nu \times\int \{\sum_m (2\pi)^4\delta^4(k-q+P_n)\langle 0|V_\mu{}^+|\pi^i m\rangle\langle \pi^i m|V_\mu{}^-|0\rangle + (\text{term with } V\to A)\}\frac{d^3k}{(2\pi)^3 2k_0}\,. \quad (5.5)$$

The integration over the momentum of the soft pion can be written in terms of invariants

$$\frac{dk^3}{(2\pi)^3} = \frac{1}{4\pi^2}\frac{1}{M_W{}^2}[(k\cdot q)^2 - M_W{}^2\mu^2]^{1/2} d(k\cdot q)\,, \quad (5.6)$$

where μ is the mass of the pion. By using (5.1)–(5.6) and evaluating the equal-time commutators, we obtain the results

$$4\pi^2 \lim_{(k\cdot q)\to 0} [(k\cdot q)^2 - M_W{}^2\mu^2]^{-1/2}\frac{d\Gamma(W^-\to \text{“}m\text{”}+\pi^-)}{d(k\cdot q)} = \left(\frac{1}{f_\pi}\right)^2 \frac{g^2}{2M_W}[\rho_{B1}{}^V(q^2)+\rho_{B1}{}^A(q^2)] = \left(\frac{1}{f_\pi}\right)^2 \Gamma(W^-\to \text{all bosons})\,, \quad (5.7)$$

$$4\pi^2 \lim_{(k\cdot q)\to 0} [(k\cdot q)^2 - M_W{}^2\mu^2]^{-1/2}\frac{d\Gamma(W^-\to \text{“}m\text{”}+\pi^0)}{d(k\cdot q)} = \left(\frac{1}{f_\pi}\right)^2 \Gamma(W^-\to \text{all bosons})\,, \quad (5.8)$$

$$4\pi^2 \lim_{(k\cdot q)\to 0} [(k\cdot q)^2 - M_W{}^2\mu^2]^{-1/2} \times\frac{d\Gamma(W^-\to \text{“}m\text{”}+\pi^+)}{d(k\cdot q)} = 0\,. \quad (5.9)$$

These results are independent of the asymptotic properties derived in Sec. II. As it is always the case with soft-pion theorems, they involve an extrapolation in k^2 and $k\cdot q$. The extrapolation in k^2 is from μ^2 to zero. The extrapolation in $k\cdot q$ is much more subtle, because for physical values, $k\cdot q > (\sqrt{q^2})\mu$ and thus the range of extrapolation increases with q^2. The last relation (5.9) provides the easiest test for the validity of the extrapolation. The other two state that the decay width into a soft π^- or π^0 and any number of bosons is proportional to the decay width into all bosons.

VI. CONCLUSIONS

The study of the asymptotic behavior of the spectral functions in several models indicates that asymptotically they all approach constant values. For $\Delta S=0$ transitions, they are related in the simple manner summarized by Eq. (2.20). Similar results are satisfied by the spectral functions of the strangeness-violating currents.

From the experimental point of view, the most important result is the branching ratio of the IVB. Using our assumptions concerning the e^+e^- cross section, the branching ratio cannot be much smaller than unity. The most questionable of the assumptions is the small contribution from the isoscalar part of the electromagnetic current. $SU(3)$ arguments would indicate

[23] R. E. Taylor, in Ref. 2.

that $\rho_{em}^{I=0}$ is about one third of $\rho_{em}^{I=1}$. However, if, as has been discussed by Pais and Treiman,[20] the soft-pion techniques are valid at $q^2 \approx M_W^2$, then we can extract the isovector contribution $\rho_{em}^{I=1}(q^2)$ directly from experiments. The presence of an isoscalar contribution can easily be incorporated into our results.

A sizable branching ratio makes possible the search for the IVB through its hadronic decay modes. Our estimates imply that about half of the W events will decay into hadrons. The background from inelastic neutrino-nucleon scattering, discussed by Cline, Mann, and Rubbia,[11] is still manageable, because a complete analysis should also include two additional, compensating effects: (1) a decrease in the number of events due to the smaller branching ratio and (2) an increase in the number of events arising from the contribution of inelastic W production. The total cross section for the inelastic W production has already been calculated[24] and is almost a third of the total elastic W production. Furthermore, preliminary calculations[25] indicate that the angular distribution of the inelastic events is very similar to the angular distribution of the elastic events. Therefore we conclude that a branching ratio equal or greater than the one estimated in this paper makes searches of the IVB through its hadronic modes very attractive.

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The branching ratio is greatly modified in the particular case considered by Lee,[12] where the hadronic decay modes overwhelmingly dominate over the leptonic modes. In this case, searches through the hadronic modes become necessary and the results of Secs. II and V still hold.

Finally, if the intermediate boson exists, the soft-pion theorem provides a new test of PCAC. This can be accomplished by a single experiment, where one observes the decay rate into bosons and also identifies those events which contain at least one soft pion.

ACKNOWLEDGMENTS

One of us (E. A. P.) wishes to thank Dr. E. Goldwasser and the National Accelerator Laboratory for their kind hospitality at the 1970 Summer Study that originated his interest on this topic. We both wish to thank Professor A. Pais for his interest in this work and for helpful conversations.

PHYSICAL REVIEW D VOLUME 3, NUMBER 5 1 MARCH 1971

Direct-Channel Resonance Model of Deep-Inelastic Electron Scattering. I. Scattering on Unpolarized Targets*

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(Received 17 November 1970)

We construct a resonance model of deep-inelastic electron scattering. Using semiempirical rules for the form of the nucleon spectrum and a universality hypothesis for the transition form factors, we obtain explicit expressions for the structure functions W_1 and W_2 in the Bjorken limit. The ratio of the longitudinal to transverse photoabsorption cross sections vanishes for large momentum transfers, and νW_2 becomes scale invariant. A one-parameter fit is obtained to νW_2 (proton), and a zero-parameter prediction is made of νW_2 (proton)-νW_2 (neutron). There is good agreement between the theory and experiment in the region where scale invariance is well established.

I. INTRODUCTION

DEEP-INELASTIC scattering of charged leptons on hadrons provides valuable information about hadron structure. Perhaps the most striking feature of the data is *scale invariance*, i.e., that the structure functions essentially depend on the ratio ν/q^2 only, where ν is the energy loss of the electron, and q^2 is the invariant momentum transfer squared.

* Research supported in part by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-4076.

There exist several models[1–8] which describe the basic features of the data more or less correctly, and are also able to make certain predictions. In these papers we

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