Quark Model for Double-Charge-Exchange Meson-Baryon Scattering

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In an $SU(3)$-symmetric quark model, double-charge-exchange scattering processes of the type $PN \to P'B$, $\Delta Q(P,P') = \pm 2$, are investigated. The physical baryon octet is taken as $\mathbf{8}(\text{physical}) = \mathbf{8}' \cos\theta + \mathbf{8} \sin\theta$, where $\mathbf{8}$ and $\mathbf{8}'$ arise, respectively, from $\mathbf{3}\otimes\bar{\mathbf{3}}$ and $\mathbf{3}\otimes\mathbf{6}$ contained in $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$. In terms of the mixing angle θ, a number of predictions are made.

I. INTRODUCTION

IN recent years, there has been considerable interest in the study of the double-charge-exchange processes

$$K^- + p \to \pi^+ + \Sigma^- \quad (1a)$$
$$\to \pi^+ + Y_1^{*-} \quad (1b)$$
$$\to K^+ + \Xi^- \quad (1c)$$
$$\to K^+ + \Xi^{*-}, \quad (1d)$$
$$\pi^- + p \to \pi^+ + \Delta^- \quad (1e)$$
$$\to K^+ + \Sigma^- \quad (1f)$$
$$\to K^+ + Y_1^{*-}, \quad (1g)$$
$$\pi^+ + n \to \pi^- + \Delta^{++}. \quad (1h)$$

Some data are available on these reactions, and more data are being obtained at the present time.[1-3]

In quark models, under the assumption of impulse approximation (lowest order), these reactions are not allowed. Of course, in higher orders, one does obtain nonzero matrix elements. Quark-model calculations on these processes have been performed by Białas and Zalewski.[4] However, since there exist many versions of the quark model,[5] we would like to calculate the matrix elements for these double-charge-exchange reactions in our $SU(3)$-symmetric quark model.[6] In this paper, our aim is neither to try to list all the available papers on this subject nor to compare our calculations with those of others. Our task is simply to compare our results with the available experimental data. The mechanism we use is the same as has been used in Refs. 4 and 7.

In this paper, we use the notation of Okubo,[8] according to which Q_i stands for the quark ($i = 1, 2, 3$ refer to the p_0, n_0, and λ_0 quarks, respectively) and Q^i stands for the antiquark. The pseudoscalar-meson octet $P_j{}^i$, as in other quark models, is a composite of $Q\bar{Q}$. The baryon states are contained in the direct product

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3} = (\mathbf{3}\otimes\bar{\mathbf{3}}) \oplus (\mathbf{3}\otimes\mathbf{6}) = (\mathbf{8}\oplus\mathbf{1}) \oplus (\mathbf{10}\oplus\mathbf{8}').$$

We shall write the baryon wave functions in terms of

$$B_{ijk} = Q_i Q_j Q_k.$$

Our quark-model wave functions for the pseudoscalar mesons and the baryons are listed in Ref. 9. The basic difference between other quark models and ours lies in the baryon wave functions. In our case, the wave functions are generated by a third-rank tensor, with the properties of the quark fields left free.

In the $SU(3)$ quark model, there are two baryon octets $\mathbf{8}$ and $\mathbf{8}'$, whereas experimentally there is only one baryon octet. Therefore, we choose our physical baryon octet as

$$\mathbf{8}(\text{physical}) = \mathbf{8}' \cos\theta + \mathbf{8} \sin\theta, \quad (2)$$

where for the sake of simplicity θ is taken as real. In earlier work, we have found that $\theta = 20°$ explains reasonably well the available high-energy data on meson-baryon scattering and the photoproduction of pseudoscalar mesons.[6,10] Therefore, our main aim here is to determine the importance of $\theta = 20°$ for the reactions (1). We may mention that, in principle, there exists another baryon octet: $-\mathbf{8}' \sin\theta + \mathbf{8} \cos\theta$, which is orthogonal to our $\mathbf{8}$(physical). However, because experimentally only one baryon octet is known, we set the coefficient of the orthogonal octet equal to zero.

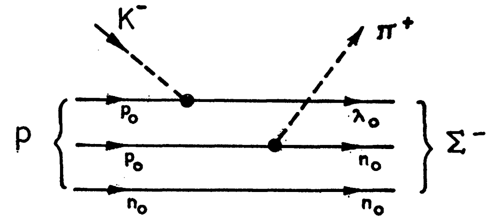

FIG. 1. Typical diagram in lowest-order quark model for reactions (1).

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TABLE I. Calculated (as a function of θ) and experimental values of the cross sections at $\sqrt{s}=2.33$ GeV $[p(Kp)_{\text{lab}}=2.24$ GeV/c, $p(\pi N)_{\text{lab}}=2.41$ GeV/$c]$. K^-p data are taken from Ref. 1 and πp data from Fig. 1 of Ref. 2. $r\equiv|A_{10}/A_8|^2$; $\sigma(K^-p\to\pi^+\Sigma^-)\equiv1$.

	Expt.	$\theta=15°$	$\theta=18°$	$\theta=20°$	$\theta=22°$	$\theta=30°$
$\sigma(K^-p\to\pi^+Y_1^{*-})$	0.7 ± 0.4	$0.88r$	$0.80r$	$0.75r$	$0.71r$	$0.59r$
$\sigma(K^-p\to K^+\Xi^-)$	1.4 ± 0.5	1.49	1.31	1.20	1.11	0.80
$\sigma(K^-p\to K^+\Xi^{*-})$	0.3 ± 0.2	$0.61r$	$0.55r$	$0.52r$	$0.49r$	$0.41r$
$\sigma(\pi^-p\to\pi^+\Delta^-)$	$\cdots$	$2.73r$	$2.47r$	$2.32r$	$2.20r$	$1.82r$
$\sigma(\pi^-p\to K^+\Sigma^-)$	0.5 ± 0.2	0.12	0.07	0.05	0.03	0
$\sigma(\pi^-p\to K^+Y_1^{*-})$	$\cdots$	$0.70r$	$0.63r$	$0.60r$	$0.57r$	$0.47r$
$\sigma(\pi^+n\to\pi^-\Delta^{++})$	$\cdots$	$2.73r$	$2.47r$	$2.32r$	$2.20r$	$1.82r$

In Sec. II, we calculate the matrix elements. Our results are discussed in Sec. III.

II. CALCULATIONS

The mechanism we use to calculate the matrix elements for the reactions (1) is shown in Fig. 1. Of course, we make the usual assumptions about the additivity of the two quark amplitudes, etc. However, we must note that the additivity assumption is fulfilled in a high-energy region (lab momentum $p_L\gtrsim5$ GeV/c). At low energies, the contribution from the direct-channel resonances dominates, with the result that the additivity assumption is no longer valid. In order to avoid unnecessary complications, we shall in this paper concentrate our attention only on the *high-energy* meson-baryon inelastic scattering processes. Therefore, the effective Lagrangian corresponding to Fig. 1 can be written as

$$L_{\text{eff}}=-6A_m\bar{B}_{abc}B_{ade}\bar{P}_e{}^cP_b{}^d\,, \tag{3}$$

where the overhead bar denotes the incoming state. The numerical factor (-6) is introduced for convenience. The complex amplitude A_m is the integral over space-time variables and contains all the spin and kinematic dependencies, etc. The spin coupling is taken into account by the subscript m; $m=8$ or 10, depending on whether the final baryons belong to the $SU(3)$ octet or the decuplet, respectively.

Using the values of the pseudoscalar-meson and baryon wave functions from Ref. 9, we obtain the following matrix elements:

$$\langle\pi^+\Sigma^-|K^-p\rangle=(1+a_+)A_8\,, \tag{4a}$$

$$\langle\pi^+Y_1{}^{*-}|K^-p\rangle=dA_{10}\,, \tag{4b}$$

$$\langle K^+\Xi^-|K^-p\rangle=d^2A_8/2\,, \tag{4c}$$

$$\langle K^+\Xi^{*-}|K^-p\rangle=dA_{10}\,, \tag{4d}$$

$$\langle\pi^+\Delta^-|\pi^-p\rangle=\sqrt{3}dA_{10}\,, \tag{4e}$$

$$\langle K^+\Sigma^-|\pi^-p\rangle=(1+a_-)A_8\,, \tag{4f}$$

$$\langle K^+Y_1{}^{*-}|\pi^-p\rangle=dA_{10}\,, \tag{4g}$$

$$\langle\pi^-\Delta^{++}|\pi^+n\rangle=-\sqrt{3}dA_{10}\,, \tag{4h}$$

where

$$a_\pm=\cos2\theta\pm\sqrt{3}\sin2\theta \tag{5a}$$

and

$$d=2\sqrt{2}\cos\theta\,. \tag{5b}$$

The equality of $\pi^+\Delta^-$ and $\pi^-\Delta^{++}$ rates is a direct consequence of charge symmetry. For the sake of brevity, the word "physical" has been dropped from the baryon octet in the relations (4).

III. DISCUSSION AND CONCLUSIONS

In order to compare our calculations with the available experimental data, we first note the relation between the measured cross section σ and the square of the matrix elements $|\langle f|i\rangle|^2$ given by Meshkov *et al.*[11]:

$$\sigma=(1/s)|\langle f|i\rangle|^2p_f/p_i\,, \tag{6}$$

where s, p_i, and p_f denote (in the c.m. system) the total energy squared, the incident momentum, and the final momentum, respectively. However, since the matrix elements will in general be energy dependent, it is necessary to compare the cross sections *at the same value of s*. We are assuming that the matrix elements depend on the invariant quantity s and *not* on the incident momentum p_i. Of course, it is seldom possible to have experimental data available on the various reactions at the same value of s.

In this paper, we shall ignore mass differences between particles belonging to the same isospin multiplet. Also, we shall normalize our cross sections to

$$\sigma(K^-p\to\pi^+\Sigma^-)\equiv1\,. \tag{7}$$

Using (6) and (7) in (4), we obtain the values of the cross sections as a fraction of $\sigma(K^-p\to\pi^+\Sigma^-)$. These branching ratios as a function of θ are listed in Table I.

Unfortunately, at energies at which our calculations are expected to be reasonable ($p_L\gtrsim5$ GeV/c), there are no experimental data available in the published literature on these double-charge-exchange processes. However, to get a feel for the numbers, we also present in Table I the experimental data at low energy, $\sqrt{s}=2.33$ GeV. Of course, the comparison between our calculated numbers and the experimental data can only be approximate. In any event, this comparison shows that except for $\sigma(\pi^-p\to K^+\Sigma^-)$, the remaining data are consistent with the calculations for θ in the range of 15–22° and with r in the approximate range of 0.3–1.0. However, for a meaningful comparison, it is essential to have accurate experimental data available at high energies. We may mention that data have been reported by Dauber *et al.*[12] on $\sigma(\pi^-p\to\pi^+\Delta^-)$ at 4 GeV/c laboratory momentum. However, because the matrix elements will in general be energy dependent, these data are not

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useful for our purposes. In any event, it is our hope that accurate experimental data at high energies ($p_L \gtrsim 5$ GeV/c) will soon be available on all these double-charge-exchange reactions.

Lastly, in our calculations, strict $SU(3)$ invariance, with the exception of physical masses for mesons and baryons, has been assumed. However, if necessary, $SU(3)$ symmetry-breaking effects can be easily introduced into the calculations by (e.g.) treating the strange quark differently from the nonstrange quarks. At the present time, we do not see any necessity of introducing $SU(3)$ symmetry-breaking effects into the calculations.

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Asymptotic Single-Particle Distributions in the Multiperipheral Model*

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The conjectures of Feynman and of Benecke, Chou, Yang, and Yen on the high-energy limit of single-particle distributions are studied in the framework of the multiperipheral model. It is found that classes of multiperipheral diagrams add to give limiting single-particle distributions.

I. INTRODUCTION

THE great complexity of inelastic hadronic interactions at high energies has led both theorists and experimentalists to focus their attention on inclusive experiments.[1] They offer the advantages of being easy to perform and relatively simple to describe from a theoretical point of view. This simplicity is a consequence of the summation over all the unobserved channels, which tends to average out the details of the matrix element and to exhibit only its dominant features.

The examples of inclusive experiments that we discuss here are single-particle distributions. Feynman[1] has recently proposed to describe these distributions in the center-of-mass system by means of the double differential cross section $d^2\sigma/dxdq_\perp$, where $x=2q_{\parallel}(s)^{-1/2}$; here $s^{1/2}$ is the total energy, and $q_\perp$ and $q_\parallel$ are the transverse and longitudinal components of the momentum of the observed particle. The cross section is then written in the form

$$\frac{d^2\sigma}{dxdq_\perp}=\frac{q_\perp}{\bar{x}}f(x,q_\perp,s)\,, \tag{1.1}$$

where

$$\bar{x}=[x^2+(q_\perp{}^2+\mu^2)/\tfrac{1}{4}s]^{1/2} \tag{1.2}$$

and μ is the mass of the observed particle. Feynman's conjecture is that at very high s the function f becomes energy independent, i.e.,

$$\frac{d^2\sigma}{dxdq_\perp}\underset{s\to\infty}{\longrightarrow}\frac{q_\perp}{\bar{x}}f(x,q_\perp)\,. \tag{1.3}$$

A similar hypothesis was independently formulated by Benecke *et al.*,[2] who describe the same process at finite momenta in the laboratory and projectile frames, and conjecture that $d^2\sigma/dq_\parallel dq_\perp$ approaches a constant limit in those frames as $s\to\infty$. In this limit, any finite momentum in the laboratory or projectile frame transforms into a nonzero value of x in the range $-1<x<1$. Conversely, any finite momentum in the center-of-mass system, or in any "intermediate" frame reached from the c.m. frame by a boost of order s^η, $0<\eta<\frac{1}{2}$, goes to the point $x=0$. The conjecture of Benecke *et al.* turns out to be equivalent to Feynman's hypothesis for $x\neq 0$. The point $x=0$, which concentrates all the information of finite momenta in this continuum of frames, is, however, very important, and for this reason we adopt Feynman's notation in our present work.

Our main purpose is to study the high-energy limit of single-particle distributions in the multiperipheral model that was used by Caneschi and Pignotti[3] to fit experimental single-particle distributions at accelerator energies. We point out that in the description of an inclusive experiment we cannot restrict the model to the multi-Regge region of low multiplicities and large subenergies, which only accounts for a small part of the inelastic cross section, but we have to use the model for all multiplicities and throughout phase space, and we can only expect it to be meaningful in some average sense.[4] If we increase the total energy, this approximation is not improved as the additional energy

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