Furthermore, such a breakdown in either the form-factor dependence or the constancy of σ_γ implies consequences more dramatic than the rejection of vector dominance; anomalous muon behavior at high energies would be indicated.

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Hyperon Beta Decay*

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Polarized-hyperon β decay is considered in the center-of-mass frame of the outgoing leptons. Simple and exact expressions are obtained for the differential intensity distribution function under the assumption of local current-current interaction and two-component neutrinos. It is shown that by exploiting only the fully integrated data, it is possible to make a large number of tests on the structure of the $\Delta S=1$ current even with somewhat limited statistics. These include direct tests on the locality and $V-A$ nature of the interaction without additional assumptions, as well as detailed and rather stringent tests on the Cabibbo theory.

I. INTRODUCTION

THE structure of weak interactions has largely been deduced from the accumulated experimental data on leptonic and $\Delta S=0$ semileptonic decay processes. The resulting $V-A$ current-current interaction picture was naturally generalized to the $\Delta S=1$ semileptonic processes. With the additional assumption of universality expressed through the $SU(3)$ current algebra of Gell-Mann, the Cabibbo theory[1] represents a unified picture for all semileptonic processes. The predictions of this theory are consistent with the available data (mostly rates, plus some angular correlations) on various hyperon β-decay processes.[2] Because of the small branching ratios for these processes ($\sim 10^{-4}$–10^{-3}), however, detailed experimental information has not so far become available. Conclusive verification of the theoretical picture, therefore, does not exist.

With the gradual accumulation of data on β decay from polarized hyperons,[3,4] this situation may soon change. Finally, a more critical and detailed comparison of experimental data with theory seems to be within our reach. In this paper we present a compact and yet complete description of polarized hyperon β-decay processes.[5] Under only the general assumptions of locality and two-component neutrinos, we derive a simple expression for the differential intensity distribution in which the dependence on three of the four independent variables is explicitly displayed.[6] Based on this formula, we propose methods for effectively extracting important information from experimental data even with limited statistics. A series of direct tests on the locality and $V-A$ nature of the interaction without additional assumptions, as well as detailed tests of the Cabibbo theory, are proposed.

The proposed tests are particularly simple if the lepton mass is negligible as compared to the baryon mass difference. This is the case for the electron decay modes of hyperons. We therefore concentrate on this case in the main text. The muon decay modes can also be analyzed effectively with the present method. Since the results are slightly more complicated and harder

* Supported in part by the U. S. Atomic Energy Commission.
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[6] The method used in this paper is similar to that of T. P. Cheng and Wu-Ki Tung, Phys. Rev. D 3, 733 (1971), for neutrino scattering processes.

to explore experimentally, we give the detailed results in Appendix A.

II. TRANSITION AMPLITUDE

Let us consider the β-decay processes

$$A \to B + \binom{e^- + \bar{\nu}_e}{e^+ + \nu_e}, \tag{1}$$

where A and B are spin-$\frac{1}{2}$ baryons. Let the 4-momenta of A, B, e, and ν be denoted by p, p', k, and k', respectively. We also define $q = p - p' = k + k'$. The masses of the baryons and their sum and difference are denoted by M_A, M_B, M_+, and M_-, respectively; thus

$$M_\pm = M_A \pm M_B. \tag{2}$$

The initial hyperon is taken to be polarized with polarization vector $\mathbf{n}$. For definiteness, we shall treat explicitly the e^- decay modes and mention the appropriate modifications of the formulas for the e^+ decay modes at the end. To begin with, let us assume that reaction (1) is described by the usual $V-A$ current-current interaction. The transition amplitude can then be written

$$\begin{aligned} f &= (G/\sqrt{2})\langle e^-\bar{\nu}| j_\mu^\dagger(0)|0\rangle\langle B|J^\mu(0)|A\rangle \\ &= (G/\sqrt{2})\langle k\lambda;\, k'\lambda'| j_\mu^\dagger(0)|0\rangle\langle p'\sigma'|J^\mu(0)|p\sigma\rangle, \end{aligned} \tag{3}$$

where j^μ and J^μ are the leptonic and hadronic weak currents, and λ, λ', σ, and σ' are the helicity indices for A, B, e, and ν, respectively. The lepton current matrix element is, of course, explicitly known:

$$\langle k\lambda, k'\lambda'| j_\mu^\dagger(0)|0\rangle = \bar{u}_\lambda(k)\gamma_\mu(1-\gamma_5)v_{\lambda'}(k'). \tag{4}$$

We would like to seek a way of representing the transition amplitude such that the dependence on certain variables exclusively associated with the current-current interaction and the local nature of the lepton current are explicitly separated from the dependence on variables which are associated with the strong interaction dynamics of the A-B vertex. This is most easily done in the frame where the currents (with 4-momentum q) are at rest, i.e., $q^\mu = (\sqrt{q^2},0,0,0)$. This frame is also the center-of-mass frame of the lepton pair. Within this frame, we define the 3-axis to be along the direction of $\mathbf{p}$ (and $\mathbf{p}'$) and the 1-3 plane to be that defined by $\mathbf{p}$ and $\mathbf{n}$ (polarization vector of A). The angle between $\mathbf{n}$ and the 3-axis ($\mathbf{p}'$) is designated by ψ.[7] The polar angles of the electron (with 3-momentum $\mathbf{k}$) are denoted by θ and ϕ (see Fig. 1). The amplitude (3) depends on four independent variables which we take to be q^2, ψ, θ, and ϕ. It is straightforward to verify that

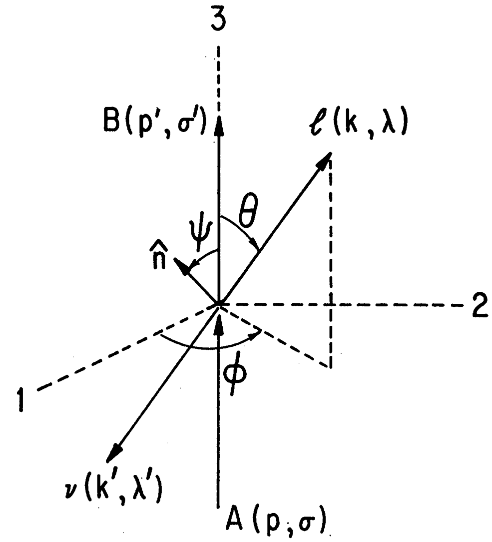

FIG. 1. Kinematics for decay process (1) in the center-of-mass frame of the lepton pair.

$$\begin{aligned} p^\mu &= (E,0,0,p), \\ p'^\mu &= (E',0,0p), \\ k^\mu &= \tfrac{1}{2}(\sqrt{q^2})(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \\ k'^\mu &= \tfrac{1}{2}(\sqrt{q^2})(1, -\sin\theta\cos\phi, -\sin\theta\sin\phi, -\cos\theta), \end{aligned} \tag{5}$$

where

$$\begin{aligned} E &= (q^2+M_A{}^2-M_B{}^2)/2\sqrt{q^2}, \\ E' &= (-q^2+M_A{}^2-M_B{}^2)/2\sqrt{q^2}, \\ p &= [(M_+{}^2-q^2)(M_-{}^2-q^2)]^{1/2}/2\sqrt{q^2}. \end{aligned} \tag{6}$$

$M_\pm$ are defined by Eq. (2) and the lepton mass has been neglected.

Let us now examine the two components of the amplitude (3). The hadronic vertex function, by definition, is a function of the invariant variable q^2 only [cf. Eqs. (5) and (6)]. We denote it by

$$\langle p'\sigma'|J^\mu(0)|p\sigma\rangle = J_{\sigma'\sigma}{}^\mu(q^2). \tag{7}$$

The leptonic vertex function can be written as

$$\begin{aligned} \langle k\lambda, k'\lambda'| j_\mu^\dagger(0)|0\rangle &= \langle 0| j_\mu(0)|k\lambda, k'\lambda'\rangle^* \\ &= \langle 0| j_\mu(0)R(\theta,\phi)|k_s\lambda, k_s'\lambda'\rangle^* \\ &= D^*(\theta,\phi)_\mu{}^\nu\langle 0| j_\nu(0)|k_s\lambda, k_s'\lambda'\rangle^*, \end{aligned} \tag{8}$$

where $k_s = \frac{1}{2}(\sqrt{q^2})(1,0,0,1)$, $k_s' = \frac{1}{2}(\sqrt{q^2})(1,0,0,-1)$, and $D(\theta,\phi)$ is the rotation matrix for the current vector. Again, the last factor in Eq. (8) is a function of q^2 only and can be designated as the leptonic form factor,

$$\langle 0| j^\mu(0)|k_s\lambda, k_s'\lambda'\rangle = j_{\lambda\lambda'}{}^\mu(q^2). \tag{9}$$

We can now substitute Eqs. (7) and (8) into (3) and obtain the expression for the transition amplitude. In doing so, we shall separate the time component (spin-0

[7] Strictly speaking, ψ is the angle between $\mathbf{n}$ and $\mathbf{p}'$ in the A rest frame, this being the frame in which the A polarization vector $\mathbf{n}$ is most unambiguously defined. We define the polarization vector $\mathbf{n}$ in the lepton c.m. frame by referring back to the A rest frame. The two frames are related by a pure Lorentz transformation along the $\mathbf{p}'$ direction.

part) from the space components (spin-1 part) of the currents, denoting the former by the superscript s (for scalar) and, expressing the latter in terms of its spherical components designated by the superscript m ($m=+1, 0, -1$), we obtain

$$f_{\lambda\sigma',\sigma}=(G/\sqrt{2})[j_\lambda^{(s)*}(q^2)J_{\sigma'\sigma}^{(s)}(q^2) - j_\lambda^{(n)*}(q^2)D_{mn}^{(1)*}(\theta,\phi)J_{\sigma'\sigma}^{(m)}(q^2)], \quad (10)$$

where $D_{mn}^{(1)}(\theta,\phi)=e^{-im\phi}d_{mn}^{(1)}(\theta)$ is now the usual rotation matrix for angular momentum 1,[8] and the neutrino helicity index is omitted since it is fixed.

Equation (10) compactly exhibits the essential features of the structure of the effective local interaction Lagrangian. The factorized form with the θ, ϕ dependence explicitly separated from the form-factor dependence on q^2 reflects the current-current interaction picture. The occurrence of the $D_{mn}^{(0)}(\theta,\phi)=\delta_{mn}$ and $D^{(1)}(\theta,\phi)$ functions explicitly reflects the vector nature of the weak current. It is easy to see that *the indices n and m have the physical meaning of being helicities of the currents j and J, respectively.* Angular momentum conservation demands

$$n=\lambda-\lambda', \quad m=\sigma-\sigma'. \quad (11)$$

The lepton current vertex can be explicitly calculated using Eq. (4). If we neglect the lepton mass and substitute the result into Eq. (10), we get

$$f_{-\frac{1}{2}\sigma',\sigma}=2G(\sqrt{q^2})e^{im\phi}d_{m-1}^{(1)}(\theta)J_{\sigma'\sigma}^{(m)}(q^2). \quad (12)$$

In this limit, the electron helicity is 100% left-handed, similar to the antineutrino, which is 100% right-handed. We note that the scalar (time component) form factors contribute only to terms proportional to the lepton mass and thus do not enter Eq. (12).

To gain some feeling about the hadron form factors $J_{\sigma'\sigma}^{(m)}(q)$, we give the connection between these and the conventional invariant form factors. We define the latter by the following equations:

$$\begin{aligned} J^\mu &= V^\mu + A^\mu, \\ \langle p'\sigma'|V^\mu|p\sigma\rangle &= \bar{u}_{\sigma'}(p')[\gamma^\mu f_1(q^2) - i(\sigma^{\mu\nu}q_\nu/M_+)f_2(q^2) + (q^\mu/M_+)f_3(q^2)]u_\sigma(p), \\ \langle p'\sigma'|A^\mu|p\sigma\rangle &= \bar{u}_{\sigma'}(p')[\gamma^\mu\gamma_5 g_1(q^2) - i(\sigma^{\mu\nu}q_\nu\gamma_5/M_+)g_2(q^2) + (q^\mu\gamma_5/M_+)g_3(q^2)]u_\sigma(p). \end{aligned} \quad (13)$$

Also, for convenience, we write,

$$\begin{aligned} [(\sqrt{q^2})/M_+M_-]J_{++}^{(0)} &= V_L - A_L, \\ [(\sqrt{q^2})/M_+M_-]J_{--}^{(0)} &= V_L + A_L, \\ [(\sqrt{q^2})/M_+M_-]J_{-+}^{(1)} &= V_T + A_T, \\ [(\sqrt{q^2})/M_+M_-]J_{+-}^{(-1)} &= V_T - A_T, \end{aligned} \quad (14)$$

where the subscripts L and T stand for *longitudinal* and *transverse*, respectively. Then we get

$$\begin{aligned} V_L &= (1-q^2/M_-^2)^{1/2}[f_1+(q^2/M_+^2)f_2], \\ V_T &= (2q^2/M_+^2)^{1/2}(1-q^2/M_-^2)^{1/2}[f_1+f_2], \\ A_L &= (1-q^2/M_+^2)^{1/2}[g_1-(q^2/M_+M_-)g_2], \\ A_T &= (2q^2/M_-^2)^{1/2}(1-q^2/M_+^2)^{1/2}[g_1-(M_-/M_+)g_2]. \end{aligned} \quad (15)$$

It is readily recognized that V_L and V_T are proportional to the usual G_E and G_M form factors widely used in electromagnetic processes for spin-$\frac{1}{2}$ baryons. The other two form factors A_L and A_T are their axial-vector counterparts. We shall refer to these form factors as the *helicity form factors*.

Before turning to Sec. III, let us make one additional remark. It should be obvious that the procedure used to derive Eq. (12) can be readily applied to local current-current interaction of the scalar and (antisymmetric) tensor types as well. Since under a rotation [see Eq. (8)] the scalar current is invariant and the antisymmetric tensor transforms as two independent vectors (of spin 1), the contributions due to these currents to the scattering amplitude are very similar to the two terms in Eq. (10). If one again calculates the lepton vertex functions (for S and T currents) explicitly, one obtains the following form for the scattering amplitude:

$$f_{\frac{1}{2}\sigma',\sigma}=2G(\sqrt{q^2})[S_{\sigma'\sigma}+e^{im\phi}d_{m0}^{(1)}(\theta)T_{\sigma'\sigma}^{(m)}], \quad (16)$$

where

$$S_{\sigma'\sigma}=\langle p'\sigma'|S(0)|p\sigma\rangle$$

and

$$\begin{aligned} T_{\sigma'\sigma}^{(0)} &= \langle p'\sigma'|(T^{03}-T^{12})|p\sigma\rangle, \\ T_{\sigma'\sigma}^{(1)} &= \langle p'\sigma'|(-T^{01}+T^{23}-iT^{02}+iT^{31})|p\sigma\rangle/\sqrt{2}, \\ T_{\sigma'\sigma}^{(-1)} &= \langle p'\sigma'|(T^{01}-T^{23}-iT^{02}+iT^{31})|p\sigma\rangle/\sqrt{2}. \end{aligned} \quad (17)$$

In comparing Eq. (16) with Eq. (12), we find that the S-T interaction gives rise to 100% right-handed electrons with resulting lepton current helicity 0 [reflected in the second index on the d-function in Eq. (16)], in contrast to the previously mentioned left-handed electron and lepton current helicity (−1) in the $V-A$ interaction. Otherwise Eqs. (12) and (16) are very similar.

III. INTENSITY DISTRIBUTION

The intensity distribution function for process (1) is

$$I=\rho_{\sigma\tau}^A[f_{\frac{1}{2}\sigma',\sigma}f_{\frac{1}{2}\sigma',\tau}^*+f_{-\frac{1}{2}\sigma',\sigma}f_{-\frac{1}{2}\sigma',\tau}^*], \quad (18)$$

where $\rho^A=\frac{1}{2}(1+p\mathbf{n}\cdot\boldsymbol{\sigma})=\frac{1}{2}(1+p\cos\psi\sigma_3-p\sin\psi\sigma_1)$ is the spin-density matrix for particle A with polarization p. Substituting Eqs. (12), (16), and the above expression for ρ^A into Eq. (18), writing out the explicit dependence on the variables ψ, θ, and ϕ, and multiplying by the differential phase-space factor, we can obtain the

[8] Specifically,

$$d^{(1)}(\theta)=\begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -(\sin\theta)/\sqrt{2} & \frac{1}{2}(1-\cos\theta) \\ (\sin\theta)/\sqrt{2} & \cos\theta & -(\sin\theta)/\sqrt{2} \\ \frac{1}{2}(1-\cos\theta) & (\sin\theta)/\sqrt{2} & \frac{1}{2}(1+\cos\theta) \end{pmatrix}.$$

detailed differential distribution,

$$d\Gamma=\frac{G^2M_-^3}{2^7\pi^4}\left(\frac{M_+}{2M_A}\right)^3[(1-q^2/M_+^2)(1-q^2/M_-^2)]^{1/2}dq^2d(\cos\psi)d(\cos\theta)d\phi$$
$$\times\{[I_1\sin^2\theta+I_2\times\tfrac{1}{2}(1+\cos^2\theta)+I_3\cos\theta]+p\cos\psi[I_4\sin^2\theta+I_5\times\tfrac{1}{2}(1+\cos^2\theta)+I_6\cos\theta]$$
$$+p\sin\psi\sin\theta\,(I_7\cos\phi+I_8\sin\phi)+p\sin\psi\sin\theta\cos\theta\,(I_9\cos\phi+I_{10}\sin\phi)\}\,,\quad(19)$$

where I_i consist of simple combinations of the form factors and are functions of q^2 only. We give the explicit expressions for $\{I_i\}$ in a later part of this paper [see Eq. (35) and Appendices A and B] and concentrate on the dependence of $d\Gamma$ on the explicitly displayed variables ψ, θ, and ϕ for the time being.

Provided the assumption about local current-current interaction is valid, Eq. (19) exhibits the maximum amount of information contained in reaction (1). In other words, if we can extract from experimental data the form of the coefficients I_i, then we know everything about reaction (1). In practice, hyperon β-decay processes have very small branching ratios and large statistics are hard to come by. The question is, therefore, how to extract the maximum amount of information with the least number of assumptions under the condition of limited statistics.

As a first step we can integrate over the q^2 variable in Eq. (19). There are two reasons for doing this: First, as just mentioned, we would like to increase the statistics; secondly, since we do not know the exact q^2 dependence of the form factors, integrating over this variable frees us from the necessity of making any assumptions about the q^2 behavior in the following discussions. The result can be written

$$\frac{d^3\Gamma}{d(\cos\psi)d(\cos\theta)d\phi}=\langle I_1\rangle\sin^2\theta+\langle I_2\rangle\tfrac{1}{2}(1+\cos^2\theta)+\langle I_3\rangle\cos\theta+p\cos\psi\,[\langle I_4\rangle\sin^2\theta+\langle I_5\rangle\tfrac{1}{2}(1+\cos^2\theta)+\langle I_6\rangle\cos\theta]$$
$$+p\sin\psi\sin\theta\,[\langle I_7\rangle\cos\phi+\langle I_8\rangle\sin\phi]+p\sin\psi\sin\theta\cos\theta\,[\langle I_9\rangle\cos\phi+\langle I_{10}\rangle\sin\phi]\,,\quad(20)$$

where

$$\langle I_i\rangle=\frac{G^2M_-^3}{2^7\pi^4}\left(\frac{M_+}{2M_A}\right)^3\int_0^{M_-^2}dq^2[(1-q^2/M_+^2)(1-q^2/M_-^2)]^{1/2}I_i(q^2)\,.\quad(21)$$

We now show how the individual coefficients $\langle I_i\rangle$ and combinations of these coefficients can be extracted from experiments by exploiting the complete data in various different ways. In Sec. IV we use this information as the basis for various tests of the nature of the basic interaction responsible for the decay process (1).

(A) The total decay rate is obtained by integrating Eq. (20) over the full phase space; thus,

$$\Gamma=(16\pi/3)(\langle I_1\rangle+\langle I_2\rangle)\,.\quad(22)$$

(B) Letting A_B be the difference of number of events with $0<\psi<\frac{1}{2}\pi$ and $\frac{1}{2}\pi<\psi<\pi$, we obtain the asymmetry of particle B with respect to the spin of particle A:

$$A_B=(8\pi p/3)(\langle I_4\rangle+\langle I_5\rangle)\,.\quad(23)$$

(C) Let us divide the phase space into four parts, characterized by $(0<\theta<\frac{1}{2}\pi,\ 0<\psi<\frac{1}{2}\pi)$, $(0<\theta<\frac{1}{2}\pi,\ \frac{1}{2}\pi<\psi<\pi)$, $(\frac{1}{2}\pi<\theta<\pi,\ 0<\psi<\frac{1}{2}\pi)$, and $(\frac{1}{2}\pi<\theta<\pi,\ \frac{1}{2}\pi<\psi<\pi)$, respectively. Denoting the integrals of Eq. (20) over these four regions by Γ_1, Γ_2. Γ_3, and Γ_4, respectively, we can form the following asymmetries:

$$A_C=\Gamma_1+\Gamma_2-\Gamma_3-\Gamma_4=4\pi\langle I_3\rangle\,,$$
$$A_C'=\Gamma_1-\Gamma_2-\Gamma_3+\Gamma_4=2\pi p\langle I_6\rangle\,.\quad(24)$$

(D) Similarly, let us divide the phase space into four parts characterized by $(|\cos\theta|<|\cos\theta_1|,0<\psi<\frac{1}{2}\pi)$, $(|\cos\theta|<|\cos\theta_1|,\ \frac{1}{2}\pi<\psi<\pi)$, $(|\cos\theta|>|\cos\theta_1|,\ 0<\psi<\frac{1}{2}\pi)$, and $(|\cos\theta|>|\cos\theta_1|,\ \frac{1}{2}\pi<\psi<\pi)$, respectively, where $\cos\theta_1$ is defined by the equation

$$3\int_0^{\cos\theta_1}d(\cos\theta)\sin^2\theta=3\cos\theta_1-\cos^3\theta_1=1\,.\quad(25)$$

Again denoting the integrals of Eq. (20) over the four sections of the phase space by Γ_1, Γ_2, Γ_3, and Γ_4, respectively, we can form the asymmetries

$$A_D=\Gamma_1+\Gamma_2-\Gamma_3-\Gamma_4=8\pi a_1\langle I_2\rangle\,,$$
$$A_D'=\Gamma_1-\Gamma_2-\Gamma_3+\Gamma_4=4\pi p a_1\langle I_5\rangle\,,\quad(26)$$

where

$$a_1=2|\cos\theta_1|-1\,.\quad(27)$$

(E) In complete analogy to (D) we can define four sections of the phase space as above, substituting θ_1 by θ_2 which satisfies

$$3\int_0^{\cos\theta_2}d(\cos\theta)(1+\cos^2\theta)=3\cos\theta_2+\cos^3\theta_2=2\,.\quad(28)$$

The corresponding asymmetries are

$$A_E=16\pi a_2\langle I_1\rangle\,,$$
$$A_E'=8\pi p a_2\langle I_4\rangle\,,\quad(29)$$

where

$$a_2=2|\cos\theta_2|-1\,.\quad(30)$$

(F) Now divide the phase space into four sections characterized by $(0<\theta<\frac{1}{2}\pi,\ \cos\phi>0)$, $(\frac{1}{2}\pi<\theta<\pi,$

$\cos\phi>0$), ($0<\theta<\frac{1}{2}\pi$, $\cos\phi<0$), and ($\frac{1}{2}\pi<\theta<\pi$, $\cos\phi<0$), respectively. As before, denoting the integrals of Eq. (20) over these sections by Γ_1, Γ_2, Γ_3, and Γ_4, we can form the asymmetries

$$A_F=\Gamma_1+\Gamma_2-\Gamma_3-\Gamma_4=\pi^2 p\langle I_7\rangle\,,\qquad A_F'=\Gamma_1-\Gamma_2-\Gamma_3+\Gamma_4=\tfrac{4}{3}\pi p\langle I_9\rangle\,. \tag{31}$$

(G) Finally, dividing the phase space into four sections characterized by ($0<\theta<\frac{1}{2}\pi$, $0<\phi<\pi$), ($\frac{1}{2}\pi<\theta<\pi$, $0<\phi<\pi$), ($0<\theta<\frac{1}{2}\pi$, $\pi<\phi<2\pi$), and ($\frac{1}{2}\pi<\theta<\pi$, $\pi<\phi<2\pi$) and forming the asymmetries as before, we get

$$A_G=\pi^2 p\langle I_8\rangle\,,\quad A_G'=\tfrac{4}{3}\pi p\langle I_{10}\rangle\,. \tag{32}$$

Obviously, there are other asymmetries that one can form. For instance, the electron and neutrino asymmetries A_e, A_ν (with respect to the spin of particle A) as well as the e-ν correlation $A_{e\nu}$ in the rest frame of particle A are commonly used quantities.[5] We shall come back to these other asymmetries in the latter part of Sec. IV.

IV. TESTS FOR VARIOUS ASPECTS OF THEORY

Assuming that the asymmetries mentioned in Sec. III are all experimentally measured, how can we use these numbers effectively as tests for various aspects of the theory? In the following, we shall proceed step by step, starting from tests of the locality and $V-A$ structure of the interaction with no assumptions and ending with detailed tests of the Cabibbo theory with certain minimal assumptions on the q^2 behavior of the form factors.

(a) *Locality.* We remind the reader that Eqs. (19) and (20) were derived under only the assumption of local current-current interaction with pointlike lepton current vertex and two-component neutrinos. Tests on the validity of these expressions are therefore direct tests on this assumption. From the decay rate and asymmetry measurements, two such tests can be made. Thus, from Eqs. (22), (26), and (29) we obtain

$$\Gamma=\frac{2}{3}\left(\frac{A_D}{a_1}+\frac{A_E}{2a_2}\right), \tag{33}$$

and from Eqs. (23), (26), and (29),

$$A_B=\frac{2}{3}\left(\frac{A_D'}{a_1}+\frac{A_E'}{2a_2}\right). \tag{34}$$

We emphasize that these tests do not depend on assumptions on time-reversal invariance or properties of the hadron form factors.

(b) *$V-A$ structure of the weak current.* If locality holds, the next question is whether the weak current is pure $V-A$ in structure as is the case for leptonic and $\Delta S=0$ semileptonic processes. To see what tests are possible, we have to know the contributions of the various currents to the coefficients I_i in Eqs. (19) and (20). For practical reasons, we shall only write out the $V-A$ contributions:

$$\begin{aligned}
I_1&=|V_L|^2+|A_L|^2\,,\\
I_2&=-I_6=|V_T|^2+|A_T|^2\,,\\
-I_3&=I_5=2\,\mathrm{Re}(V_TA_T^*)\,,\\
I_4&=-2\,\mathrm{Re}(V_LA_L^*)\,,\\
I_7&=\sqrt{2}\,\mathrm{Re}(V_TV_L^*+A_TA_L^*)\,,\\
I_8&=-\sqrt{2}\,\mathrm{Im}(V_TA_L^*+A_TV_L^*)\,,\\
I_9&=-\sqrt{2}\,\mathrm{Re}(V_TA_L^*+A_TV_L^*)\,,\\
I_{10}&=\sqrt{2}\,\mathrm{Im}(V_TV_L^*+A_TA_L^*)\,.
\end{aligned} \tag{35}$$

Note that the first four lines are diagonal in the longitudinal and transverse indices while the last four are interference terms. Similarly, I_1, I_2, I_6, I_7, and I_{10} are diagonal in V and A while the others are $V-A$ interference terms.

In view of Eq. (35), we see that measurements on the asymmetries furnish two direct tests on the $V-A$ nature of the weak current. Thus, from the second and third lines of Eq. (35) and Eqs. (21), (24), and (26), we obtain

$$A_D/A_C'=-4a_1/p \tag{36}$$

and

$$A_D'/A_C=-a_1p\,.$$

It is straightforward to show that the presence of S and/or T currents would spoil these relations. We shall give the relevant formulas in Appendix B to provide a basis for experimentally setting limits on these other currents.

More detailed tests of the $V-A$ current structure are also possible, provided additional assumptions are made. Thus, if time-reversal invariance is valid, all form factors must be real. One can then derive more relations among the I_i which, in principle, can also serve as the basis for tests of the $V-A$ interaction. But these additional relations are not preserved under the q^2 integration of Eq. (21). Consequently, comparison with experiment must be made either at fixed q^2 or under further assumptions about the q^2 dependence of the form factors. We shall come back to these tests in Subsec. (d) below in connection with comparison with Cabibbo theory.

(c) *Time-reversal invariance.* It is obvious from Eq. (35) that $I_8=I_{10}=0$ if time-reversal invariance holds. This is true even if other types of current are present. In terms of the measured asymmetries, this means

$$A_G=A_G'=0 \tag{37}$$

if time-reversal invariance is valid. This test is well known; we mention it here for completeness.

(d) *Detailed comparison with Cabibbo theory.* The Cabibbo theory correlates all $\Delta S=0$ and $\Delta S=1$ semileptonic decays in terms of a few parameters in the

TABLE I. Decay rate and 14 measurable asymmetries as functions of the form factors f_1, f_2, g_1, and g_2. The second column gives the zeroth-order term and the third column the first-order term in $\delta(=M_-/M_+)$. All quantities are in units of $\Gamma_0=(G_V \sin\theta_C f_{AB})^2 M_-^5/60\pi^3(1+\delta)^3$. All asymmetries are corrected for 100% target polarization. They are defined to be the difference in rates in the appropriately defined halves of the phase space. This definition differs by a factor of 2 with similar asymmetries defined as the coefficient of the cosine of an angle. The asymmetries in parenthesis are not independent if the tests of locality and $V-A$ interaction, Eqs. (33), (34), and (36), are satisfied. The last column gives the equation number in which the quantity is defined.

	Term of zeroth order in δ	Term of first order in δ	Eq. No.
Γ	$\|f_1\|^2+3\|g_1\|^2$	$-4\,\mathrm{Re}g_1g_2^*$	(22)
$A_{e\nu}$	$\|f_1\|^2-\|g_1\|^2$	$-2\|f_1\|^2-6\|g_1\|^2+4\,\mathrm{Re}(g_1g_2^*)$	
$\frac{1}{2}(A_e-A_\nu)$	$-\|g_1\|^2$	$-\frac{1}{3}[\|f_1\|^2+\|g_1\|^2+\mathrm{Re}(f_1f_2^*-5g_1g_2^*)]$	
$A_C'(A_D,A_E)$	$-\frac{1}{2}\|g_1\|^2$	$\mathrm{Re}(g_1g_2^*)$	(24)
A_F	$(15\pi^2/128)\|g_1\|^2$	$(15\pi^2/256)[\|f_1\|^2+\mathrm{Re}(f_1f_2^*-3g_1g_2^*)]$	(31)
A_B	$-(5/4)\,\mathrm{Re}(f_1g_1^*)$	$(5/12)\,\mathrm{Re}[2(f_1+f_2)g_1^*+f_1g_2^*]$	(23)
$\frac{1}{2}(A_e+A_\nu)$	$\mathrm{Re}(f_1g_1^*)$	$-\frac{1}{3}\,\mathrm{Re}(2f_1g_1^*+g_1f_2^*+f_1g_2^*)$	
$A_C(A_D',A_E')$	0	$-(5/4)\,\mathrm{Re}[(f_1+f_2)g_1^*]$	(24)
A_F'	$-\frac{1}{3}\,\mathrm{Re}(f_1g_1^*)$	$-\frac{1}{3}\,\mathrm{Re}(f_1g_1^*+g_1f_2^*-f_1g_2^*)$	(31)
A_G	$\frac{1}{4}\pi\,\mathrm{Im}(f_1g_1^*)$	$-\frac{1}{4}\pi\,\mathrm{Im}(f_1g_1^*-g_1f_2^*+f_1g_2^*)$	(32)
A_G'	0	$(5\pi/64)\,\mathrm{Im}(-f_1f_2^*+g_1g_2^*)$	(32)

$SU(3)$ limit. We note that in the symmetry limit, all mass differences vanish. Consequently, the range of momentum transfer squared q^2 in hyperon β-decays shrink to a point, i.e., $q^2=0$. Since definitive theoretical predictions are confined to this point, it is necessary to adopt some approximations in the general equations previously derived in order to bring about a detailed comparison between theory and experiment. Thus, we assume as usual, that the invariant form factors f_1, f_2, g_1, and g_2 are constant over the range of q^2 under consideration ($0<q^2<M_-^2$). We shall also evaluate the q^2 integral in Eq. (21) by neglecting terms which are quadratic (or of higher powers) in the small parameter $\delta=M_-/M_+$. The two approximations are consistent with each other since the terms neglected in the first approximation are expected to be of the order M_-^2/Λ^2, where Λ is some effective ($S=1$) vector or axial-vector meson mass.

Using Eqs. (15), (19), (21), and (35), we obtain an expression for $d^3\Gamma/d(\cos\psi)d(\cos\theta)d\phi$ in the form of Eq. (20) with the following coefficients:

$$\begin{aligned}
\langle I_1\rangle &= I_0[3|f_1|^2+5|g_1|^2-4\delta\,\mathrm{Re}(g_1g_2^*)],\\
\langle I_2\rangle &= -\langle I_6\rangle = 4I_0[|g_1|^2-2\delta\,\mathrm{Re}(g_1g_2^*)],\\
-\langle I_3\rangle &= \langle I_5\rangle = 5I_0\delta\,\mathrm{Re}[(f_1+f_2)g_1^*],\\
\langle I_4\rangle &= -\tfrac{5}{2}I_0[3\,\mathrm{Re}(f_1g_1^*)-\delta\,\mathrm{Re}(f_1g_2^*)],\\
\langle I_7\rangle &= \tfrac{15}{16}\pi I_0[2|g_1|^2+\delta|f_1|^2\\
&\qquad +\delta\,\mathrm{Re}(f_1f_2^*-3g_1g_2^*)],\\
\langle I_8\rangle &= -4I_0[-\mathrm{Im}(f_1g_1^*)\\
&\qquad +\delta\,\mathrm{Im}(f_1g_1^*-g_1f_2^*+f_1g_2^*)],\\
\langle I_9\rangle &= -4I_0[\mathrm{Re}(f_1g_1^*)+\delta\,\mathrm{Re}(f_1g_1^*+g_1f_2^*-f_1g_2^*)],\\
\langle I_{10}\rangle &= \tfrac{15}{16}\pi I_0\delta\,\mathrm{Im}(-f_1f_2^*+g_1g_2^*).
\end{aligned}\tag{38}$$

Here

$$I_0=\frac{(G_V\sin\theta_C f_{AB})^2}{2^6\times15\pi^4(1+\delta)^3}M_-^5,$$

where G_V is the universal Fermi coupling constant, θ_C the Cabibbo angle, f_{AB} the f-type $SU(3)$ Clebsch-Gordan coefficient, $M_-=M_A-M_B$, and $\delta=M_-/M_+$.

In Sec. III we have shown how the various experimentally measurable asymmetries yield direct information on the quantities $\langle I_i\rangle$ [cf. Eqs. (22)–(32)]. The ten equations in (38) provide a system of overconstrained equations from which the numbers f_1, f_2, g_1, and g_2 can be solved even without very high statistics. In fact, we can improve the situation even more by adding to the above equations several more, corresponding to the commonly measured electron and neutrino asymmetries A_e and A_ν (with respect to the polarization vector of particle A) as well as the e-ν correlation, both in the rest frame of A.[2]

We have, therefore, a total of 15 kinematically independent quantities (the total rate plus 14 different asymmetries) which are, in principle, experimentally measurable using integrated data alone. If the locality as well as the $V-A$ interaction conditions [Eq. (33), (34), and (36)] are satisfied, the number of independent quantities is reduced by four. In Table I we list the remaining 11 measurable quantities as functions of the four independent invariant form factors (evaluated at $q^2=0$). Note that, if time-reversal invariance holds, these form factors are real and the nine remaining nontrivial measurable numbers are functions of only four unknowns. In any specific experiment, some particular asymmetries may be hard to get because of the bias of the experimental setup. But with many possible asymmetries to choose from, it seems not very hard to measure enough numbers to allow a determination of the form factors with a reasonable confidence level.

Such determination of the values of the form factors without *a priori* prejudice certainly provides a most critical test of the Cabibbo theory.

Quite independent of the Cabibbo theory, there is the interesting question concerning the existence of second-class currents. The concept of second-class currents for $\Delta S=1$ transitions is well defined only in the $SU(3)$ symmetry limit. The existence of such currents is manifested by the nonvanishing of the invariant form factor g_2. Since it has been suggested that second-class currents may exist in nuclear β decay ($\Delta S=0$) with a rather large form factor,[9] it is most interesting to see whether they also show up in hyperon β decays and, if they do, with what magnitude.

V. CONCLUDING REMARKS

(i) The formulas in the text were derived explicitly for the $(e\bar{\nu})$ decay modes of hyperons. It is not hard to see that for the $(e^+\nu)$ decay modes, the same formulas hold with only modifications due to the fact that the leptons (and hence the lepton current) have the opposite helicities. This means, for instance, that the function $d_{m-1}^{(1)}(\theta)$ in Eq. (12) is to be replaced by $d_{m1}^{(1)}(\theta)$ and, consequently, in Eqs. (35) and (38), I_3, I_6, I_7, I_8, I_9, and I_{10} change sign.[10] The corresponding changes in Table I are that A_C, $A_{C'}$, A_F, $A_{F'}$, A_G, and $A_{G'}$ change sign.

(ii) We have emphasized the application of these considerations to the $\Delta S=1$ hyperon decays. It is obvious that the same formalism applies to all semileptonic decay processes. The case for spin-$\frac{1}{2}$ baryon decay where the lepton mass is not negligible is discussed in some detail in Appendix A.

(iii) The $\Delta S=1$ β decay which offers the most interesting possibilities at present is $\Lambda\to p+e+\bar{\nu}$. The rate and e-ν correlation data[2,11] from unpolarized Λ as well as some very crude measurement[11,12] of A_e seems to agree with Cabibbo theory. More refined experiments using polarized Λ with significantly increased statistics are underway.[3,4] Preliminary results from these experiments, though not sufficient to allow any definitive conclusions, seem to give indications of some very interesting surprises.[3,4,13] It is expected that when the complete data become available, detailed analyses such as those proposed in this paper may well be feasible. Since some of the measured quantities which are available for the first time in these new experiments offer a much more sensitive test of the theory than the previously available rate and e-ν correlation measurements, it is hoped that the very important questions concerning the structure of the $\Delta S=1$ current and the validity of the Cabibbo theory can be clarified in the very near future.

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APPENDIX A

In some β-decay processes, for instance, free neutron β decay and the muon decay modes of hyperons, the lepton mass cannot be neglected as is done in the main text of this paper. In that case we can go back to Eq. (10) (which is perfectly general) and evaluate the lepton form factors without any approximation. We get

$$\begin{aligned}-j_{1/2}^{(s)}(q^2)&=j_{1/2}^{(0)}(q^2)=2m(1-m^2/q^2)^{1/2},\\ j_{-1/2}^{(-1)}(q^2)&=2\sqrt{2}(q^2-m^2)^{1/2},\end{aligned}\tag{A1}$$

where m is the lepton mass. Substituting (A1) into (10), we get

$$\begin{aligned}f_{\frac{1}{2}\sigma',\sigma}&=\sqrt{2}mG(1-m^2/q^2)^{1/2}[J_{\sigma'\sigma}^{(s)}(q^2)\\&\qquad+e^{im\phi}d_{m0}^{(1)}(\theta)J_{\sigma'\sigma}^{(m)}(q^2)],\\ f_{-\frac{1}{2}\sigma',\sigma}&=2G(q^2-m^2)^{1/2}e^{im\phi}d_{m-1}^{(1)}(\theta)J_{\sigma'\sigma}^{(m)}(q^2).\end{aligned}\tag{A2}$$

Note that, aside from a simple factor, the second line of (A2) is the same as the old result, Eq. (12), while the first is identical to the contributions of the scalar and tensor currents, Eq. (16), with S replaced by $J^{(s)}$ and $T^{(m)}$ by $J^{(m)}$. Substituting (A2) in Eq. (18), we obtain the differential distribution function $d\Gamma$, which is again of the form Eq. (19) with a trivial change in the kinematic factor,

$$\begin{aligned}d\Gamma=&\frac{G^2M_-^3}{2^7\pi^4}\left(\frac{M_+}{2M_\Lambda}\right)^3\left[\left(1-\frac{q^2}{M_+^2}\right)\left(1-\frac{q^2}{M_-^2}\right)\right]^{1/2}\left(1-\frac{m^2}{q^2}\right)^2\\&\times dq^2d(\cos\psi)d(\cos\theta)d\phi\\&\times\{[I_1\sin^2\theta+I_2\times\tfrac{1}{2}(1+\cos^2\theta)+I_3\cos\theta]\\&+p\cos\psi\,[I_4\sin^2\theta+I_5\times\tfrac{1}{2}(1+\cos^2\theta)+I_6\cos\theta]\\&+p\sin\psi\sin\theta\,[(I_7\cos\phi+I_8\sin\phi)\\&\qquad+p\sin\psi\sin\theta\cos\theta\,(I_9\cos\phi+I_{10}\sin\phi)\},\end{aligned}\tag{A3}$$

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where

$$\begin{aligned}
I_1&=|V_L|^2+|A_L|^2+(m^2/q^2)\tfrac{1}{2}(|V_S|^2+|A_S|^2\\
&\qquad+|V_T|^2+|A_T|^2-|V_L|^2-|A_L|^2),\\
I_2&=|V_T|^2+|A_L|^2+(m^2/q^2)\\
&\qquad\times(|V_S|^2+|A_S|^2+|V_L|^2+|A_L|^2),\\
I_3&=-2\,\mathrm{Re}V_TA_T^*+(m^2/q^2)2\,\mathrm{Re}(V_SV_L^*+A_SA_L^*),\\
I_4&=-2\,\mathrm{Re}V_LA_L+(m^2/q^2)\\
&\qquad\times\mathrm{Re}(-V_SA_S^*+V_LA_L^*+V_TA_T^*),\\
I_5&=2\,\mathrm{Re}V_TA_T^*-(m^2/q^2)2\,\mathrm{Re}(V_SA_S^*+V_LA_L^*),\\
I_6&=-(|V_T|^2+|A_T|^2)-(m^2/q^2)\\
&\qquad\times 2\,\mathrm{Re}(V_SA_L^*+V_LA_S^*),\\
I_7&=\sqrt{2}\,\mathrm{Re}(V_TV_L^*+A_TA_L^*)-(m^2/q^2)\\
&\qquad\times\sqrt{2}\,\mathrm{Re}(V_SA_T^*+A_SV_T^*),\\
I_8&=-\sqrt{2}\,\mathrm{Im}(V_TA_L^*+A_TV_L^*)-(m^2/q^2)\\
&\qquad\times\sqrt{2}\,\mathrm{Im}(V_SV_T^*+A_SA_T^*),\\
I_9&=-\sqrt{2}\,\mathrm{Re}(V_TA_L^*+A_TV_L^*)(1+m^2/q^2),\\
I_{10}&=\sqrt{2}\,\mathrm{Im}(V_TV_L^*+A_TA_L^*)(1+m^2/q^2).
\end{aligned}\tag{A4}$$

Here, in analogy to Eq. (14), we have defined

$$\begin{aligned}
[(\sqrt{q^2})/M_+M_-]J_{++}^{(s)}&=V_S-A_S,\\
[(\sqrt{q^2})/M_+M_-]J_{--}^{(s)}&=V_S+A_S,
\end{aligned}\tag{A5}$$

where V_S and A_S are related to the invariant form factors f_3 and g_3 by

$$\begin{aligned}
V_S&=(1-q^2/M_+^2)^{1/2}[f_1+(q^2/M_+M_-)f_3],\\
A_S&=(1-q^2/M_-^2)^{1/2}[g_1-(q^2/M_+^2)g_3].
\end{aligned}\tag{A6}$$

Integrating Eq. (A3) over the variable q^2, one obtains a differential rate formula identical to Eq. (20) with coefficients

$$\langle I_i\rangle=\frac{G^2M_-^3}{2^7\pi^4}\left(\frac{M_+}{2M_A}\right)^3\int_{m^2}^{M_-^2}dq^2 \times\left[\left(1-\frac{q^2}{M_+^2}\right)\left(1-\frac{q^2}{M_-^2}\right)\right]^{1/2}\left(1-\frac{m^2}{q^2}\right)^2 I_i(q^2),\tag{A7}$$

where $I_i(q^2)$ are given by (A4). The total rate Γ and the asymmetries A_B, A_C, $\cdots$, A_G' are related to the coefficients $\langle I_i\rangle$ by the same formulas as given in Sec. III, Eqs. (22)–(32).

It should be quite obvious that the proposed tests for locality, Eqs. (33) and (34), as well as that for time-reversal invariance, Eq. (37), remain applicable even in this case. On the other hand, because of the similarity between the $V-A$ lepton mass correction terms [first line of Eq. (A2)] and the scalar and tensor current contributions [Eq. (16)], the simple test for $V-A$ interaction, Eq. (36), no longer holds. This is easily checked by examining Eqs. (24), (26), and (A4). More detailed tests of the $V-A$ theory can, however, be carried out following the same procedure as in Sec. IV(d). The 15 kinematically independent measurable quantities mentioned there (total rate plus 14 asymmetries) are reduced by four if the locality and time-reversal invariance conditions hold. The remaining 11 quantities can be expressed in terms of six real form factors if the current is of the usual $V-A$ type. Because of the more involved relations between measured quantities and the invariant form factors, however, a detailed comparison between experiment and theory necessarily needs much more refined data than in the previous case. The relevant formulas corresponding to Eq. (38) and Table I for the present case shall not be explicitly given here. They can be obtained in a straightforward manner from Eqs. (A4)–(A7) and Eqs. (22)–(32) by any interested reader.

APPENDIX B

In connection with the proposed tests of the $V-A$ structure of the weak current, Eq. (36), we give here the contributions of the S and T form factors to the relevant coefficients $\{I_i\}$ of the distribution functions Eqs. (19) and (20). Since the tests involve I_2, I_3, I_5, and I_6 we shall list only these four coefficients. From Eqs. (16), (18), and (19) we obtain

$$\begin{aligned}
I_2&=2(|S|^2+|P|^2+|T_L|^2+|\tilde{T}_L|^2),\\
I_3&=4\,\mathrm{Re}(ST_L^*+P\tilde{T}_L^*),\\
I_5&=-4\,\mathrm{Re}(SP^*+T_L\tilde{T}_L^*),\\
I_6&=-4\,\mathrm{Re}(S\tilde{T}_L^*+PT_L^*),
\end{aligned}\tag{B1}$$

where, in analogy to Eq. (14), we have defined

$$\begin{aligned}
[(\sqrt{q^2})/M_+M_-]S_{++}&=S-P,\\
[(\sqrt{q^2})/M_+M_-]S_{--}&=S+P,\\
[(\sqrt{q^2})/M_+M_-]T_{++}^{(0)}&=T_L-\tilde{T}_L,\\
[(\sqrt{q^2})/M_+M_-]T_{--}^{(0)}&=T_L+\tilde{T}_L.
\end{aligned}\tag{B2}$$

From Eqs. (35) and (B1) we see that the following quantities can be used as measures of deviation from the $V-A$ structure of the weak current:

$$\frac{\langle I_2\rangle+\langle I_6\rangle}{\langle I_2\rangle-\langle I_6\rangle},\quad\frac{\langle I_3\rangle+\langle I_5\rangle}{\langle I_3\rangle-\langle I_5\rangle}.\tag{B3}$$