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Photonuclear Interactions of High-Energy Muons*

G. L. CASSIDAY
University of Utah, Salt Lake City, Utah 84112
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A calculation of the photonuclear energy-loss coefficient b_n is made by extrapolating recent SLAC inelastic muon scattering data to the 10^{12}-eV energy region. Previous methods of calculation are discussed, and attention is drawn to the q^2 dependence of the inelastic cross section. It is also noted that b_n should be A dependent. This is predicted from vector dominance and indicated by the SLAC data on photoproduction off complex nuclei. The value of b_n obtained is 0.21×10^{-6} g^{-1} cm^2. The relationship of b_n to observed cosmic-ray muon intensities is discussed, along with the implications of higher b_n values than are predicted here.

I. INTRODUCTION

A KNOWLEDGE of the energy loss in matter of muons with energies of the order of 1–10 TeV (10^{12}–10^{13} eV) is of great importance to deep-mine cosmic-ray experiments which use the rock cover as an energy analyzer. The determination of muon energies by their range has many advantages over direct spectrograph methods or burst measurements, but the method is limited by uncertainties in the photonuclear energy losses of muons. Although these losses are generally thought to be small compared with the energy losses due to bremsstrahlung and pair production, there are some experimental indications of an anomalously large rate of energy loss,[1] perhaps attributable to the poorly known photonuclear interaction.

We present here a calculation, based both on current theoretical models[2] and on SLAC data on muon-proton inelastic scattering,[3] which indicates that the actual value of the photonuclear energy loss rate is, if anything, somewhat smaller than the conventionally accepted value.[4] Consequently, the source of any anomalous photonuclear losses must be sought outside the framework of the best current models and data.

In addition to its importance as a tool for interpreting muon intensities underground, a knowledge of the photonuclear cross section at these high energies is of great intrinsic interest. Indeed, muon experiments carried out deep underground offer the possibility of exploring the photonuclear interaction at energies several decades higher than those available at present accelerators.

II. RATE OF ENERGY LOSS

The rate of energy loss by cosmic ray muons is expressed by the relation

$$-dE/dx=a+bE,$$

where the parameters a and b are slowly varying functions of energy.[5] The first term, a, gives the contribution from ionization and excitation. The second term, bE, represents the combined contributions from bremsstrahlung, pair production, and photonuclear interactions and may be expressed as $b=(b_b+b_{pp}+b_n)$. At muon energies less than 1 TeV, the dominant mechanism is ionization and excitation, but at higher energies the second set of processes dominates. It is currently believed[4,6] that the value of b is about 3.6×10^{-6} g^{-1} cm^2. The electromagnetic parts are in principle calculable from quantum electrodynamics (QED), but in practice the calculations have proven to be difficult. The generally accepted value of b_b+b_{pp} is 3.3×10^{-6} g^{-1} cm^2, although recent work by Erlykin[7] indicates that b_b+b_{pp} might be as large as 4.0×10^{-6} g^{-1} cm^2. Further calculations to confirm this higher value would be desirable.

That part of b which rests on shakiest ground, both theoretically and experimentally, is b_n. Its value is generally believed[6] to be about 0.3×10^{-6} g^{-1} cm^2, but some estimates[4] are as high as 1.5×10^{-6} g^{-1} cm^2. These estimates, coupled with the calculated value of b_b+b_{pp}, indicate that the over-all b value could lie anywhere within the range of 3.6×10^{-6}–5.5×10^{-6} g^{-1} cm^2. The calculation of b_n presented here should help clear up some of this difficulty.

III. RELATION OF b TO MUON INTENSITIES UNDERGROUND

The integral muon spectrum at sea level can be expressed in the following way:

$$M(>E)=M_0E^{-\gamma},$$

where $M(>E)$ is the muon intensity for energies $>E$ and γ is the measured spectral index at sea level and

* Research supported by the National Science Foundation.

[1] H. E. Bergeson, J. W. Keuffel, M. O. Larson, G. W. Mason, and J. L. Osborne, Phys. Rev. Letters **21**, 1089 (1968).

[2] For example, see the discussions by W. T. Toner and F. J. Gilman, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, 1970).

[3] M. L. Perl, T. Braunstein, J. Cox, F. Martin, W. T. Toner, B. D. Dieterle, T. F. Zipf, W. L. Lakin, and H. C. Bryant, Phys. Rev. Letters **23**, 1191 (1969).

[4] P. J. Hayman, N. S. Palmer, and A. W. Wolfendale, Proc. Phys. Soc. (London) **275A**, 391 (1963).

[5] P. H. Barrett, L. M. Bollinger, G. Cocconi, Y. Eisenberg, and K. Greisen, Rev. Mod. Phys. **24**, 133 (1952).

[6] K. Kobayakawa, Nuovo Cimento **47**, 156 (1967).

[7] A. D. Erlykin, in *Proceedings of the Ninth International Conference on Cosmic Rays, 1965* (The Institute of Physics and the Physical Society, London, 1966), p. 999.

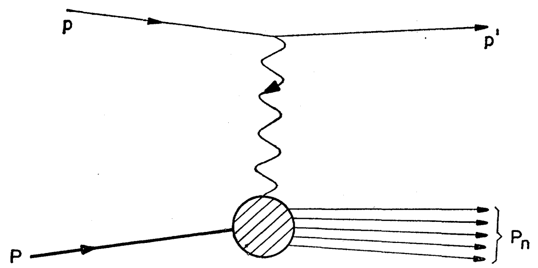

FIG. 1. One-photon-exchange diagram for inelastic muon scattering.

is believed to be about 2.1–2.9.[8,9] A solution of the energy-loss expression of Sec. II gives

$$E=(a/b)(e^{bh}-1).$$

In other words, only muons whose energies are greater than E will survive at depth h underground.[5] (Of course, we are neglecting straggling, but here we only intend to show roughly how b relates to measured underground muon intensities.) Consequently, at depths greater than 2000 hg/cm^2 [1 hectogram (hg) $=10^2$ g], where $e^{bh}\gg 1$, the depth-intensity relation becomes

$$M(h)\simeq Ae^{-\gamma bh}.$$

Since the logarithmic slope of the depth-intensity curve is known, we see that $\gamma\propto 1/b$. Consequently, knowledge of b enables us to make meaningful statements about the muon energy spectrum. Alternatively, direct measurements of the muon energy spectrum (coupled with improved calculations of the electromagnetic energy losses) would yield experimental information about the muon photonuclear interaction at ultrarelativistic energies.

IV. PREVIOUS METHODS OF CALCULATING b_n

The process with which we are concerned is represented by the Feynman amplitude shown in Fig. 1. In the past the cross section for the above process has been calculated by estimating the flux of exchanged virtual photons and coupling this flux to the real total photoproduction cross section.[10] The Williams-Weizsäcker (WW) treatment involves a decomposition of the muon's electromagnetic field into its specific frequency components. If the resulting flux is then multiplied by a constant real total photoproduction cross section σ_γ of about 125 μb and integrated over all possible energy losses ν, one obtains a b_n value of

$$b_n=(2N\alpha/\pi)\sigma_\gamma\simeq 0.35\times 10^{-6}\ \text{g}^{-1}\ \text{cm}^2,$$

where N is Avogadro's number.

The Kessler-Kessler (KK) treatment employs a QED calculation of the muon-photon vertex[10] in the amplitude of Fig. 1, but takes the zero-q^2 limit in calculating the photon flux. The value for b_n then obtained is

$$b_n=(2N\alpha/\pi)\sigma_\gamma[\tfrac{2}{3}\ln(E/\mu)-29/39]\simeq 1.9\text{–}2.6\times 10^{-6}\ \text{g}^{-1}\ \text{cm}^2,$$

where E is the lab energy of the muon and μ is its rest mass.

These values of b_n differ by an order of magnitude in the TeV energy region. There has been some confusion as to which one should be used. Each of these methods neglects q^2 dependencies in the cross section (indeed, each takes $q^2=0$), and thus avoids the question of how nucleon structure affects the process.

Both Diayasu *et al.*[11] and Fowler[12] have realized the deficiencies in the WW and KK treatments and have performed calculations of the cross sections which do allow for q^2 dependencies. Diayasu's method is quite analogous to calculations performed by Drell and Walecka[13] on inelastic electron scattering in which arguments are presented which explicitly show that in all electrodynamic processes connected by single photon exchange with nuclei, two inelastic form factors which are functions of two variables (q^2 and ν, for example) always appear. Diayasu's result for the inelastic cross section is

$$d^2\sigma/dq^2d\nu=N(q^2,\nu)[L(q^2,\nu)+L'(q^2,\nu)],$$

where N is the virtual photon flux, and L and L' are the structure functions. N is dependent only upon what occurs on top of the photon line in Fig. 1 and is presumably well known on the basis of QED. All the structure contained within the nucleon blob is incorporated in Diayasu's L and L' functions. He assumed two possibilities for L and L':

$$\text{(a)}\quad L(q^2,\nu)=(4\pi/\nu)\sigma_\gamma,\qquad L'=0$$

$$\text{(b)}\quad L(q^2,\nu)=\frac{4\pi}{\nu}\left(\frac{1}{1+|q^2|/0.365}\right)^2,\quad L'=0.$$

In each case, taking $L'=0$ is equivalent to setting the cross section for longitudinally polarized photons equal to zero. Assumption (a), the "corelike assumption," is the statement that the nucleon has no structure and is simply a point. Assumption (b), the "cloudlike assumption" is the statement that the nucleon is "soft" or has a q^2-dependent structure. This particular q^2 dependence $(1+|q^2|/0.365)^{-2}$, chosen by Diayasu, basically represents the form factor for an exponential charge distribution of the proton with a radius of 1.4 F.

[8] See the rapporteur talk by A. W. Wolfendale, in *Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, 1969* (Central Research Institute for Physics, Budapest, 1970).
[9] J. C. Osborne, N. S. Palmer, and A. W. Wolfendale, Proc. Phys. Soc. (London) **84**, 911 (1964).
[10] D. Kessler and P. Kessler, Nuovo Cimento **4**, 601 (1956).
[11] K. Diayasu, K. Kobayakawa, T. Murota, and T. Nakano, Suppl. J. Phys. Soc. Japan **17**, 344 (1962).
[12] A. D. Crossland and G. N. Fowler, Nucl. Phys. **53**, 273 (1964).
[13] S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) **28**, 18 (1964).

Diayasu's analysis has been applied to the underground cloud chamber data of Higashi *et al.*,[14] and it is found that the q^2-dependent assumption (b) best explains the muon-energy-transfer distribution. However, the pointlike assumption (a) best explains the observed angular distribution data. The best fit to the q^2 data is 10% pointlike and 90% cloudlike, but the point like effect is not in evidence until $|q^2|$ values of about 1 $(\mathrm{GeV}/c)^2$ are reached, where there are only a few data points. Furthermore, it is reasonable that the pointlike effect should not be seen in the energy-transfer data since most of the energy transfer is due to events of very low $|q^2|$. If a value for b_n is calculated using the Diayasu method and a fit to the Higashi q^2 data, a value of $b_n \sim 0.3\times 10^{-6}$ $\mathrm{g}^{-1}\,\mathrm{cm}^2$ is obtained.

Fowler[12] has calculated the cross section and his results are expressed in terms of $\sigma_L(W^2,q^2)$ and $\sigma_T(W^2,q^2)$, the photoproduction cross sections for production of states of c.m. energy W^2 by longitudinally and transversely polarized photons. However, a model must be chosen in order to calculate these cross sections. He considers the photonuclear process to be dominated by the leading Regge trajectory, in this case the ρ trajectory. His model employs the production of an excited nucleon via this ρ trajectory, and it is assumed that this mechanism is responsible for most of the muons produced in the process. He is not able to distinguish between σ_L and σ_T but shows that their sum (it is their sum which enters into the over-all inelastic cross section) has a form factor dependence of $(1+|q^2|/1.0)^{-2}$. He reaches this conclusion by a Regge parametrization of both the q^2 and the energy-transfer data of Higashi. Furthermore, the value of 1.0 $(\mathrm{GeV}/c)^2$ in the above q^2 factor is very close to what is obtained by using the vector-dominance approach to the problem. Also, Fowler's model fits both distributions of the Higashi experiment equally well, whereas Diayasu's does not. However, Fowler obtains 14.7 μb for σ_γ from his fit to the Higashi data, which differs by a factor of 5 from the value of 72 μb obtained by applying Diayasu's analysis to that very same data. Furthermore, Fowler's value is about ten times less than the value of 125 μb obtained from the SLAC data.[3] Thus, it would seem that the application of Diayasu's method to the energy-loss problem might be preferred. However, the application of Fowler's fit yields a b_n value of about 0.35×10^{-6} $\mathrm{g}^{-1}\,\mathrm{cm}^2$, which agrees closely with the Diayasu result; but in view of the above inconsistencies, further calculation is necessary.

V. PRESENT ESTIMATE OF INELASTIC CROSS SECTION

More recent calculations of the differential muon inelastic scattering cross section have been made by several authors.[3,13] The result is (letting $t=|q^2|$, the

[14] S. Higashi, T. Kitamura, Y. Mishima, S. Miyamoto, Y. Watase, and H. Shibata, Nuovo Cimento **38**, 107 (1965).

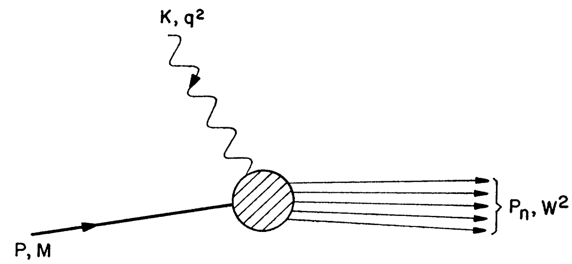

FIG. 2. Real photoproduction diagram for a final-state hadron system having an invariant mass identical to that of the final-state hadron system produced in the inelastic muon scattering diagram of Fig. 1.

absolute value of the 4-momentum transfer)

$$d^2\sigma/dtdK = \Gamma_T(t,K)[\sigma_T(t,K)+\epsilon\sigma_S(t,K)],$$

where $K=(W^2-M^2)/2M=\nu-t/2M$, W^2 is the c.m. energy of the final-state hadron system, M is the proton mass, and K represents the momentum of the virtual photon in c.m. system of the final-state hadrons. It is to be noted that if we compare the amplitude of Fig. 1 with that real photoproduction process (shown in Fig. 2) which results in identical values of W^2 for the final-state hadron system, then K is the relevant variable for comparison and not ν, as has been previously used.[15] $\Gamma_T(t,K)$ is the flux of transversely polarized virtual photons defined by

$$\Gamma_T(t,K)=\frac{\alpha}{2\pi t}\left(\frac{K}{p^2}\right)\left(1-\frac{2\mu^2}{t}+\frac{2EE'-\frac{1}{2}t}{(E-E')^2+t}\right).$$

ϵ is the ratio of the flux of scalar to transverse photons,

$$\epsilon=\left(\frac{2EE'-\frac{1}{2}t}{(E-E')^2+t}\right)\left(1-\frac{2\mu^2}{t}+\frac{2EE'-\frac{1}{2}t}{(E-E')^2+t}\right)^{-1}.$$

$E(E')$ and $p(p')$ are the lab energy and momentum of the incident (scattered) muon. $\sigma_T(t,K)$ and $\sigma_S(t,K)$ are the cross sections for transverse and scalar photons on nucleons (σ_S is related to Fowler's σ_L by a gauge transformation).

The main difficulty in this calculation, as in Fowler's, stems from our lack of a model for σ_T and σ_S. Several theories have been employed in calculating these cross sections (e.g., models based on Regge poles, partons, and vector dominance),[2] but none have met with complete success.

Considerations by Sakurai[16] concerning vector dominance lead to the predictions

$$\sigma_T(K,t)=(1+t/m_\rho^2)^{-2}\sigma_T(K,\,t=0)$$

and

$$R=\frac{\sigma_S(K,t)}{\sigma_T(K,t)}=\left(\frac{t}{m_\rho^2}\right)\left(\frac{K}{K+t/2M}\right)^2\zeta(K),$$

[15] L. N. Hand, Phys. Rev. **129**, 1834 (1963).
[16] J. J. Sakurai, Phys. Rev. Letters **22**, 981 (1969).

where $\sigma_T(K, t=0)$ is equal to $\sigma_\gamma(K)$, the real total photon-nucleon cross section corresponding to the process represented by Fig. 2, m_ρ is the rest mass of the ρ meson, and $\zeta(K)$ is the ratio of the ρ-nucleon total cross sections with helicity states 0 and ± 1, respectively. At high energies, $\zeta \simeq 1$ and the above model then predicts a value of R which increases linearly with t. Also note that

$$\sigma_T + \epsilon\sigma_S = \left(1 + \frac{t}{m_\rho^2}\right)^{-2} \times \left[1 + \frac{t}{\epsilon m_\rho^2}\left(\frac{K}{K+t/2M}\right)^2 \zeta(K)\right]\sigma_\gamma(K).$$

At high energies and small momentum transfers, it is seen that

$$\sigma_T + \epsilon\sigma_S \simeq (1+t/m_\rho^2)^{-1}\sigma_\gamma(K).$$

It is observed that this inverse linear dependency fits the SLAC data[3] quite well, and thus seems to bear out the vector-dominance prediction. However, the SLAC inelastic electron data at large t are in disagreement with the vector-dominance prediction[2] that R should rise linearly with t. One would expect that this will be true of the muon data also, but this remains to be seen. At any rate, it is not necessary to use any specific model in order to calculate the energy loss at these ultra-relativistic energies. Instead, we simply take for the q^2 dependence an empirical fit to the SLAC muon data and extrapolate to our energy region. The best estimate of the inelastic muon-nucleon cross section is then

$$\frac{d^2\sigma}{dtdk} = \Gamma_T(t,K)\left\{\frac{1}{1+t/m_\rho^2}\right\}\sigma_\gamma(K).$$

A final point which has been neglected in the previous calculations and should be mentioned is the possibility of an A dependence of the cross section. Vector dominance predicts that the cross section for real photons off complex nuclei, $\sigma_{\gamma A}$, should go like $A^{2/3}\,\sigma_\gamma$ at high energy.[17–21] On the other hand, a long mean free path of photons in nuclear matter implies a cross section which goes as A^1. This apparent paradox is resolved if the photon couples to the ρ meson which has a mean free path in nuclear matter comparable to the size of nucleons; shadowing would then be expected since the interaction should be confined mostly to the nuclear surface. The total photonuclear cross sections measured by Caldwell *et al.* at SLAC[22] tend to support this idea, although not quite to the full extent of the vector-dominance prediction. Nonetheless, the shadowing effect appears to be increasing with energy. For example, at 20 GeV $\sigma_{\gamma A} \sim \sigma_\gamma A^{0.9}$, and at cosmic ray energies, this effect should be complete.

VI. CALCULATED ENERGY-LOSS COEFFICIENT

Recalling the relation between K, ν, t, we can express the cross section $d^2\sigma/dtdK(t,K)$ as $d^2\sigma/dtdK(\nu,t)$. b_n is then

$$b_n = \frac{N}{AE}\int_{\nu_{\min}}^{\nu_{\max}}\int_{t_{\min}}^{t_{\max}} \nu\frac{d^2\sigma}{dtdK}(\nu,t)dtd\nu\,,$$

where

$$\frac{d^2\sigma}{dtdK}(\nu,t) = \Gamma_T(t,\nu)\left(1+\frac{t}{m_\rho^2}\right)^{-1}\sigma_\gamma(K)A^{f(K)}.$$

The above form shows the separation of the cross section into three distinct parts: (a) the virtual photon flux, (b) the form factor (ff) or q^2 dependence due to the ρ propagator (or nucleon structure), and (c) the real photonuclear cross section. We have written σ_γ as a function of K to emphasize that K is the relevant variable for comparison to the real photonucleon cross section. Furthermore, the A dependence of the real photonucleon cross section is explicitly exhibited in the factor $\sigma_{\gamma A}(K) = \sigma_\gamma(K)A^{f(K)}$.

For $\sigma_\gamma(K)$ we have used a fit to the data of Cone *et al.*[23] for $0 \le K \le 1.5$ GeV. This includes the resonance contribution. For $K > 1.5$ GeV we have considered $\sigma_\gamma(K)$ to be a constant and equal to 125 μb as taken from the SLAC data[3] and in accordance with the vector-dominance prediction. For the exponent $f(K)$ we have used the following fit:

$$f(K) = 1\,, \qquad K \le 1.5 \text{ GeV}$$
$$f(K) = 0.33e^{(1.5-K)/37.5} + 0.67\,, \quad K > 1.5 \text{ GeV}.$$

There is no theoretical justification for this particular fit. It does yield agreement with the observed A dependence of the SLAC data[22] and it tends toward $\frac{2}{3}$ at large K.

Elastic scattering kinematics as well as inelastic have been considered in obtaining the limits of integration:

(i) $\nu_{\min}$ has been chosen equal to M_π, the pion rest mass, since $\sigma_\gamma(K)$ is zero for $K < M_\pi$.

(ii) $\nu_{\max}$ is calculated from the kinematics of an elastic head-on collision and the result is

$$\nu_{\max} = E[1-(M/2E)(1+\mu^2/M^2)].$$

[17] F. J. Gilman, SLAC Report No. SLAC-PUB-589, 1969 (unpublished); in Proceedings of the Conference on Particle Interactions at High Energies, University of Toronto, 1969 (unpublished).
[18] K. Gottfried and D. R. Yennie, Phys. Rev. **182**, 1595 (1969).
[19] L. Stodolsky, Phys. Rev. Letters **18**, 135 (1967).
[20] S. J. Brodsky and J. Pumplin, Phys. Rev. **182**, 1794 (1969).
[21] G. von Bochmann, B. Margolis, and C. L. Tang, Phys. Rev. Letters **24**, 483 (1970).
[22] D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, F. V. Murphy, and D. E. Yount, Phys. Rev. Letters **23**, 1256 (1969).
[23] A. A. Cone, K. W. Chen, J. R. Dunning, Jr., G. Hartwig, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. **156**, 1490 (1967).

(iii) t_{min} is calculated from the kinematics of forward scattering with the result $t_{min}=\mu^2\nu^2/E(E-\nu)$.

(iv) t_{max} is calculated from the kinematics of elastic scattering when $K=0$. In other words, $W^2=M^2$, and no particles are produced. The limit obtained is $t_{max}=2M\nu$.

In order to evaluate the above double integral, a numerical integration was performed using the method of Gaussian quadrature. By using ten points, 0.1% accuracy is obtained. The results of the calculation are presented in the first line of Table I. Also are presented are those results which would be obtained under the assumptions that the cross section is independent of q^2 and varies linearly with A. These latter results are presented primarily for comparison with the results of other workers, where either or both of those assumptions were used.

The value of b_n obtained in this calculation for this energy region is $0.24–0.21\times10^{-6}$ g^{-1} cm^2. However, if $\sigma_{\gamma A}$ is linear in A at high energies, then a value for b_n as large as 0.57×10^{-6} g^{-1} cm^2 could be obtained. The $A^{2/3}$ variation has not been experimentally verified although, as mentioned before, for photon energies up to 20 GeV there is exhibited a definite tendency for $\sigma_{\gamma A}$ to vary less than linearly with A. These results then imply an over-all b value of 3.5×10^{-6} g^{-1} cm^2.

TABLE I. Values of the energy loss coefficient b_n as a function of energy. The WW and KK results are quoted for a value for σ_γ of 125 μb as are the results of the present calculation. The Diayasu and Fowler results are quoted for values of σ_γ of 72 μb and 14.7 μb, respectively. All results are based upon an A for standard rock of 22.5.

	Entries are values of b_n (10^{-6} g^{-1} cm^2) for a muon energy E (TeV) of					
Fits	1	5	10	15	20	25
Present method						
$A^{2/3}$, inverse linear q^2	0.24	0.21	0.21	0.21	0.21	0.21
A^1, no q^2	1.35	1.44	1.52	1.56	1.60	1.62
A^1, inverse linear q^2	0.57	0.57	0.57	0.57	0.57	0.57
Previous methods						
WW	0.35	0.35	0.35	0.35	0.35	0.35
KK	1.87	2.24	2.41	2.50	2.57	2.62
Diayasu	0.29	0.30	0.31	0.31	0.31	0.31
Fowler	0.31	0.36	0.38	0.40	0.41	0.41

VII. DISCUSSION

Several comments on Table I are in order. (a) It is obvious why the KK method yields such large values of b_n: The q^2 dependence has been neglected, which is what would be expected for point nucleons. (b) The present work with an inverse linear q^2 fit yields a value somewhat larger than those obtained using the methods of Diayasu and Fowler. This is to be expected since both of these methods are based on the experimental data of Higashi and use an inverse quadratic q^2 fit (although the Diayasu method uses a 10% point nucleon fit). Furthermore, the Diayasu method, in order to fit the Higashi data, has used a value for σ_γ of 72 μb, whereas 125 μb has been used in this calculation. Using the 125 μb figure, the Diayasu method would yield a b_n value of about 0.52×10^{-6} g^{-1} cm^2, which compares favorably with our results using an A^1 dependence and an inverse linear q^2 fit. It is not possible to make such a direct comparison with the results obtained using the Fowler method, since σ_γ is one of four parameters which cannot be chosen independently of each other and which appear naturally as a result of fitting the Higashi data. (c) The WW method is in reasonable agreement with the q^2-dependent methods. In light of our present discussion, this should be seen merely as a chance result. It is most certainly due to the manner in which the WW spectrum is cut off.

Early results of the Utah group[1] seemed to call for a b value of $\sim6\times10^{-6}$ g^{-1} cm^2 in order to reconcile underground muon intensities with their flat muon production spectrum. However, new and more comprehensive data are consistent with a conventional b value, particularly in view of uncertainties in the primary proton spectral index,[24] and hence, in the muon production spectrum. Somewhat higher b values are still not ruled out. Indeed, in addition to the Utah effect, observations of large horizontal air showers by the INS group at Tokyo[25] and measurements deep underground at the Kolar Gold Fields[26] indicate possible anomalous muon behavior which could result in a larger rate of energy loss than is predicted here. Clearly, a measurement of the muon spectral index in the TeV region at sea level should resolve this difficulty. Alternatively, a direct measurement of the muon inelastic cross section at these energies would clear up the b value situation. Consideration is being given to making this measurement at Utah.

If the b value is indeed as large as was earlier stated by Utah, most probably the increase would be in b_n. But it would have to be larger than 0.21×10^{-6} g^{-1} cm^2 by about a factor of 10. Such a large b value would imply that either σ_γ must increase with energy, or at q^2 values greater than 1 (GeV/c)^2 the nucleon inelastic form factor must increase with q^2 or $\sigma_{\gamma A}$ must vary more strongly with A than $A^{2/3}$ at high energy. The occurence of any combination of these possibilities is sufficient to rule out the idea of vector dominance.

[24] P. Kiraly and A. W. Wolfendale, J. Phys. A. (to be published); J. W. Keuffel (private communication).

[25] K. Mizutani and K. Mori, Inst. Nucl. Study, Univ. Tokyo, INS J.-114 (1970).

[26] M. G. K. Menon, S. Naranan, V. S. Narasimham, K. Hinotani, N. Ito, S. Miyake, R. Craig, D. R. Creed, J. L. Osborne, and A. W. Wolfendale, Proc. Roy. Soc. (London) **A301**, 137 (1967).

Furthermore, such a breakdown in either the form-factor dependence or the constancy of σ_γ implies consequences more dramatic than the rejection of vector dominance; anomalous muon behavior at high energies would be indicated.

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Hyperon Beta Decay*

PAUL H. FRAMPTON

Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637
and
CERN, Geneva, Switzerland†

AND

WU-KI TUNG

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637

(Received 16 November 1970)

Polarized-hyperon β decay is considered in the center-of-mass frame of the outgoing leptons. Simple and exact expressions are obtained for the differential intensity distribution function under the assumption of local current-current interaction and two-component neutrinos. It is shown that by exploiting only the fully integrated data, it is possible to make a large number of tests on the structure of the $\Delta S=1$ current even with somewhat limited statistics. These include direct tests on the locality and $V-A$ nature of the interaction without additional assumptions, as well as detailed and rather stringent tests on the Cabibbo theory.

I. INTRODUCTION

THE structure of weak interactions has largely been deduced from the accumulated experimental data on leptonic and $\Delta S=0$ semileptonic decay processes. The resulting $V-A$ current-current interaction picture was naturally generalized to the $\Delta S=1$ semileptonic processes. With the additional assumption of universality expressed through the $SU(3)$ current algebra of Gell-Mann, the Cabibbo theory[1] represents a unified picture for all semileptonic processes. The predictions of this theory are consistent with the available data (mostly rates, plus some angular correlations) on various hyperon β-decay processes.[2] Because of the small branching ratios for these processes ($\sim10^{-4}$–10^{-3}), however, detailed experimental information has not so far become available. Conclusive verification of the theoretical picture, therefore, does not exist.

With the gradual accumulation of data on β decay from polarized hyperons,[3,4] this situation may soon change. Finally, a more critical and detailed comparison of experimental data with theory seems to be within our reach. In this paper we present a compact and yet complete description of polarized hyperon β-decay processes.[5] Under only the general assumptions of locality and two-component neutrinos, we derive a simple expression for the differential intensity distribution in which the dependence on three of the four independent variables is explicitly displayed.[6] Based on this formula, we propose methods for effectively extracting important information from experimental data even with limited statistics. A series of direct tests on the locality and $V-A$ nature of the interaction without additional assumptions, as well as detailed tests of the Cabibbo theory, are proposed.

The proposed tests are particularly simple if the lepton mass is negligible as compared to the baryon mass difference. This is the case for the electron decay modes of hyperons. We therefore concentrate on this case in the main text. The muon decay modes can also be analyzed effectively with the present method. Since the results are slightly more complicated and harder

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† Present address.
[1] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[2] For a review as well as original references, see H. Filthuth, in Topical Conference on Weak Interactions, CERN Report No. 69-7, 1969 (unpublished).
[3] K. H. Althoff *et al.* (CERN-Heidelberg Collaboration), in Fifteenth International Conference on High Energy Physics, Kiev, 1970 (unpublished).
[4] J. Lindquist *et al.* (Argonne–Chicago–Ohio State–Washington Collaboration), in Fifteenth International Conference on High Energy Physics, Kiev, 1970 (unpublished).
[5] For previous treatments of this problem, see, for example, D. R. Harrington, Phys. Rev. 120, 1482 (1960); J. M. Watson and R. Winston, *ibid.* 181, 1907 (1969); M. Nieto, Rev. Mod. Phys. 40, 140 (1968); V. Linke, Nucl. Phys. B12, 669 (1969); B23, 376 (1970).
[6] The method used in this paper is similar to that of T. P. Cheng and Wu-Ki Tung, Phys. Rev. D 3, 733 (1971), for neutrino scattering processes.