Possible Test of the $\Delta S = \Delta Q$ Rule in $K_{\mu 3}{}^0$ Decay in a Regeneration Experiment*

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The time dependence of the transverse component of muon polarization in $K_{\mu 3}{}^0$ decay is calculated under the assumption that the $\Delta S = \Delta Q$ rule is violated. The initial beam is taken to be an arbitrary superposition of $K_L{}^0$ and $K_S{}^0$ states. Four form factors, $f_\pm$ for the $\Delta S = \Delta Q$ amplitude and $g_\pm$ for the $\Delta S = -\Delta Q$ amplitude, enter into the description of the decay. It is shown that when $f_\pm$ and $g_\pm$ are not proportional, the transverse polarization is significantly different from zero for the first few lifetimes of $K_S{}^0$ even when there is no T violation in the decay. The possibility of experimentally observing this transverse polarization with muon decays from a regenerated $K_L{}^0$ beam is examined. It is found that the present experimental limits on $f_\pm$ and $g_\pm$ allow a transverse polarization as large as 15% at the exit face of the regenerator. This transverse polarization can, therefore, be used to test the $\Delta S = \Delta Q$ rule in $K_{\mu 3}{}^0$ decay.

INTRODUCTION

THE experimental status of the $\Delta S = \Delta Q$ rule in $K_{l3}{}^0$ decay is still unclear.[1] Several experimental groups[2] have attempted to test this selection rule by measuring the time dependence of the decay rates in $K_{e3}{}^0$ and $K_{\mu 3}{}^0$ decays. The results from these experiments range from no violation of the $\Delta S = \Delta Q$ rule to a $\Delta S = -\Delta Q$ amplitude that is almost 20% of the $\Delta S = \Delta Q$ amplitude.[3] In view of this wide range in the experimental results based on the observation of the decay rates, it is of interest to determine whether the $\Delta S = \Delta Q$ rule in $K_{l3}{}^0$ decay can be tested more accurately in some other way. Moreover, since the values of the $\Delta S = \Delta Q$ form factors based on the muon polarization measurements and on the branching ratio measurements are discrepant, it would be desirable to have measurements for the $\Delta S = -\Delta Q$ form factors that do not depend directly on the decay rate.

We present here another method for obtaining information about the $\Delta S = -\Delta Q$ amplitude in the decay $K^0 \to \pi^- \mu^+ \nu$.[4] The method is based on the time dependence of the transverse component[5] of muon polarization resulting from an interference between the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ amplitudes. The transverse polarization of the muon corresponds to the expectation value of the operator $\Omega = \boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{q})$, where $\boldsymbol{\sigma}$ is the muon spin operator, and $\mathbf{q}$ and $\mathbf{k}$ are the muon and neutrino three-momenta, respectively. Since spins and momenta are odd under time reversal, the operator Ω is also odd under time reversal. Consequently, if final-state interaction is neglected,[6] the expectation value of Ω would normally be zero if there is no T violation in the decay.[7] However, for a state that is a superposition of $K_L{}^0$ and $K_S{}^0$ states, the expectation value of Ω can, in general, be nonzero, even when there is no T violation, provided there is a violation of the $\Delta S = \Delta Q$ rule.[8] This nonvanishing transverse polarization can, therefore, be used to test the $\Delta S = \Delta Q$ rule in $K_{\mu 3}{}^0$ decay.

A proposal similar to the present one was made some years ago by Kenny and Sachs.[9] Their proposal, however, envisaged an experimental situation where the beam at $t = 0$ was a pure K^0 beam. Since the transverse polarization is appreciable only during the first few lifetimes of $K_S{}^0$, this situation can be realized when the K^0 is produced in a strong-interaction process, and the experiment is performed close to the production site of the K^0. Such an experimental setup would encounter serious background problems, and this type of experiment has not been performed thus far.

We have examined the possibility of observing the time dependence of the transverse polarization starting with a regenerated $K_L{}^0$ beam. An experiment of this type would have fewer background problems compared to a pure K^0 beam experiment. Furthermore, because of the nonzero value of the strong-interaction regeneration phase, there emerges the interesting and experimentally exploitable result that the transverse polarization is, in general, maximum and nonvanishing at

* Research (Yale Report No. 2726-569) supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1) 2726.

[1] W. J. Willis, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (North-Holland, Amsterdam, 1968); J. W. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968); C. Rubbia, in Proceedings of the Topical Conference on Weak Interactions, CERN, 1969 (unpublished); K. Tipsis, in MIT Report No. 563, 1969 (unpublished).

[2] L. S. Littenberg, J. H. Field, O. Piccioni, W. A. M. Welhop, S. S. Murty, P. H. Bowles, and T. H. Burnett, Phys. Rev. Letters **22**, 654 (1969); B. R. Webber, LRL Report No. UCRL 19226 (unpublished); K. Tipsis (Ref. 1); B. R. Webber, F. T. Solmitz, F. S. Crawford, Jr., and M. Alston-Garnjost, Phys. Rev. Letters **21**, 498 (1968).

[3] See K. Tipsis, Ref. 1.

[4] For definiteness we consider only the $\pi^- \mu^+ \nu$ mode in this paper. None of our results will be changed if we considered the $\pi^+ \mu^- \nu$ mode instead.

[5] We define the transverse direction as $\mathbf{T} = \mathbf{q} \times \mathbf{k} / |\mathbf{q} \times \mathbf{k}|$, where $\mathbf{q}$ and $\mathbf{k}$ are the muon and neutrino three-momenta, respectively.

[6] The contribution of the final-state electromagnetic interaction to the transverse polarization is of the order of 0.2%. See N. Byers, S. W. MacDowell, and C. N. Yang, in *Proceedings of the International Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (IAEA, Vienna, 1965), p. 953.

[7] J. J. Sakurai, Phys. Rev. **109**, 980 (1958).

[8] The present calculation is a demonstration that this is so. A formal proof is also possible, and will be given elsewhere.

[9] B. G. Kenny and R. G. Sachs, Phys. Rev. **138**, B943 (1965).

$t=0$, even when there is no T violation in the decay. This is in contrast to the case studied by Kenny and Sachs, where, without T violation, the transverse polarization always vanished at $t=0$.

We find that, unless the $\Delta S=\pm\Delta Q$ form factors are proportional, the present experimental limits on the $\Delta S=\pm\Delta Q$ form factors together with experimentally realizable regeneration allow a transverse polarization as large as 15% at the exit face of the regenerator. Since such a transverse polarization is well within the present experimental capabilities,[10] a search for this transverse polarization offers a feasible way to test the $\Delta S=\Delta Q$ rule in $K_{\mu 3}{}^0$ decay, and thus to improve the present experimental limits on the $\Delta S=-\Delta Q$ form factors.

CALCULATION

We define the $\Delta S=\pm\Delta Q$ decay amplitudes as[11]

$$\mathfrak{M}(K^0\to\pi^-\mu^+\nu)=(G/\sqrt{2})\bar{u}_\nu(k)(1+\gamma^5)[(\gamma\cdot Q+\gamma\cdot P)f_+ +(\gamma\cdot Q-\gamma\cdot P)f_-]v_\mu(q)\,,\quad(1)$$

$$\mathfrak{M}(\bar{K}^0\to\pi^-\mu^+\nu)=(G/\sqrt{2})\bar{u}_\nu(k)(1+\gamma^5)[(\gamma\cdot Q+\gamma\cdot P)g_+ +(\gamma\cdot Q-\gamma\cdot P)g_-]v_\mu(q)\,,\quad(2)$$

where G is the Fermi constant, $f_\pm$ and $g_\pm$ are dimensionless form factors for the $\Delta S=\Delta Q$ and the $\Delta S=-\Delta Q$ transitions, respectively, and Q, P, q, and k are the kaon, pion, muon, and neutrino four-momenta, respectively. In general, $f_\pm$ and $g_\pm$ are functions of the square of the momentum transferred between the hadrons. However, we shall assume that $f_\pm$ and $g_\pm$ are constants.[1] Time-reversal invariance requires that $f_\pm$ and $g_\pm$ be relatively real,[12] and the $\Delta S=\Delta Q$ rule requires that $g_+=g_-=0$.

We start with a general superposition of $K_L{}^0$ and $K_S{}^0$ states[13] at $t=0$ of the form

$$K_\rho{}^0=(K_L{}^0+\rho K_S{}^0)/(1+|\rho|^2)^{1/2}\,,\quad(3)$$

where $\rho=|\rho|e^{i\phi_\rho}$ is a complex regeneration parameter. Then from Eqs. (1)–(3), the time dependence of the decay amplitude for $K_\rho{}^0\to\pi^-\mu^+\nu$ is given by

$$\mathfrak{M}(K_\rho{}^0\to\pi^-\mu^+\nu)(t)=G\bar{u}_\nu(k)(1+\gamma^5)[\gamma\cdot QA_1(t)+MA_2(t)]v_\mu(q)\,,\quad(4)$$

where

$$A_1(t)=[\rho F_1e^{-im_St-\lambda_St/2}+F_2e^{-im_Lt-\lambda_Lt/2}]/(1+|\rho|^2)^{1/2}\,,$$

$$A_2(t)=[\rho G_1e^{-im_St-\lambda_St/2}+G_2e^{-im_Lt-\lambda_Lt/2}]/(1+|\rho|^2)^{1/2}\,,$$

$$F_1=(f_++rg_+),\quad F_2=(f_+-rg_+)\,,$$

$$G_1=(m/2M)[(f_+-f_-)+r(g_+-g_-)]\,,$$

$$G_1=(m/2M)[(f_+-f_-)+r(g_+-g)]\,,$$

$m_{S,L}$ and $\lambda_{S,L}$ are the mass and decay rate for the $K_{S,L}{}^0$ meson; r is a complex parameter indicating the effect of CP violation; and M and m are the kaon and muon masses, respectively.

The time dependence of the transverse component of muon polarization at a given point on the Dalitz plot can now be calculated from Eq. (4). The result, referred to the kaon rest system, is[14]

$$\mathcal{P}_T(t)=\frac{2|\mathbf{k}\times\mathbf{q}|\operatorname{Im}[A_1(t)A_2(t)^*]}{|A_1(t)|^2(E_\mu E_\nu+\mathbf{q}\cdot\mathbf{k})+|A_2(t)|^2(E_\mu E_\nu-\mathbf{q}\cdot\mathbf{k})-2mE_\nu\operatorname{Re}[A_1(t)A_2(t)^*]}\,.\quad(5)$$

For the special case when T invariance holds,[15] Eq. (5) reduces to

$$\mathcal{P}_T(t)=(2m/M)(f_+g_--f_-g_+)|\rho|e^{-(\lambda_S+\lambda_L)t/2}\times\sin(\Delta mt+\phi_\rho)|\mathbf{k}\times\mathbf{q}|/\Delta(t)\,,\quad(6)$$

where

$$\begin{aligned}\Delta(t)=&|\rho|^2e^{-\lambda_St}[(E_\mu E_\nu+\mathbf{q}\cdot\mathbf{k})F_1{}^2+(E_\mu E_\nu-\mathbf{q}\cdot\mathbf{k})G_1{}^2-2mE_\nu F_1G_1]\\&+e^{-\lambda_Lt}[(E_\mu E_\nu+\mathbf{q}\cdot\mathbf{k})F_2{}^2+(E_\mu E_\nu-\mathbf{q}\cdot\mathbf{k})G_2{}^2-2mE_\nu F_2G_2]+2|\rho|e^{-(\lambda_S+\lambda_L)t/2}\cos(\Delta mt+\phi_\rho)\\&\times[(E_\mu E_\nu+\mathbf{q}\cdot\mathbf{k})F_1F_2+(E_\mu E_\nu-\mathbf{q}\cdot\mathbf{k})G_1G_2-mE_\nu(F_1G_2+F_2G_1)]\,.\end{aligned}$$

We note from Eq. (6) the interesting phenomenon, first noted by Kenny and Sachs, that even when there there is no T violation, the transverse polarization does not generally vanish, provided that there is a violation of the $\Delta S=\Delta Q$ rule. However, if T invariance holds and the $\Delta S=\pm\Delta Q$ form factors satisfy the relation $(f_+g_--f_-g_+)=0$, the transverse polarization will vanish even though there may be a violation of the $\Delta S=\Delta Q$ rule. The condition $(f_+g_--f_-g_+)\neq 0$ for nonvanishing transverse polarization can be understood simply if one notes that it is equivalent to the requirement that for two amplitudes to interfere it is necessary that they be linearly independent. Two special cases of interest when the relation $(f_+g_--f_-g_+)=0$ holds are (a) $g_+=g_-=0$, in which case there is no violation of the $\Delta S=\Delta Q$ rule, and (b) $f_-=g_-=0$, which is the exact $SU(3)$ limit. In all cases when the relation $(f_+g_--f_-g_+)=0$ holds, the method suggested here will not provide a test for the $\Delta S=\Delta Q$ rule.

RESULTS AND DISCUSSION

It is convenient for the parametrization of our results to define combinations of $f_\pm$ and $g_\pm$ in the form $\xi=f_-/$

[10] I thank Professor Willis for information on the current experimental capabilities.

[11] Our notation and conventions are identical to those in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), Appendix A.

[12] We neglect final-state electromagnetic interactions.

[13] We define $K_S{}^0=(K^0+r\bar{K}^0)/\sqrt{2}$, and $K_L{}^0=(K^0-r\bar{K}^0)/\sqrt{2}$, where r indicates the effect of CP violation, and equals unity when CP is conserved.

[14] N. Cabibbo and A. Maksymowicz, Phys. Letters **9**, 352 (1964); **11**, 360(E) (1964); **14**, 72(E) (1965).

[15] We also set $r=1$. and define $\Delta m=(m_L-m_S)$.

f_+, $\delta=g_+/f_+$, and $\zeta=g_-/f_+$. The measurement of muon polarization in $K_{\mu 3}$ decay gives values of ξ around -1.0, and measurement of the branching ratio in $K_{l3}{}^0$ decay gives values of ξ around zero.[1] Time dependence of the decay rate in $K_{e3}{}^0$ decay gives $|\delta|\lesssim 0.2$, and time dependence of the decay rate in $K_{\mu 3}$ decay gives a functional relation between ξ, δ, and ζ.[16] These experimental limits on ξ, δ, and the $\Delta S=-\Delta Q$ amplitude in $K_{\mu 3}{}^0$ decay restrict the range of values of ζ to $|\zeta|\lesssim 1.5$.[17] The regeneration parameter ρ has been measured as a function of kaon momentum for a copper regenerator.[18] We have taken $|\rho|=0.1$ and $\phi_\rho=-45°$ which approximate the value of ρ for a "thin" copper regenerator and kaon momentum of around 2 GeV/c.[19]

For numerical computation we have chosen $E_\mu=200$ MeV and $E_\pi=160$ MeV. This is a point in the region of the Dalitz plot where the transverse polarization is kinematically enhanced. Then from Eq. (6) we have computed $\mathcal{P}_T(t)$ for $\xi=-1$, 0; $\delta=-0.1$, 0.1; $\zeta=-1.0$, -0.5, 0, 0.5, 1.0.[20] The results are shown in Figs. 1 and 2, where we have plotted percentage transverse polarization against $\tau=\lambda_S t$.

The common feature of all the graphs is the relatively large transverse polarization at $\tau=0$ and a secondary maximum for τ between 3 and 4. Also, except for small oscillations, the transverse polarization reaches its vanishing asymptotic value (as expected for real form factors) around $\tau=8$ .The magnitude of the transverse polarization is largest for $\xi=-1.0$, $\delta=0.1$, and $\zeta=1.0$, in which case it reaches a value of around 15% at $\tau=0$ and 3% at the secondary maximum. We also note that for $-1<\xi<0$, and $|\zeta|>|\delta|$, the sign of the transverse polarization is opposite to the sign of ζ.

The magnitude of the transverse polarization at $\tau=0$ depends crucially on the magnitude and phase of the regeneration parameter. From Eq. (6) we see that for

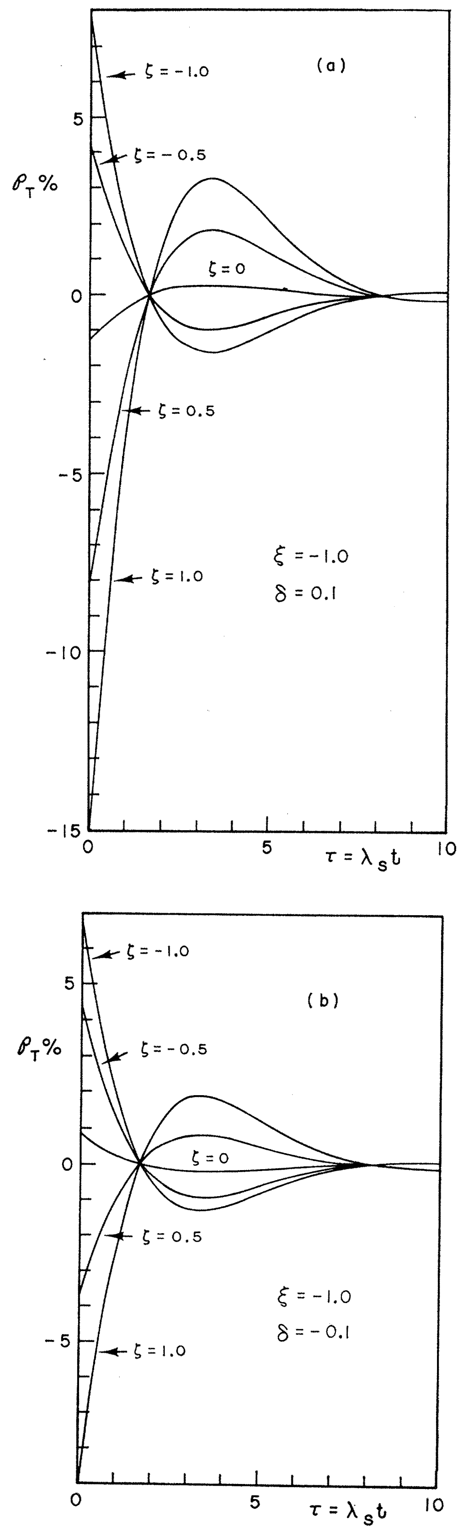

FIG. 1. Time dependence of the transverse component of muon polarization for $E_\mu=200$ MeV, $E_\pi=160$ MeV, and $\xi=-1.0$; and (a) $\delta=0.1$, (b) $\delta=-0.1$. The regeneration parameter is taken as $|\rho|=0.1$, $\phi_\rho=-45°$.

[16] B. R. Webber *et al.*, Ref. 2.

[17] Explicitly, if x is the ratio of the $\Delta S=-\Delta Q$ and $\Delta S=\Delta Q$ amplitude in $K_{\mu 3}{}^0$ decay, then $|x|^2=(a|\delta|^2+b|\zeta|^2+c\,\mathrm{Re}\delta\zeta^*)/(a+b|\xi|^2+c\,\mathrm{Re}\xi)$, where $a=0.6265$, $b=0.0181$, and $c=0.1210$. In obtaining the limits on ζ, we have assumed that ξ, δ, and ζ are real.

[18] H. Faissner, H. Foeth, A. Staude, K. Tittel, P. Darriulat, K. Kleinknecht, C. Rubbia, J. Sandweiss, M. I. Ferrero, and C. Grosso, Phys. Letters **30B**, 204 (1969); P. Darriulat, K. Kleinknecht, C. Rubbia, J. Sandweiss, H. Foeth, A. Staude, K. Tittel, M. I. Ferrero, and C. Grosso, *ibid.* **30B**, 209 (1969).

[19] We assume here that the correction due to incoherent regeneration has been taken into account. The two sources of incoherence will be inelastic and diffractive scattering. The amount of incoherent regeneration will depend on experimental parameters such as beam momentum, thickness of the regenerator, and the angular resolution. The correction due to incoherent regeneration has been studied, e.g., by M. W. Strovink, Princeton University Report No. PURC-2137-23, 1970 (unpublished). He finds that for a beam momentum of 2.2 GeV/c, an angular resolution of 25 mrad, and a copper regenerator 9.125 in. thick, the correction to the regeneration phase due to diffractive and inelastic regeneration is $-10.8°\pm 4.6°$. It is interesting to note that the correction due to incoherent regeneration increases the absolute value of ϕ_ρ, and consequently the magnitude of the transverse polarization at $\tau=0$ will increase.

[20] We have used the values $\tau_S=0.862\times 10^{-10}$ sec, $\tau_L=5.38\times 10^{-8}$ sec, and $\Delta m=(m_L-m_S)=0.469\ \tau_S{}^{-1}$ sec^{-1} from the compilation by the Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

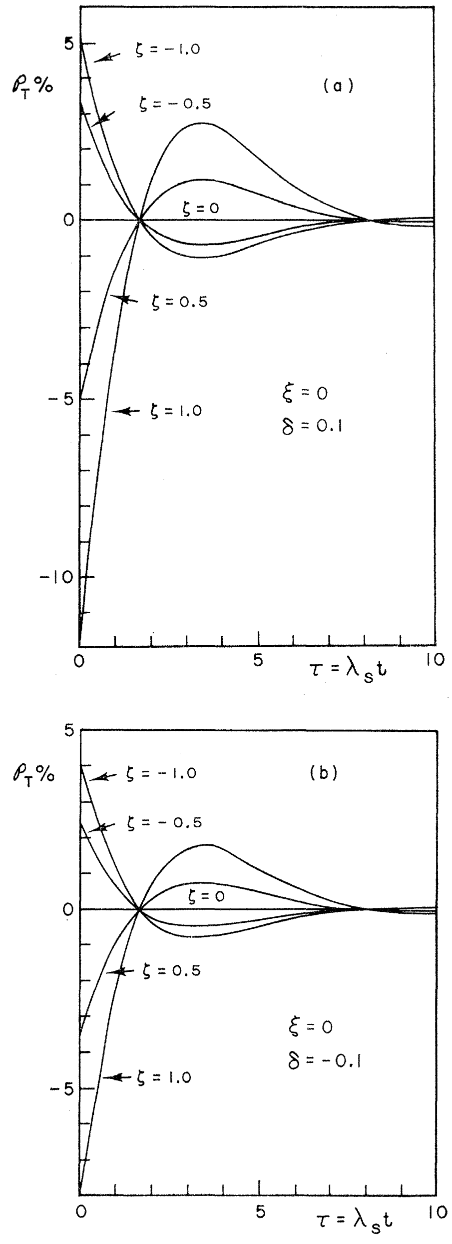

FIG. 2. Time dependence of the transverse component of muon polarization for $E_\mu = 200$ MeV, $E_\pi = 160$ MeV, and $\xi = 0$; and (a) $\delta = 0.1$, (b) $\delta = -0.1$. The regeneration parameter is taken as $|\rho| = 0.1$, $\phi_\rho = -45°$.

$\tau = 0$ the transverse polarization is proportional to $\sin\phi_\rho$. The dependence on $|\rho|$ is complicated by the dependence of $\Delta(t)$ on $|\rho|$. However, for $E_\mu = 200$ MeV and $E_\pi = 160$ MeV, $\Delta(t)$ varies by about 5% as $|\rho|$ varies from 0.1 to 0.2. Consequently, for the point on the Dalitz plot under consideration and for $|\rho| \lesssim 0.2$, the transverse polarization varies approximately linearly with $|\rho|$. Experimentally, therefore, it is advantageous to arrange for regeneration such that $|\rho| \sin\phi_\rho$ is maximum.

TABLE I. Approximate limits on ζ set by a 2 and 3% time-averaged transverse polarization experiment for various values of ξ and δ. Note that the sign of the transverse polarization will be opposite to the sign of ζ for $-1 < \xi < 0$ and $|\zeta| > |\delta|$.

		$\delta = -0.1$	$\delta = 0$	$\delta = 0.1$
$\xi = -1.0$	2%	$-0.4 < \zeta < 0.4$	$-0.4 < \zeta < 0.3$	$-0.4 < \zeta < 0.2$
	3%	$-0.5 < \zeta < 0.5$	$-0.5 < \zeta < 0.4$	$-0.5 < \zeta < 0.3$
$\xi = 0$	2%	$-0.7 < \zeta < 0.5$	$-0.6 < \zeta < 0.4$	$-0.5 < \zeta < 0.3$
	3%	$-1.2 < \zeta < 0.7$	$-1.0 < \zeta < 0.6$	$-0.8 < \zeta < 0.5$

From Eq. (6), we see that the transverse polarization is proportional to $\zeta - \xi\delta$. Consequently, if both ξ and ζ are zero, the transverse polarization will vanish even though δ may be different from zero. On the other hand, if ζ is zero, and ξ and δ are different from zero, the transverse polarization will be nonvanishing. However, in this latter case, the magnitude of the transverse polarization at $\tau = 0$ is of the order of 1% for $|\delta| = 0.1$ (see Fig. 1). To measure such a polarization would require a precision an order of magnitude better than currently possible. In the absence of such a precision, the nonvanishing transverse polarization would appear to be useful only if ζ is significantly different from zero.

In general, the measurement of the transverse polarization will give a functional relation between ξ, δ, and ζ. Thus, while a nonvanishing transverse polarization for $\tau \lesssim 8$, and a vanishing transverse polarization for $\tau \gtrsim 8$ would constitute an unmistakable evidence for a violation of the $\Delta S = \Delta Q$ rule,[21] this by itself would not yield unambiguous values for ξ, δ, and ζ. To obtain a value of ζ, for instance, from a transverse polarization measurement, one must know the value of ξ and δ from other experiments.

From the experimental point of view, the most interesting feature of the time dependence is the relatively large transverse polarization at $\tau = 0$. However, in the event that both δ and ζ tend toward their maximum positive values allowed by present experiments ($\delta = 0.2$, $\zeta = 1.5$), the secondary maximum may also be observable. Assuming that a time-averaged transverse polarization of the order of 2–3% per lifetime of K_S^0 near $\tau = 0$ is detectable, a search for such a polarization would permit more stringent limits on the $\Delta S = -\Delta Q$ form factors than is presently possible in experiments on the time dependence of the decay rate in K_{e3}^0 and $K_{\mu 3}^0$ decays. We illustrate this in Table I, where we give the approximate limits a 2 and 3% time-averaged transverse polarization experiment would set on the value of ζ for various values of ξ and δ.

We have also studied the time dependence of the components of muon polarization in the decay plane. The time variation of these components, however, is not large enough to be experimentally significant at the present time. We have also investigated the effect of T violation on the time dependence of the transverse

[21] The converse is not true. See discussion following Eq. (6).

polarization. In particular, we have considered the possibility that the entire $\Delta S=-\Delta Q$ amplitude is T violating (i.e., δ and ζ are purely imaginary) as proposed by Sachs.[22] If we require that the asymptotic transverse polarization be less than 2%, as suggested by present experiments,[23] then we find that the general features of the time dependence of the transverse polarization remain unchanged, although significant modifications occur. To detect these modifications, however, would require a greater experimental precision than is currently possible. Further details concerning this will be presented separately.

[22] R. G. Sachs, Phys. Rev. Letters 13, 288 (1964).
[23] K. K. Young, M. J. Longo, and J. A. Helland, Phys. Rev. Letters 18, 806 (1967).

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Double-Regge Analysis of Single-Pion Production in $\pi^- p$ Interactions at 8 GeV/c*

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A double-Regge model (DRM) has been applied to $n\pi^+\pi^-$ and $p\pi^-\pi^0$ final states produced in $\pi^- p$ interactions at 8 GeV/c incident beam momentum. Application of the model has been limited to the energy region where all two-body final-state masses are large. Some difficulty exists in trying to understand the qualitative features of all the data in terms of the model. Parameters of the model have been determined for those events which display features consistent with the DRM. These parameters are used to predict distributions in various kinematic variables.

I. INTRODUCTION

THERE has been much interest recently in describing strong interactions involving three or more final-state particles in terms of various multiparticle-exchange mechanisms. There have been several detailed formulations of these models. Among these are the un-Reggeized double-particle-exchange model of Joseph and Pilkuhn[1] and the double-Regge-pole-exchange models (DRM) of Chan *et al.*,[2] Bali *et al.*,[3] and Zachariasen and Zweig.[4] Experimental consistency with the DRM has been reported in describing $\pi\pi N$ and KKN final states[2] as well as other quasi-three-body final states.[5–8] Reggeized multiperipheral models have also been used to give qualitative agreement with experiment for higher-multiplicity final states.[9] The attractive feature of the DRM is that one can use information from two-body processes to describe interactions with three bodies in the final state.

We have employed the DRM to study the reactions

$$\pi^- p \longrightarrow n\pi^+\pi^- \quad (1)$$

and

$$\pi^- p \longrightarrow p\pi^-\pi^0 \quad (2)$$

at 8 GeV/c. The formulation of the DRM and the form used for the amplitude are discussed in Sec. II. In Sec.

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[2] (a) Chan Hong-Mo, K. Kajantie, and G. Ranft, Nuovo Cimento 49A, 157 (1967); (b) Chan Hong-Mo, K. Kajantie, G. Ranft, W. Beusch, and E. Flaminio, *ibid.* 51A, 696 (1967).
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[4] F. Zachariasen and G. Zweig, Phys. Rev. 160, 1322 (1967); 160, 1326 (1967).
[5] R. Lipes, G. Zweig, and W. Robertson, Phys. Rev. Letters 22, 433 (1969); see also G. Zweig, ANL Report No. ANL/HEP 6909, 1968 (unpublished).
[6] For a brief survey of double peripheral models with references up to January 1968, see S. Ratti, in *Proceedings of the Topical Conference on High-Energy Collisions of Hadrons, CERN, 1968* (Scientific Information Service, Geneva, 1968), p. 611; see also reports by O. Czyzewski and Chan Hong-Mo, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).
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