# Questions of Lorentz Invariance in Field Theories with Indefinite Metric*

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It is pointed out that, contrary to the view expressed by Nakanishi, there is no real reason why one should rigidly follow the so-called "real space-momentum" prescription when "pinching" occurs in a field theory with indefinite metric. Since by adopting the modification introduced by Cutkosky *et al.*, a relativistic and unitary $S$ matrix can be obtained, that should be the obvious choice.

THE paper by Nakanishi[1] makes a valid technical point about a suggestion made in one of our papers, but derives from it conclusions far beyond what the author has proved. The following comments seem all the more necessary, since one of us, in a certain sense, has been directly cited as a witness (albeit perhaps unintentionally) by the closing sentences of the above-mentioned paper. Here, then, is our opinion.

The question revolves around the proper definition of integration paths in Feynman integrals in a field theory with indefinite metric; since the usual "$i\epsilon$" prescription fails, one has to find a suitable generalization compatible with unitarity, relativistic invariance, etc. The problem was posed, but not solved, in our first two papers on the subject,[2] since no attempt was made there to discuss relativistic invariance. Later on, a simple solution, or so it seemed at the time, was suggested by one of us.[3] Let us refer to this, in view of later experience, as the "naive" prescription. Soon afterwards, it was pointed out in a paper by Cutkosky *et al.*[4] that, due to pinching, this naive prescription would lead to violations of relativistic invariance in some higher-order diagrams. A manifestly covariant, but more sophisticated, prescription for defining the Feynman integrals when pinching occurs in our theory was then described by CLOP in the same paper.

Section II of Nakanishi's paper discusses again the question of relativistic invariance of the naive prescription; it shows that noninvariance difficulties can occur in some simpler graphs than the ones pointed out earlier in the CLOP paper.[5] In our finite theory of quantum electrodynamics,[6] the simplest case that corresponds to Nakanishi's example would be some of the fourth-order $e^+e^-$ scattering diagrams; for these diagrams, violation of relativistic invariance occurs only if one includes effects due to the square of the width of the $B^0$ quantum, which is in turn proportional to $\alpha^2$. Previously, a comparison between the two above-mentioned prescriptions has been made[4] for the same type of diagrams, but only to first order in the width. To that order, these two prescriptions do give identical results. Owing to our incorrect impression that there was no discrepancy between the two prescriptions in these fourth-order diagrams, the naive form was allowed to persist in the discussion of these diagrams, given in Sec. V of our recent paper,[6] simply because, for pedagogical reasons, it seemed easier to present. This particular discussion should now, of course, be stricken out completely; whenever pinching occurs, one should follow the CLOP prescription. Fortunately, this has no effect on any other parts of that paper, nor does it affect any of the few actual calculations that have been performed so far, such as the vacuum-polarization calculation given in Sec. VI.

In the Introduction and in Sec. III of Nakanishi's paper, however, some very general conclusions are stated, for which no real foundation has been laid. The notion of a "nonanalytic barrier" with a "definite shape" following from the Hamiltonian or Lagrangian structure of the theory seems rather nebulous. True, for a relativistic field theory with complex masses, the contour deformation required by the CLOP prescription has not been derived from the Lagrangian formulation (though heuristic arguments can be given concerning their compatibility[7]). For that matter, neither has the naive "real space-momentum" prescription. It may be worthwhile to recall that this naive prescription was originally suggested by one of us, based only on an *ad hoc* application of the usual quantization rule to the Fourier components of a field with complex masses. Even for the

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[1] N. Nakanishi, this issue, Phys. Rev. D **3**, 811 (1971).

[2] T. D. Lee and G. C. Wick, Nucl. Phys. **B9**, 209 (1969); **B10**, 1 (1969).

[3] T. D. Lee, in *Quanta*, edited by P. G. O. Freund, C. J. Goebel and Y. Nambu (Chicago U. P., Chicago, 1970), p. 260.

[4] R. E. Cutkosky, P. V. Landshoff, D. Olive, and J. C. Polkinghorne, Nucl. Phys. **B12**, 281 (1969). This paper will be referred to as CLOP hereafter. In fairness to these authors, it should be pointed out that their involvement in the present controversy is entirely unintentional, the purposes of their investigation, as clearly stated in the introduction to their paper, being rather different from ours.

[5] The existence of difficulties was kindly mentioned to us by R. Cutkosky prior to the appearance of the CLOP paper (private communication). Nakanishi, however, in Ref. 6 of his paper, denies the validity of the CLOP counter-examples; although we do not quite follow his reasoning, we shall not argue this (by now somewhat academic) point any further.

[6] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).

[7] See, e.g., lectures by T. D. Lee, in *Proceedings of the International School of Physics "Ettore Majorana," Erice, Italy, 1970*, edited by A. Zichichi (Academic, New York, to be published). The formal argument given there, which is based on Lagrangian field theory, leads automatically to the completely symmetrized limit of the CLOP prescription, and thereby resolves the ambiguity encountered in the CLOP paper for the so-called "double ice cream cone" diagram.

free field, these Fourier components (i.e., plane-wave solutions) would lead to an explicitly noncovariant representation in which the three-momentum is always real, but the energy complex. In the coordinate space, the equation of a free field with complex masses is, of course, manifestly covariant; however, its plane-wave solutions diverge exponentially in the asymptotic region. (One notes that, even for those solutions that diverge only along the time direction in a specific Lorentz frame, the same solutions viewed in other systems of reference would diverge in the asymptotic region along the spatial directions as well, thus violating the condition for the validity of the Fourier theorem.) The mathematical procedure of applying the usual quantization rules to these Fourier components must, therefore, be regarded as a purely formal one; its general validity is clearly questionable. Thus, both the naive and the CLOP prescriptions are, at present, merely recipes for evaluating the $S$ matrix, connected only heuristically to the Lagrangian field theory.

Between these two prescriptions, the one that leads to a relativistically invariant unitary $S$ matrix is clearly to be preferred.[8] The alternative raised by Nakanishi, *either* to sacrifice the Lorentz invariance *or* the Lagrangian field-theoretical formulation, appears to be an artificially created issue. His novel suggestion that there may be some merit in a "very slightly" noninvariant $S$ matrix, obtained by rigidly adhering to the original naive prescription, is not, at any rate, a line of thought we would like to encourage.

[8] See also the remarks given near the end of Sec. V of Ref. 6.

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# Comments on Eikonalization at High Energy*

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We previously showed how $s$-channel iteration of general kinds of exchanges in high-energy $e$-$e$ scattering in quantum electrodynamics leads to eikonalization. In this note we clarify this result and briefly discuss its implications.

IN a previous publication,[1] we demonstrated that under certain conditions, $s$-channel iteration of general kinds of exchanges in high-energy $e$-$e$ scattering in quantum electrodynamics (QED) leads to eikonalization.[2] In this note we would like to clarify this result and briefly discuss its implications. For a more complete discussion of the terms and methods we use, we refer the reader to the original article.

Consider a high-energy two-body → two-body scattering process which proceeds by the multiple exchange of *any* connected unit, as in Fig. 1. Then the conditions under which eikonalization takes place, for both $e$-$e$ scattering in QED and for a $\varphi^3$ theory, are twofold: First, the external particles retain their respective large momenta; that is, the left-hand particle has a large plus component both before, during, and after the collision, and the right-hand particle similarly retains its large minus component. This condition implies that $t/s$ remains small, which is a commonly stated eikonalization condition. Second, we require that there be no vertex corrections on the external lines.

Under these two conditions we may consider the exchange of an arbitrary connected unit, keeping both the

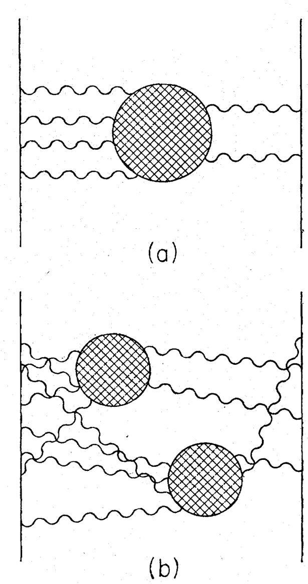


FIG. 1. Multiple exchange of connected units: (a) an example of single exchange of a connected unit; (b) one of the second-order iterations of diagram (a).

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[1] S.-J. Chang and Paul M. Fishbane, Phys. Rev. D 2, 1104 (1970).

[2] Similar results have been obtained by R. Sugar in $\varphi^3$ theory. We would like to thank Professor R. Blankenbecler (private communication).