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Does Multiple Reggeon Exchange Produce an Eikonal-Type Formula?

I. J. MUZINICH*
Brookhaven National Laboratory, Upton, New York 11973

AND

GEORGE TIKTOPOULOS† AND S. B. TREIMAN†
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

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An attractive possibility, much discussed recently, suggests that multiple Reggeon exchange produces a scattering amplitude of the eikonal form. We investigate this matter on a simple example. The anticipated eikonal structure does not emerge.

A POPULAR theoretical model for the discussion of Regge behavior bases itself on the amplitude corresponding to a series of simple ladderlike Feynman graphs, sometimes called a "tower." An unsatisfactory feature of this model for two-body scattering is the lack of unitarity. This manifests itself, for example, in the fact that by an appropriate choice of the coupling constant one can make the real part of the leading Regge trajectory greater than 1 and thereby violate the Froissart bound.

A number of authors[1] have considered "unitarity corrections" to the simple Regge asymptotic behavior by using an "eikonal" formula (similar to the one introduced by Molière[2] in potential scattering), in which the "potential" is essentially the simple Regge amplitude. Thus in expanding the amplitude in powers of the potential the first term corresponds to a Regge pole, and the higher ones to multiple exchanges of Regge poles, i.e., a series of Regge cuts. This semiphenomenological proposal was recently advocated also by Cheng and Wu[3] on the basis of their extensive work on high-energy limits in quantum electrodynamics (QED). They consider certain N-tower-exchange diagrams. These consist of N off-mass-shell tower amplitudes being exchanged in all possible ways between two energetic "fermion" lines (heavy lines in the example of Fig. 1). Each blob in Fig. 1 represents a Regge tower characterized by its order $(e^4)^{n_i}$ in the coupling constant. For a given choice of the set $n_1, n_2, \ldots, n_N$, Cheng and Wu[3] retain the leading term in an expansion of the amplitude in powers of $\ln s$. They then sum these leading terms over the set n_i; and finally they sum over N, i.e., over all towers.

Motivated by the work of Chen and Wu, we have considered an alternative prescription, according to which each blob represents a *full* tower summed over all orders in the coupling constant. That is, each blob is immediately represented by an off-mass-shell Regge amplitude. In order to obtain a result equivalent to that of Cheng and Wu for the over-all amplitude corresponding to multitower exchanges it is sufficient to neglect, in the fermion propagators, all terms which are quadratic in the momenta q_i and q_i' of the tower legs. By means of a remarkable identity,[4] it is then straightforward to obtain an eikonal-type formula of the form

$$is\int d^2x_\perp e^{i\mathbf{\Delta}\cdot\mathbf{x}_\perp}\{e^{is^{\alpha-1}f(x_\perp^2)}-1\}. \qquad (1)$$

Here s is the c.m. energy squared, $\mathbf{\Delta}$ is the momentum transfer, and f is a complicated function which falls exponentially, like $e^{-\mu|\mathbf{x}_\perp|}$, at large $|\mathbf{x}_\perp|$. Whereas a single Regge tower[5] would behave like s^α and therefore would violate the Froissart bound for $\alpha>1$, the eikonal form behaves like s^α for $\alpha<1$ but changes to $s\,(\ln s)^2$ for $\alpha>1$ since in that case the important contribution to the integral comes from the region $s^{\alpha-1}e^{-\mu|\mathbf{x}_\perp|}\sim 1$. Thus the Froissart bound is saturated.

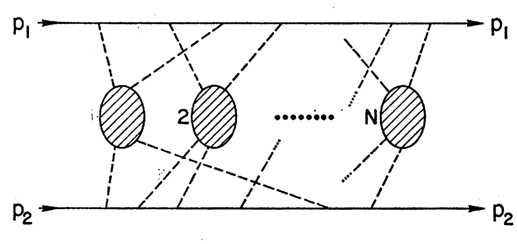

FIG. 1. General N-tower diagram.

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[1] R. C. Arnold, Phys. Rev. **153**, 1523 (1967); R. C. Arnold and M. L. Blackmon, *ibid.*, **176**, 2082 (1968); See also Chan Hong-Mo, in *Proceedings of the Fourteenth International Conference on High-Eenrgy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 391, rapporteur's talk on work by S. Frautschi and B. Margolis, C. B. Chiu and J. Finkelstein, and A. A. Anselm and I. T. Dyatlov. For a review of the topic and detalied references, see. C. B. Chiu, Rev. Mod. Phys. **41**, 640 (1969).

[2] G. Molière, Z. Naturforsch. **2a**, 133 (1947).

[3] H. Cheng and T. T. Wu, Phys. Rev. Letters **24**, 1456 (1970); see also S. J. Chang and T. M. Yan, *ibid.* **25**, 1586 (1970).

[4] M. Lévy and J. Sucher, Phys. Rev. **186**, 1656 (1969).

[5] In the work of Cheng and Wu, the high-energy behavior of a tower is dominated by a fixed Regge cut rather than a pole, so that their formula differs from Eq. (1) by certain factors of $\ln s$. This difference is not essential for our present discussion.

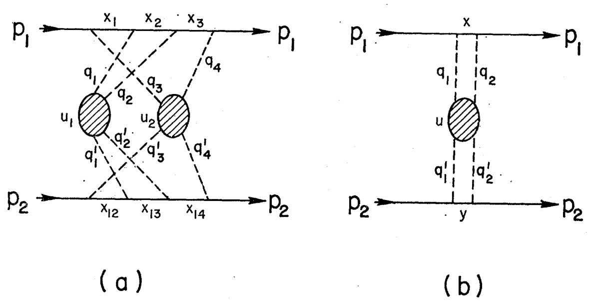

FIG. 2. (a) Two-tower diagram. (b) One-tower diagram. The Feynman parameters of certain lines mentioned in the text are indicated.

We are here concerned with the question of whether, in general, the crucial step of dropping quadratic terms is justified. In fact, we present an argument against it in the context of a theory with spinless electrons and spinless photons, where we consider the exchange of one and two blobs, Figs. 2(a) and 2(b). First we consider the two-tower exchange diagram of Fig. 2(a) for forward scattering. The "blobs" represent towers of Reggeons for which we simply take the Regge behavior $[(q_1-q_1')^2]^\alpha$ and $[(q_4-q_4')^2]^\alpha$. It can be shown that inclusion of Regge-residue factors depending on the appropriate momentum transfers and off-shell masses does not affect our conclusions. In order to facilitate the discussion we take α to be a *fixed* power independent of momentum transfer (as, e.g., in the case of QED). The momentum integrations can then be carried out if one first uses the Feynman identity in a generalized fractional-power form. The resulting amplitude is of the form[6]

$$F_2(s)=\int\frac{A(u_1u_2)^{-\alpha-1}du_1du_2dx_1dx_2\cdots dx_{14}\delta(u_1+u_2+\sum x_i-1)}{[h-(au_1+bu_2+cu_1u_2+d_1d_2)s-i\epsilon]^{4-2\alpha}}. \tag{2}$$

Here u_1 and u_2 are the Feynman parameters corresponding to the factors $[(q_1-q_1')^2]^\alpha$ and $[(q_4-q_4')^2]^\alpha$, respectively, and A, h, a, b, c, d_1, and d_2 are polynomials in the remaining Feynman parameters x_1, x_2, $\ldots$, x_{14}; A and h also depend on u_1 and u_2. What is crucial here is that, because of the nonplanar topology of the graph, d_1 and d_2 are nondefinite; i.e., they change sign in the interior of the integration domain.

To obtain the large s behavior of the integral, we first introduce the variables $v_1=u_1s$, $v_2=u_2s$ and drop certain terms of order $1/s$ in the integrand, to obtain

$$F_2(s)\underset{s\to\infty}{\sim}s^{2\alpha}\int\frac{\bar{A}(v_1v_2)^{-\alpha-1}dv_1dv_2dx_1dx_2\cdots dx_{14}\delta(\sum x_i-1)}{(\bar{h}-av_1-bv_2-d_1d_2s-i\epsilon)^{4-2\alpha}},$$

where the bar indicates that for the large-s limit we have set $v_1/s=v_2/s\to 0$. The leading behavior of this last integral comes from that branch of the hypersurface $\{d_1=0,\ d_2=0\}$ which lies in the interior of the integration domain. The final result is, for $\mathrm{Im}\,s\gtrless 0$,

$$F_2(s)\underset{s\to\infty}{\sim}\pm\frac{2\pi is^{2\alpha-1}}{3-2\alpha}\int\frac{\bar{A}(v_1v_2)^{-\alpha-1}\delta(d_1)\delta(d_2)dx_1\cdots dx_{14}dv_1dv_2\delta(\sum x_i-1)}{(\bar{h}-av_1-bv_2)^{3-2\alpha}}.$$

This $s^{2\alpha-1}$ behavior corresponds to the well-known position of the Mandelstam J-plane branch point[7] at $J=2\alpha-1$.

A similar result already emerges at the level of one tower or one Reggeon exchange [Fig. 2(b)]. After performing the momentum integrations and making a change of variables in the Feynman parameters, we obtain[6]

$$F_1(s)=\int\frac{Bu^{-\alpha-1}dudxdy\cdots}{(P-xyus-i\epsilon)^{2-\alpha}}, \tag{3}$$

where u is the Feynman parameter associated with the factor $[(q_1-q_1')^2]^\alpha$, and x and y are the Feynman parameters associated with the energetic "electron" lines. The functions B and P are polynomials in the Feynman parameters.

The leading behavior at large s comes from small values of u in the above expression; introducing the variable $v=us$ and dropping terms of order $1/s$, we obtain for the large-s behavior ($\alpha<0$)

$$F_1(s)\underset{s\to\infty}{\sim}s^\alpha\int\frac{\bar{B}v^{-\alpha-1}dvdxdy\cdots}{(\bar{P}-xyv-i\epsilon)^{2-\alpha}}.$$

The s^α factor corresponds to the familiar Regge behavior which is expected of diagrams of the type in Fig. 2(b) on general grounds. What is of interest to us here is the fact that the leading high-energy behavior did not come from vanishingly small values of the Feynman parameters x_1, x_2, x_3 and x_{12}, x_{13}, x_{14} of the energetic "electron" line in the two-tower case or from $x=y\to 0$ in the one-tower case. Therefore the dropping of quadratic terms in the momenta q_i and q_i' in those lines would *modify* the quantities $\bar{A}$, $\bar{h}$, a, b, d_1, d_2, $\bar{B}$, and

[6] Strictly speaking, the integrals in Eqs. (2) and (3) exist only for $\mathrm{Re}\,\alpha<0$. It is, however, straightforward to discuss the case $\mathrm{Re}\,\alpha>0$ by, for example, analytic continuation. Our conclusion is not affected.

[7] S. Mandelstam, Nuovo Cimento 30, 1148 (1963).

$\bar{P}$ (by the omission of certain monomials in them) and thus would lead to a different coefficient of $s^{2\alpha-1}$ (or of s^{α} in the one-tower case). This means that the "eikonal" approximation of dropping quadratic terms does not here give the correct leading asymptotic behavior for these Regge-type graphs, although it happens to give the correct power of s. This means that we have no way to perform the correct summation or to anticipate the form of the result. Obviously, exponentiation rests crucially upon the N dependence of the coefficient of the general term $s^{N\alpha-N+1}$, N being the number of towers exchanged.

The essence of our argument remains valid in the case of QED with vector photons. In view of the recently announced calculations by Cheng and Wu,[3] in which an eikonal form was obtained by summing over only the leading logarithmic terms of individual ladder diagrams, we must assume that, in general, terms other than the leading ones sum to a result of comparable importance in its energy dependence.

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Divergences of Massive Yang-Mills Theories*

S. L. Glashow and J. Iliopoulos
Brookhaven National Laboratory, Upton, New York 11973
and
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

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In an earlier paper, we suggested a model of weak interactions where the divergences of higher-order amplitudes are properly ordered. In this paper, we show that the introduction of Yang-Mills self-couplings preserves this ordering. We show, in particular, that the worst divergences of Feynman amplitudes are like $(g^2\Lambda^2)^n$, despite the fact that individual Feynman diagrams diverge like $(g^2\Lambda^4)^n$.

I. INTRODUCTION

In a previous paper,[1] a new model of weak interactions was introduced for which the divergences of higher-order weak interactions are properly ordered. That is to say, divergent Feynman amplitudes of order $(G\Lambda^2)^n$ induce no symmetry breaking, and those of order $G(G\Lambda^2)^n$ satisfy the observed selection rules of weak interactions. Only a single charged intermediate vector meson is needed; the ordering of divergences is accomplished by means of the introduction of a fourth quark.

A more symmetric model of weak interactions is obtained with the introduction of a third intermediate vector meson coupled to the neutral current corresponding to the commutator of the weak charge with its adjoint. The three weak charges satisfy the algebra of the rotation group, thereby explicitly implementing the notion[2] of algebraic universality. The ordering of divergences persists, and the predicted neutral lepton processes are compatible with experiment.

However, in this model, the weak currents are not partially conserved, i.e., they are conserved in the limit of vanishing quark and lepton mass, but only to zeroth order in G. To make the symmetry exact in the massless limit, it is necessary to introduce Yang-Mills self-couplings of the three intermediate vector mesons. The purpose of the present paper is to demonstrate that the introduction of these self-couplings does not upset the proper ordering of divergences.[3] Although individual Feynman diagrams will have a degree of divergence worse than $(G\Lambda^2)^n$, we shall show that the sum of all diagrams contributing to a given amplitude, to each order in G, diverges no worse in the presence of the self-couplings than in their absence. Thus, we may introduce a Yang-Mills model of weak interactions which preserves the observed selection rules.

II. THEOREM

First, we review the analysis in the absence of Yang-Mills self-couplings. We envisage the existence of certain fundamental massive fermion fields ψ (quarks and leptons) which are symmetrically coupled to a triplet of W's with an interaction Lagrangian:

$$g\mathbf{W}\cdot\bar{\psi}\gamma_\mu\mathbf{C}\psi\,, \tag{1}$$

where the C involve a factor of $1+\gamma_5$ and are otherwise numerical matrices satisfying the commutation relations

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[1] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[2] S. L. Glashow (unpublished); see M. Gell-Mann, in *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience, New York, 1960), pp. 508–513.

[3] This question was also considered by T. Appelquist and C.-E. Carlson, Phys. Rev. **187**, 2119 (1969).