parts of the amplitude are in the ratio 1:2. The implications of this are that we cannot neglect the $I=\frac{3}{2}$ component in doing the analysis for the $K\pi$ system in the low $K\pi$ mass region. Whether the $K\pi$ asymmetry can be explained by including the $I=\frac{3}{2}$ component is completely unclear.

V. CONCLUSIONS

(1) (P,π) exchange dominates the reaction $K^+p \rightarrow K^+\pi^-\Delta^{++}{}_{1236}$ at 9 GeV/c for $M(K^+\pi^-) \gtrsim 1540$ MeV. In general the model agrees with the data fairly well for $-t_{KK} < 0.5(\text{GeV}/c)^2$ and $-t_{p\Delta}$ $0.5(\text{GeV}/c)^2$. The validity of the model above these t cuts is definitely in doubt.

(2) The introduction of an empirical Toller-angle dependence at the interval vertex helps to improve the confidence level to be more uniform over the distribution of all the variables considered except that the fit to the Toller-angle distribution itself has not been improved much. In the small-t region, the Toller-angle distribution [as shown in Fig. 9(c)] indicates a large discrepancy between the model and the data. Further investigation of Toller-angle dependence is necessary.

(3) With the present knowledge of the Regge parameters determined by the data from the two-body final states, the many possibilities of the exchange pairs, and the statistical limitation of our data, the values of the Regge parameters we used are subject to quite large uncertainties. However, this should not affect the conclusion that the contribution from the extrapolation is large. By comparing tke data with the result from the extrapolation to the small $K\pi$ mass region, we find that the latter agrees with Harari's postulate that Pomeranchukon exchange is responsible for the background only.

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Regge-Pole-Cut Analysis of Total Cross Sections with $SU(3)$ and Exchange Degeneracy*

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A phenomenological study of $SU(3)$ and exchange degeneracy is made in a Regge-pole-cut model. The high-energy data of pion-nucleon, kaon-nucleon, nucleon-nucleon, and nucleon-antinucleon total cross sections, including the new Serpukhov data, are analyzed by means of least-squares fits with parameters corresponding to the P, P', ρ, A_2, and ω Regge poles and subtractive vacuum cut. The data are fitted adequately with the assumptions of $SU(3)$ and exchange degeneracy. For completeness, the data are analyzed with the assumptions of $SU(3)$ alone, exchange degeneracy alone, and also without either of these assumptions.

I. INTRODUCTION

PRESENTED is a Regge-phenomenological study of $SU(3)$ and exchange degeneracy, utilizing the available high-energy total-cross-section data of meson-nucleon, nucleon-nucleon, and nucleon-antinucleon reactions, using a model consisting of Regge poles and a vacuum cut. A similar study, although involving only Regge poles and with slightly different pole assumptions, was made by Ahmadzadeh[1] some four years ago, with notable success. The recent Serpukhov data[2] with its surprises makes a study similar to that of Ref. 1 [i.e., using Regge poles with $SU(3)$ and exchange degeneracy], but including a vacuum cut, of interest. Phenomenological studies of the total cross sections have already been made by Barger and Phillips[3] and by Lendyel and Ter-Martirosyan.[4] However, they did not include $SU(3)$ or exchange degeneracy in their analyses.

II. FORMALISM

Specifically, the five leading Regge poles, the P (Pomeranchukon), P', ω, ρ, and A_2, are included in the present analysis. The ϕ and f' poles are assumed to de-

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[1] A. Ahmadzadeh, Phys. Rev. Letters 16, 952 (1966).

[2] IHEP-CERN Collaboration, Phys. Letters 30B, 500 (1969).

[3] V. Barger and R. J. N. Phillips, Phys. Rev. Letters 24, 291 (1970).

[4] A. I. Lendyel and K. A. Ter-Martirosyan, in Proceedings of the International Conference on High-Energy Physics and Theory of Elementary Particles, Kiev, 1969 (unpublished).

couple from the nonstrange baryons. This assumption follows from the quark model, in which the ϕ and f', each consisting mainly of a strange quark-antiquark system, do not couple to a nucleon-antinucleon pair. The exchange-degenerate $\rho-A_2$ trajectory and the exchange-degenerate $\omega-P'$ trajectory are taken from the Chew-Frautschi plot as $\alpha(t)=0.5+\alpha' t$, where $\alpha'=0.9$ $(\text{GeV}/c)^{-2}$. In addition, we include a vacuum cut which is parametrized in a manner that states only its energy dependence. Least-squares fits to the data are made assuming $SU(3)$ alone, exchange degeneracy alone, both $SU(3)$ and exchange degeneracy, and without either of these assumptions. For completeness, fits are made also without the assumption of a vacuum cut.

In addition to the usual Regge-pole energy-dependence expression

$$(s-\tfrac{1}{2}\sum_{i=1}^{4} m_i^2)^{\alpha(t)},$$

an expression of the form $(\alpha' s)^{\alpha(t)}$ is utilized for reasons to be explained. Thus, letting

$$Z=s-\tfrac{1}{2}\sum_{i=1}^{4} m_i^2 \quad \text{or} \quad Z=\alpha' s,$$

we define a quantity

$$R=(\sqrt{\pi})Z^{\alpha(0)}/m_N p_{\text{lab}}$$

which is related to the imaginary parts of the forward amplitudes for the P', ω, ρ, and A_2 Regge poles. For the Pomeranchukon pole we have

$$R_P=Z^{\alpha_P(0)}/m_N p_{\text{lab}},$$

where $\alpha_P(0)=1$. Here m_N is the nucleon mass, p_{lab} is the beam momentum in the laboratory frame, and s and t are, respectively, the squares of the total energy and four-momentum transfer. We define

$$P=2\Gamma_{(\pi,K,N)}\Gamma_N R_P,$$
$$P'=\gamma_{(\pi,K,N)P'}\gamma_{NP'}R,$$
$$\omega=\gamma_{(\pi,K,N)\omega}\gamma_{N\omega}R,$$
$$\rho=\gamma_{(\pi,K,N)\rho}\gamma_{N\rho}R,$$
$$A_2=\gamma_{(\pi,K,N)A_2}\gamma_{NA_2}R,$$

where the Γ's and γ's are the factorized positive residue constants. $\Gamma_{(\pi,K,N)}$ means Γ_π, Γ_K, or Γ_N and $\gamma_{(\pi,K,N)}$ means γ_π, γ_K, or γ_N, depending upon the vertex under consideration. The vacuum cut is written as[5]

$$\text{cut}=\frac{\beta_{(\pi,K,N)}\Gamma^2{}_{(\pi,K,N)}Z\ln Z}{m_N p_{\text{lab}}[(\ln Z)^2+\frac{1}{4}\pi^2]},$$

where $\beta_{(\pi,K,N)}$ means β_π, β_K, or β_N, depending upon the process under consideration; the β's are positive constants. (Note that the inclusion of $\Gamma_{(\pi,K,N)}{}^2$ is suggestive of the cut's being a product in some sense of two Pomeranchukon poles, with common factor $\Gamma_N{}^2$ absorbed into the β's.) The expressions for the total cross sections are found in the literature.[6] For example, the K^-n total cross section is written in this notation as

$$\sigma(K^-n)=P+P'+\omega-\rho-A_2-\text{cut}.$$

With no further assumptions, there are a total of 16 Γ's, γ's, and β's.

From the assignment of the vector mesons and tensor mesons to nonets with specified mixing of ϕ-ω and of P'-f', the usual $SU(3)$-invariant interaction Lagrangians can be constructed which give the $t=0$ Regge-pole residues for pseudoscalar-meson and baryon interactions.[7] With the quark-model assumption of ϕ and f' decoupling to the nonstrange baryons, these relations reduce the number of free parameters to 12.

Exchange degeneracy of the ρ-A_2 residues and that of the ω-P' residues can be inferred from the absence of an exchange potential or from application of the finite-energy sum rules to K^+N and NN processes, for which there are no ordinary (and apparently no exotic) direct-channel resonances. When exchange degeneracy is combined with $SU(3)$, the number of free parameters is reduced to nine.[6] The $SU(3)$ and exchange-degeneracy predictions $\gamma_{KA_2}\gamma_{NA_2}=\gamma_{K\rho}\gamma_{N\rho}=\frac{1}{2}\gamma_{\pi\rho}\gamma_{N\rho}$ have been tested phenomenologically by the author in a study involving reactions for which only the ρ and A_2 quantum numbers can be exchanged, and these relations have been found to be in agreement with the data.[8]

About the energy dependence: Consider the K^+p, K^+n, pp, and pn total cross sections. They depend only on the Pomeranchukon pole (and vacuum cut, if included in the analysis), if one assumes exact exchange degeneracy. Assuming exchange degeneracy, no cut, and the traditional energy-dependence expression, these four cross sections are proportional to

$$(s-\tfrac{1}{2}\sum_{i=1}^{4} m_i^2)/p_{\text{lab}},$$

which is very nearly a constant, even in the lower-energy region from 6 to 20 GeV/c. The K^+N experimental cross sections[9] are essentially constant in this region, and hence are fitted well with the above assumptions. The experimental pp and pn total cross sections[9] have a more negative slope than is predicted. This fact would seem to argue against exact exchange degeneracy (even with no cut); however, the experimental uncertainties are large enough that the exchange-degen-

[5] We owe our specific logarithmic energy-dependence expression to the model of S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968).

[6] See, for example, V. Barger and D. Cline, *Phenomenological Theories of High Energy Scattering* (Benjamin, New York, 1969), Chap. 5.

[7] See, for example, V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966); see also Ref. 1.

[8] J. C. Jackson, Phys. Rev. **174**, 2098 (1968).

[9] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).

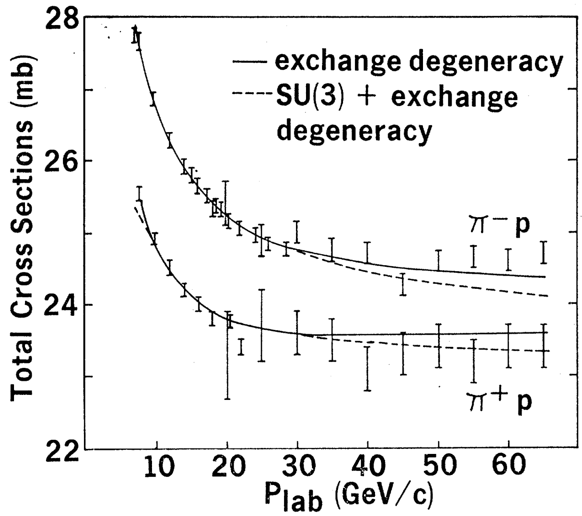

FIG. 1. Regge-pole-cut-model fits to pion-nucleon total cross sections. Curves for $SU(3)$ alone lie slightly below those for exchange degeneracy alone in the region of highest energy.

eracy assumption is compatible with experiment. Now, if a subtractive vacuum cut is included in the analysis (still assuming exchange degeneracy and using the traditional energy-dependence expression), its effect is to cause these four cross sections to rise with energy, by an amount depending on the strength of the cut. If the cut is made strong enough to satisfy the Serpukhov data, then the predicted K^+p and pp total cross sections

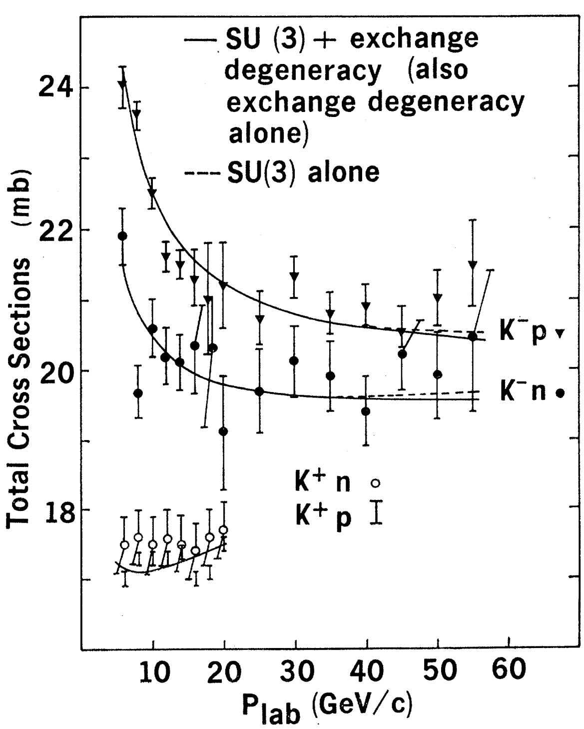

FIG. 2. Regge-pole-cut model fits to kaon-nucleon total cross sections.

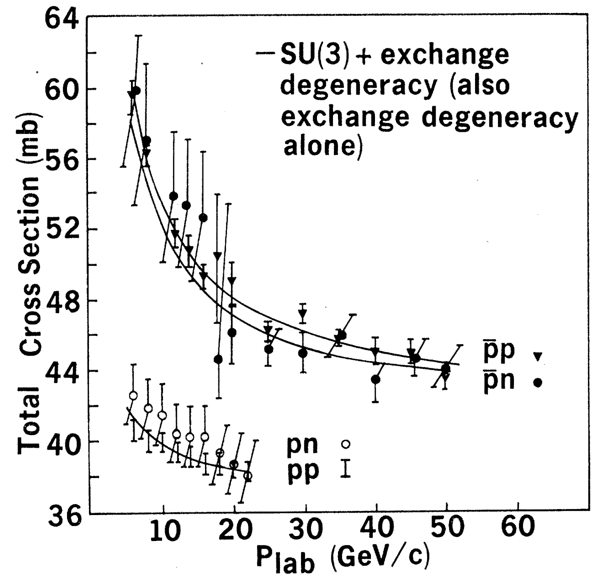

FIG. 3. Regge-pole-cut model fits to nucleon-antinucleon and nucleon-nucleon total cross sections. Nucleon-antinucleon curves for $SU(3)$ alone lie slightly above the curves shown in the region of highest energy.

are totally inconsistent with the accurate experimental data. Hence, exact exchange degeneracy is incompatible with the above assumptions of a subtractive vacuum cut and the traditional energy-dependence expression

$$(s-\tfrac{1}{2}\sum_{i=1}^{4} m_i^2)^{\alpha(0)}.$$

If, on the other hand, one assumes exchange degeneracy, no vacuum cut, and a Veneziano-type expression $(\alpha' s)^{\alpha(0)}$, then the K^+p, K^+n, pp, and pn total cross sections are proportional to s/p_{lab}, which decreases noticeably with increasing energy in the lower-energy range, below 20 GeV/c. The predicted slopes for all four of these reactions are more negative than the experimental data indicate. If we now include a negative vacuum cut in the analysis, the predicted slopes for

TABLE I. χ^2 values obtained by fitting total-cross-section data with various assumptions. Exchange degeneracy is abbreviated as exd.

Number of parameters	Assumptions	χ^2 (135 data points) $Z=0.9s$	$Z=s-\frac{1}{2}\sum_{i=1}^{4} m_i^2$
6	$SU(3)$, exd, no cut	950	385
7	$SU(3)$, exd, 1 cut of same magnitude for all reactions	202	
9	exd, no cut	607	371
9	$SU(3)$, exd, vacuum cuts	185	317
10	universality, exd, vacuum cuts	185	316
12	exd, vacuum cuts	150	198
12	$SU(3)$, vacuum cuts	160	157
16	no assumptions, vacuum cuts	140	137

TABLE II. Parameter values obtained by fitting total-cross-section data with pole-cut model, and assuming both $SU(3)$ and exchange degeneracy, $SU(3)$ alone, exchange degeneracy alone, and also without either assumption.

s dependence	0.9s				$s-\frac{1}{2}\sum_{i=1}^{4} m_i^2$
Assumptions	$SU(3)$, exd, cuts	exd, cuts	$SU(3)$, cuts	cuts	$SU(3)$, cuts
χ^2 (135 points)	185	150	160	140	157
χ^2 (πN) (44 points)	85	52	57	52	50
χ^2 (KN) (47 points)	78	76	77	70	79
χ^2 ($NN+\bar{N}N$) (44 points)	22	22	26	18	28
$\gamma_{\pi P'}\,\gamma_{NP'}$	14.0 $\pm$0.4	31.8 $\pm$0.3	26$\pm$4	31.8	42$\pm$4
$\gamma_{\pi\rho}\,\gamma_{N\rho}$	3.24$\pm$0.06	3.23$\pm$0.01	3.3 $\pm$0.1	3.23	3.18$\pm$0.05
$\gamma_{KP'}\,\gamma_{NP'}$	7.0 $\pm$0.2	7$\pm$1	13$\pm$2	14.5	21$\pm$2
$\gamma_{K\omega}\,\gamma_{N\omega}$	7.0 $\pm$0.2	7$\pm$1	6.8 $\pm$0.3	6.6	6.8 $\pm$0.1
$\gamma_{K\rho}\,\gamma_{N\rho}$	1.62$\pm$0.03	1.6 $\pm$0.1	1.64$\pm$0.05	1.9	1.59$\pm$0.03
$\gamma_{KA_2}\,\gamma_{NA_2}$	1.62$\pm$0.03	1.6 $\pm$0.1	1.7 $\pm$0.2	1.4	1.71$\pm$0.06
$\gamma_{NP'}^2$	21$\pm$1	21$\pm$5	57$\pm$12	18	95$\pm$17
$\gamma_{N\omega}^2$	21$\pm$1	21$\pm$5	20$\pm$3	20	20$\pm$3
$\gamma_{N\rho}^2$	1.10$\pm$0.02	1.0 $\pm$0.1	1.17$\pm$0.02	2.0	1.10$\pm$0.02
γ_{NA_2}	1.10$\pm$0.02	1.0 $\pm$0.1	0.9 $\pm$0.1	0.0	0.64$\pm$0.03
$\Gamma_\pi\Gamma_N$	18.1 $\pm$0.5	22.9 $\pm$0.1	21.2 $\pm$0.5	22.9	21.7 $\pm$0.6
$\Gamma_K\Gamma_N$	16.3 $\pm$0.5	16.3 $\pm$0.1	18.2 $\pm$0.5	18.6	16.7 $\pm$0.5
Γ_N^2	30.0 $\pm$0.8	30.0 $\pm$0.2	40$\pm$1	29.4	41$\pm$2
$\beta_\pi\Gamma_\pi^2$	27$\pm$2	90$\pm$1	69$\pm$5	90	105$\pm$8
$\beta_K\Gamma_K^2$	36$\pm$2	36.4 $\pm$0.7	60$\pm$5	65	65$\pm$5
$\beta_N\Gamma_N^2$	34$\pm$6	33$\pm$2	170$\pm$15	24	230$\pm$30

these four reactions are brought into agreement with experiment, and at the same time the Serpukhov data for the K^-N and $\bar{N}N$ reactions are satisfied. Hence, exact exchange degeneracy is compatible with the assumptions of a subtractive vacuum cut and an energy-dependence expression of the form $(\alpha' s)^{\alpha(0)}$.

III. RESULTS

We have fitted 135 total-cross-section data points[2,9,10] for the ten reactions. For completeness we have studied the reactions assuming no cuts as well as including cuts. Our findings are summarized in Tables I and II. Table I shows the assumptions made, number of parameters involved, and the χ^2's for each case. Table II lists for the more interesting cases the coupling constants and the χ^2's for the various types of reactions. When looking at the values for χ^2, one should keep in mind that the eight K^+p data points have extremely small uncertainties and cannot be fitted quantitatively well by any straight or simple curved line. Hence the smallest χ^2 we obtained for these eight points was $\chi^2(K^+p)\simeq 30$. Figures 1–3 show the data with the curves corresponding to exact $SU(3)$ alone, exact exchange degeneracy alone, and both of these assumptions combined. The energy-dependence expression is $Z=0.9s$, and vacuum cuts are included. In some cases where two curves are nearly coincident [e.g., the curves for $SU(3)$ alone and for exchange degeneracy alone in πN cross sections], only one is shown.

[10] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967).

Some comments should be made concerning the results. The only significant differences in the results for the case of $SU(3)$ plus exchange degeneracy and for the case of exchange degeneracy alone are that when $SU(3)$ is relaxed, the coupling of the P' in the pion-nucleon reactions approximately doubles and the corresponding cut triples in size. All other couplings remain essentially unchanged. The χ^2 for the 44 pion-nucleon data points is 85 when both $SU(3)$ and exchange degeneracy are assumed. Perhaps one can reduce this χ^2 by modifying the cut expression. Another result of interest is the approximate equality of the three vacuum cuts when both $SU(3)$ and exchange degeneracy are assumed. This equality might be a coincidence. But in any event, if the vacuum cut is an integral over the product of two Pomeranchukon poles, then one might expect the constants β to be equal. This definitely is not so in any of the cases studied. Furthermore, note that the χ^2's do not significantly improve when neither $SU(3)$ nor exchange degeneracy is assumed. This is a good argument for the validity of $SU(3)$ and exchange degeneracy themselves. Finally, in the case of $SU(3)$ alone, the particular choice of energy-dependence expression has no effect on the quality of the fit, although the exchange degeneracy is necessarily broken more strongly when the traditional energy-dependence expression is used.

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