Center-of-mass correction and magnetic moment of a fermion consisting of confined quarks

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The value of the center-of-mass correction $I_c^{-1}(R_0)$ depends on the radius of confinement R_0 in such a way that $R_0 I_c^{-1}$ shows only a slow variance with R_0. If one assumes that a composite fermion has the mass of an electron, the value of $R_0 I_c^{-1}$ is constant in the range $0.001 < R_0 < 0.5$ fm. For such a "mock electron" the value of the magnetic moment is about 1 Bohr magneton. For the proton ($R_0 \sim 0.65$ fm) the theoretical values for the magnetic moment and the axial-vector coupling constant are improved.

I. INTRODUCTION

Both the center-of-mass correction[1–3] (CMC) and the recoil correction[4–6] (RC) influence the calculation of the nuclear form factors based on a static model for the confinement of quarks.[1,7–11] It would therefore be useful to devise a formalism which could include both corrections at least in the limit of zero momentum transfer ($q \to 0$). Emphasis will be on the CMC. In the approximation developed here, the RC and CMC factorize; hence it is possible to use the existing results for the RC.

Additional interest in the calculation of the CMC stems from the dynamical problems arising from the composite models for the lepton structure. Halprin and Kerman[12] have argued that the center-of-mass correction can reconcile a baglike confinement of the constituents with the known electromagnetic properties of the leptons. That is obviously possible only if the magnetic moment of the composite object is not exactly proportional to the confinement radius R_0. The following calculation indeed shows that CMC depends strongly on the characteristic radius R_0. The magnetic moment is proportional to $R_0 I_c^{-1}(R_0)$, where I_c is the CMC integral. One finds $R_0 I_c^{-1}(R_0) > R_0$ for $R_0 < 1$ fm. CMC also improves the calculated values of the proton magnetic moment and the axial-vector coupling constant g_A.

All CMC results are obtained for the model in which three fermions (for example, quarks) are confined in a harmonic-oscillator (HO)[9–11] potential. It seems[1–3] that CMC calculated in such a model, with suitably selected parameters, closely approximates CMC for the MIT bag model.

Implications for a composite model of elementary particles are of a qualitative nature. The exact values must depend on the character of the model which need not contain three fermions but might contain, for example, a fermion and a boson instead. It is interesting that in the mock model for a fermion whose physical mass equals m_e (electron mass) one obtains "mock-electron" magnetic moment of about 1 Bohr magneton.

II. FORMALISM

The formalism is developed by combining the results of Refs. 2 and 3. One begins with a moving bag, considered as a wave packet[2] with a nonzero net momentum $\vec{p}$, i.e.,

$$|p\rangle_B = \int d^3k\, W^{-1/2}(k) W^{-1/2}(k-p)\phi(k-p)\,|k\rangle\ . \tag{2.1}$$

Here ϕ is the wave packet and W is the normalization factor.

The matrix element of a current between moving bag states is

$$_B\langle p'|J_i(x)|p\rangle_B = \int d^3k\, d^3k'[W(k)W(k')W(k-p)W(k'-p')]^{-1/2}\phi(k-p)\phi(k'-p')\langle k'|J_i(x)|k\rangle,\ \ i=1,2,3\ . \tag{2.2}$$

By using

$$\langle k'|J_i(x)|k\rangle = e^{i(\vec{k}-\vec{k}')\vec{x}} f_i(k,k') \tag{2.3}$$

one can deduce for $\vec{q} = \vec{p}\,' - \vec{p}$

$$\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\,_B\langle p'|J_i(x)|p\rangle_B = (2\pi)^3 \int d^3k[W(k)W(k+q)W(k-p)W(k+q-p')]^{-1/2}\phi(k-p)\phi(k+q-p')f_i(k,k-q)\ . \tag{2.4}$$

The left-hand side (LHS) of Eq. (2.4) is the one commonly used in the calculation of recoil corrections to the magnetic moment.[4–6] It is convenient to use the Breit frame of reference

$$\vec{p}\,'=-\vec{p}=\vec{q}/2,\quad q=(0,\vec{q})\ .$$

Leading terms in the expansion of Eq. (2.4) in powers of momentum transfer determine either μ or q_A. In order to find CMC one has to study

$$\int d^3x\, e^{i\vec{q}\cdot\vec{x}}{}_B\langle \vec{q}/2\,|\,J_i(x)\,|-\vec{q}/2\rangle_B=(2\pi)^3\int d^3k[W(k)W(k+q)W^2(k+q/2)]^{-1/2}\phi^2(k+q/2)f_i(k,k-q) \tag{2.5}$$

for small q. Since the leading q powers for μ and q_A (i.e., $\vec{q}$ and 1) originate from f_i the q dependence of W's and ϕ can be dropped. The function $\phi(k)$ is spherically symmetric in k so all vectors of $\vec{k}$ originating from f_i are integrated out. This function has been given by Wong[3] as

$$\phi^2(k)=(2\pi)^3W(k)I(k),\quad W(k)=E(k)/M\ .$$

Here E and M represent baryon energy and mass.

As shown in the Appendix, one finds for the electromagnetic current

right-hand side

$$\to iG_M(0)\chi^\dagger(\vec{\sigma}\times\vec{q}\,)/2M\chi\int d^3k\frac{M}{E(k)}I(k)\ . \tag{2.6}$$

The CMC for the magnetic moment $[\mu=G(0)]$ is found by dividing the LHS of (2.4) by

$$I_c(R_0)=\int d^3k\frac{M}{E(k)}I(k)\ . \tag{2.7a}$$

It is relatively easy to calculate this integral in a model where fermions are confined in a harmonic-oscillator (HO) potential in such a way that the 1s quark wave function in the bag is approximated very well by a simple Gaussian expression.[1,3,13]

The function $I(k)$ from (2.7) is then determined by the Fourier transform of the Hill-Wheeler overlap function for three 1s quarks, i.e.,

$$I_c(R_0)=\int d^3k\frac{M}{E(k)}\int d^3r\, e^{i\vec{k}\cdot\vec{r}}I_3(r)\ . \tag{2.7b}$$

In the numerical calculation one uses the expression[1,3]

$$I_3(r)=\left[e^{-r^2/4R_0{}^2}\left[1-c\frac{r^2}{R_0{}^2}\right]\right]^3,\qquad c=\beta^2/(4+6\beta^2),\quad \beta=0.36\ . \tag{2.7c}$$

The choice of parameter β is such that the HO-model quark wave function approximates a 1s massless-quark wave function in a bag of radius $R=1.51\ R_0$.

A useful check for numerical accuracy follows by the deletion of the factor $M/E(k)$ from (2.7b). One finds

$$1=\frac{2}{\pi}\int dk\,k^2\int dr\,r^2j_0(kr)I_3(r)\ . \tag{2.8}$$

In our model the changes in the order of integration, necessary to deduce (2.4), can be justified *a posteriori* by the very nice uniform convergence of all the integrals.[14]

The static magnetic moment μ^s can be found by neglecting the momentum dependence of the bag states on the LHS of (2.5):

$$|\vec{q}/2\rangle_B\to|0\rangle_B\ .$$

In order to substantiate the following discussion one can use values found for simple models with massless quarks:

$$\mu_H^s=2.863R_0\mu_N\ ,\qquad \mu_B^s=2.886R_0\mu_N\ . \tag{2.9}$$

Here the subscripts H and B refer to the HO model[1,3] and the MIT bag model,[7] respectively. (The R and R_0 should be given in fermis and $\mu_N=e/2M_p$, where M_p is the proton mass.) The recoil-corrected magnetic moment μ^R can be found from (2.5) by keeping the momentum dependence of the bag states. A general form of the result is[6]

$$\mu^R=\left[1-\frac{\epsilon}{M}\right]\mu^s+\frac{e}{2M}\frac{3}{5}G^A\ . \tag{2.10a}$$

Here G^A is the theoretical expression for the axial-vector form factor. By taking into account that quarks are confined one can estimate $\epsilon/M\sim0.25$ for the MIT bag with massless quarks.[6] This ratio is independent of the bag radius R since $\epsilon=\omega R^{-1}$ and $M=4\epsilon$.

The theoretical values for G^A are[15]

$$G_H^A=1.305\ ,\qquad G_B^A=1.088\ . \tag{2.10b}$$

At first glance it might seem that the G^A term in (2.10) is independent of R_0, so if $I_c(R_0)$ were to decrease with R_0, μ^A would be sharply increasing with R_0. However, this is a false conclusion. One has to stay really within the model framework by which

$$\frac{e}{2M}\frac{3}{5}G^A\to\frac{e}{8\epsilon}\frac{3}{5}G^A=\frac{eR}{8\omega}\frac{3}{5}G^A$$

so that

$$\mu^R\sim\text{const}\times R_0$$

as required for the argument of Halprin and Kerman.[12] As shown in the Appendix the CMC for g_A follows from the expression

right-hand side

$$\to g_A\chi^\dagger\vec{\sigma}\chi\int d^3k\frac{E(k)+2M}{3E(k)}I(k)\ . \tag{2.11a}$$

The LHS of (2.5) has to be calculated for the axial-vector current and then divided by

$$J_c(R_0)=[2I_c(R_0)+1]/3\ . \tag{2.11b}$$

As the g_A form factor is not sensitive to the RC[5] one has

$$g_A \cong G^A J_c^{-1} . \tag{2.11c}$$

The expressions for I_c and J_c can easily be compared with the corresponding corrections (13) and (14) of Ref. 2. By expanding square roots in the powers of k^2/M^2 one finds

$$\begin{aligned} I_c^{-1} &= \left[\int d^3k\, I(k)(1-\tfrac{1}{2}k^2/M^2+\cdots) \right]^{-1} \\ &\cong 1+\tfrac{1}{2}\langle k^2\rangle/M^2 , \\ J_c^{-1} &= \Big[\int d^3k\, I(k)(1+\tfrac{1}{6}k^2/M^2+\cdots) \\ &\quad \times(1-\tfrac{1}{2}k^2/M^2+\cdots) \Big]^{-1} \\ &\simeq 1+\tfrac{1}{3}\langle k^2\rangle/M^2 , \\ \langle k^2\rangle &= \int d^3k\, I(k)k^2 . \end{aligned} \tag{2.12}$$

III. NUMERICAL RESULTS AND DISCUSSION

The inspection of Table I reveals that the behavior of CMC is determined by the product R_0M. For $MR_0 \ll 1$ one has $I_c = \text{const} \times R_0$ so that μ does not change at all with R_0. If $MR_0 \ll 1$ the integration over k in (2.7) yields maximum contribution for $k^2 \gg M^2$ so that one has $I_c \sim MR_0$. For $M = M_p$ (proton mass) such a situation is found for $R_0 < 0.01$ fm. At the physically acceptable radius $R_0 = 0.65$ fm, which corresponds to the bag radius $R = 0.98$ fm ($=4.97\ \text{GeV}^{-1}$) the CMC is quite important. It increases the magnetic moment by $I_c^{-1} = 1.205$, i.e., by about 20%. Of the similar order of magnitude is the CMC for G_A where $J_c^{-1} = 1.128$. One finds, for example,

$$\begin{aligned} &\mu_B^s = 1.88\ \mu_N,\quad \mu_B^s(\text{corr}) = 2.27\ \mu_N , \\ &\mu_B^R = 2.06\ \mu_N,\quad \mu_B^R(\text{corr}) = 2.48\ \mu_N , \\ &G_B^A = 1.09,\quad g_B^A = 1.23 . \end{aligned} \tag{3.1}$$

It is obvious that the CMC is important as it has already been pointed out.[2] The approximation used by Ref. 2 works only for the case $MR_0 > 1$, which is certainly valid for the proton. The advantage of a more complex approach adopted here is a possibility to deal with the RC in a simple way.

However, things change if one wishes to study R_0 dependence of CMC. For $MR_0 \ll 1$ the expansion (2.12) is no longer valid. One has to use the complete expression for I_c. An interesting check of the internal consistency of the values of I_c from Table I comes from the fact that $I_c^{-1}(MR_0)MR_0$ does not depend on M if $MR_0 \ll 1$. Thus for $R_0 = 0.001$ and 0.01 (and all $R_0 < 1.0$ for $M = m_e$) one has

$$I_c^{-1}(M_p)M_pR_0 = I_c^{-1}(m_e)m_eR_0 = 1.62 . \tag{3.2}$$

Moreover, for $M = m_e$ the Halprin-Kerman[2] argument, which requires

$$I_c^{-1}R_0 = \text{const} , \tag{3.3}$$

is completely justified.

One can construct, as a kind of toy, a mock-electron model. One has to assume[12] that the correct value of the electron mass can be obtained somehow by adjusting the model parameters such as the zero-point energy and QCD-type coupling strength. Once that is taken for granted, one can introduce $M = m_e$ in all formulas and thus find the mock-electron magnetic moment $\hat{\mu}_e$. By expressing the formulas (2.9) in Bohr magnetons ($\mu_B = e/2m_e$) and by using $I_c^{-1}R_0$ from Table I one obtains highly suggestive values

$$\begin{aligned} &\hat{\mu}^s_{e,M}(\text{corr}) = 1.588 \times 10^{-3} I_c^{-1} R_0 \mu_B = 0.97\mu_B , \\ &\hat{\mu}^s_{e,B}(\text{corr}) = 1.570 \times 10^{-3} I_c^{-1} R_0 \mu_B = 0.98\mu_B . \end{aligned} \tag{3.4}$$

It might be premature to make any far-reaching conclusions on the basis of this almost correct theoretical reproduction of the physical μ_e in the framework of the mock electron model.

However, with all the examples given in this paper, one

TABLE I. Center-of-mass correction as a function of R_0 and M.

R_0 (fm)	$M=M_p$ $I_c(R_0)$	$R_0I_c^{-1}$ (fm)	$M=m_e$ $10^3I_c(R_0)$	$10^{-2}R_0I_c^{-1}$ (fm)
0.001	0.002 935	0.3407	0.0016	6.25
0.01	0.029 32	0.3411		
0.1	0.267 0	0.3745	0.1600	6.25
0.2	0.462 3	0.4326	0.3201	6.25
0.3	0.598 6	0.5012	0.4801	6.25
0.4	0.693 9	0.5765	0.6402	6.25
0.5	0.761 5	0.6566	0.8002	6.25
0.6	0.810 6	0.7402		
0.65	0.830 0	0.7831		
0.7	0.846 8	0.8266	1.120	6.25
0.8	0.874 1	0.9152		
0.9	0.895 0	1.0056		
1.0	0.911 4	1.0972	1.600	6.25
10.0	0.998 0	10.020		

is compelled to realize that the CMC, based here on an adaptation[3] of the Peierls-Yoccoz formalism,[16] plays an important role in the baglike model of quark confinement.

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APPENDIX: CURRENT MATRIX ELEMENTS

For a vector-current matrix element one has to study the relation

$$\begin{aligned} S_1 &= \bar{u}(p_2)\vec{\gamma}u(p_1) \\ &= N_1 N_2 \chi^\dagger \left[\frac{\vec{\sigma}(\vec{\sigma}\cdot\vec{p}_1)}{E_1+M} + \frac{(\vec{\sigma}\cdot\vec{p}_2)\vec{\sigma}}{E_2+M} \right] \chi \,, \\ N_i{}^2 &= (E_i+M)/2M \,, \end{aligned} \tag{A1}$$

for

$$p_2 = k+q,\quad p_1 = k,\quad q\to 0\,,$$

$$E_1 = E_2 = E(k) = (k^2+M^2)^{1/2};\quad N_1 = N_2 = N(k)\,,$$

and (A2)

$$S_1 = \frac{N^2(k)}{E(k)+M}\chi^\dagger(2\vec{k}+i\vec{\sigma}\times\vec{q})\chi\,.$$

The first form in (A2) does not contribute to the integral in (2.5). One finds analogously

$$S_2 = \frac{i}{2M}\bar{u}(p_2)\vec{\sigma}_{j\nu}q^\nu u(p_1) = \frac{1}{2M}\chi^\dagger(\vec{\sigma}\times\vec{q})_j\chi\,. \tag{A3}$$

The combination of (A2) and (A3) leads immediately ($G_M = F_1 + F_2$) to the expression (2.6).

In order to find (2.11a) one has to use

$$\bar{u}(p_2)\vec{\gamma}\gamma_5 u(p_1) = N_1 N_2 \chi^\dagger \left[\vec{\sigma} + \frac{(\vec{\sigma}\cdot\vec{p}_2)\vec{\sigma}(\vec{\sigma}\cdot\vec{p}_1)}{(E_1+M)(E_2+M)} \right] \chi\,. \tag{A4}$$

The second term in (A4) is

$$(\vec{\sigma}\cdot\vec{p}_2)\vec{\sigma}(\vec{\sigma}\cdot\vec{p}_1) = \vec{\sigma}\cdot(\vec{k}+\vec{q}\,)\vec{\sigma}\vec{\sigma}\cdot\vec{k} \to -\tfrac{1}{3}k^2\vec{\sigma}\,. \tag{A5}$$

The contribution proportional to $\vec{q}$ drops out after the integration over k. The spherical symmetry in k leads to the final result.

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