

Charge radii of proton and $M1$ radiative transitions of hadrons in a bag model with variable bag pressure

P. K. Chatley and C. P. Singh

Department of Physics, Panjab University, Chandigarh 160014 India

M. P. Khanna*

Department of Physics, University of Alberta, Edmonton, Alberta T6G 2J1, Canada

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Employing the idea of a state-dependent bag pressure, we compute the charge radii of the proton and $M1$ radiative decay widths for baryons and mesons.

I. INTRODUCTION

The MIT bag model¹ provides a satisfactory dynamical framework for treating hadrons as systems of confined quarks. In this model, the quarks are confined to some region of space (the bag) by a pressure term B . The origin of B is not explained in the theory and in the phenomenological applications of the model; it is treated as a universal parameter. The usual bag parametrization has met with limited success,²⁻⁴ however. Its biggest failure is that it gives a very small value of the proton magnetic moment. In this model, $2\mu_p M_p = 1.9$ not 2.79 (as has been found experimentally). This happens because the MIT model yields a smaller value for the proton radius.³ Also, in this model, $M1$ radiative transitions⁴ for baryons and mesons come out in poor agreement with the experimental results. In order to improve upon the MIT results, Hackman, Deshpande, Dicus, and Teplitz,⁵ have added to the energy of the bag a new term $c |N_q - N_{\bar{q}}|$, where c is a constant and $N_q (N_{\bar{q}})$ is the number of quarks (antiquarks), and have adjusted the value of c so as to regain the magnetic moment of the proton correctly. Such an attempt, though a good working principle to lessen the gap between theory and experiment, is however *ad hoc* in character and lacks a firm theoretical basis. The original MIT bag model, therefore, needs to be supplemented with new ideas.

Recently, Callan, Dashen and Gross⁶ have proposed a theory of hadronic structure, which, using QCD principles, leads to a baglike picture of hadrons. According to this theory, the QCD vacuum can exist in two distinct phases: a dense or normal phase, which is highly paramagnetic due to densely packed instanton and meron pairs with a large permeability, and a dilute phase where instanton effects are relatively small, and which is in equilibrium with the dense phase. The bag pressure B is then determined as the zero-field difference in the free-energy density between the dense and the dilute phases. It is implicit in their approach that B is essentially energy-density dependent.

The agreement observed in the implementation of the bag model can be reasonably improved by considering B as state dependent. Recently, Joseph and Nair⁷ have cal-

culated the masses of light hadrons and the magnetic moments of SU(3)-octet baryons using such an approach, and the fit obtained is quite satisfactory. So, in the present work, we argue in favor of a bag model in which the bag pressure B varies with the density of hadronic matter constituting each particle, keeping the radius of the bag constant. Our aim here is to investigate further how the idea of a state-dependent bag pressure modifies the MIT results concerning the various hadronic properties. For this, we recalculate axial-vector and electromagnetic charge radii of the proton and the $M1$ transition rates of $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+ + \gamma$ and $1^- \rightarrow 0^- + \gamma$ decays in the new approach.

Interestingly, our analysis yields results comparable with those obtained by Hackman *et al.*⁵ for the $M1$ transitions. Furthermore, our ratio for the axial and electromagnetic charge radii of proton comes out to be the same as that obtained by Donoghue and Golowich.⁸ These studies, together with those of Ref. 7, indicate that the new bag phenomenology, with a state-dependent bag pressure as its essential ingredient, is successful in reproducing several features of hadrons, such as their masses, magnetic moments, radiative decays, etc. It has the added advantage that now these properties arise in a more natural fashion from the theory, without invoking any additional *ad hoc* assumption.

The paper is organized as follows. In Sec. II, we write down the essentials of the original MIT bag and then give the new bag Hamiltonian. In Sec. III, we calculate the axial-vector and electromagnetic charge radii of the proton. The $M1$ transition rates for $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+ + \gamma$ and $1^- \rightarrow 0^- + \gamma$ are calculated in Sec. IV. In Sec. V, we conclude briefly.

II. PRELIMINARIES

A. The MIT bag Hamiltonian

The expression for the original MIT bag Hamiltonian has been discussed in detail at several places.¹⁻⁵ We simply summarize here various terms contributing to it as follows. First,

$$E_Q = \sum_i N_i w_i \quad (2.1)$$

is the kinetic-energy term. N_i is the number of quarks and antiquarks of the i th flavor. A quark of mass m moving in a spherical bag of radius R has a kinetic energy w given by

$$w(mR) = \frac{1}{R} [x^2 + (mR)^2]^{1/2}. \quad (2.2)$$

The bag volume energy

$$E_V = \frac{4}{3} \pi R^3 B \quad (2.3)$$

is the energy due to bag pressure B . Next,

$$E_0 = -\frac{Z_0}{R} \quad (2.4)$$

is a term accounting for zero-point fluctuations. The term E_M contains the residual gluonic interactions between the quarks and will be proportional to α_c , the strong coupling constant. This interaction is found to be magnetic in character:

$$E_M = \sum_{i>j} \lambda \vec{\sigma}_i \cdot \vec{\sigma}_j M_{ij}, \quad (2.5)$$

where

$$M_{ij} = 8\alpha_c \frac{\mu'(m_i R) \mu'(m_j R)}{R^3} I(m_i R, m_j R), \quad (2.6)$$

$$\mu'(mR) = \frac{R}{6} \left[\frac{4wR + 2mR - 3}{2wR(wR - 1) + mR} \right], \quad (2.7)$$

λ is 1 for a baryon and 2 for a meson, $\vec{\sigma}_i$ and $\vec{\sigma}_j$ are spins of the i th and j th quarks, and $I(m_i R, m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$ which is ≈ 1 .

The total energy or mass of a hadron is obtained by combining all of the above ingredients. Accordingly, we have

$$E(R) = \frac{C}{R} + \frac{4}{3} \pi B R^3, \quad (2.8)$$

where C contains kinetic, zero-point, and gluonic interaction energies.

In the MIT bag model, permanent confinement of each quark within the hadron bag is accomplished by means of two surface boundary conditions:

$$in \cdot \gamma \psi_\alpha(x) = \psi_\alpha(x), \quad (2.9)$$

and

$$\sum_\alpha n \cdot \partial [\bar{\psi}_\alpha(x) \psi_\alpha(x)] = 2B, \quad (2.10)$$

where n^μ is the interior unit four-normal to the bag's surface. The linear boundary condition (2.9) implies an eigenvalue equation

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}}. \quad (2.11)$$

The nonlinear boundary condition (2.10) is equivalent to minimizing the total energy with respect to radius of the bag^{1,2}: $\partial E / \partial R = 0$, which when applied to Eq. (2.8) gives

$$E = \frac{4}{3} (4\pi B)^{1/4} C^{3/4},$$

and

$$B = \frac{1}{4} \epsilon, \quad (2.12)$$

where $\epsilon = E/V$ is the energy density of the bag.

B. The new bag Hamiltonian

We now discuss the possibility of taking bag pressure term B as state-dependent, i.e., we consider that it is not a universal constant as in the original MIT model, but instead its value varies from hadron to hadron. While investigating the ground-state properties of quark gas, Chaplin and Nauenberg,⁹ using a renormalized coupling constant α_c that depends upon the Gibbs energy per quark, have demonstrated that the effective bag pressure B may depend upon quark momentum. It lends support to the idea of a state-dependent bag pressure. Making use of relation (2.12), and putting

$$\epsilon = B + \rho,$$

where ρ represents the contribution to the hadronic mass from all sources except the volume tension, we get

$$B = \frac{1}{3} \rho. \quad (2.13)$$

We may then write the total bag Hamiltonian as

$$M = \frac{4}{3} \left[\sum_i N_i w_i + \sum_{i<j} \lambda (\vec{\sigma}_i \cdot \vec{\sigma}_j) M_{ij} - \frac{Z_0}{R} \right]. \quad (2.14)$$

In Eq. (2.14), the factor $\frac{4}{3}$ is due to the addition of the bag volume energy, necessary to confine the quarks within the bag, to the remaining terms of the Hamiltonian.

As the bag-model results are not very sensitive to small variations in the bag size, we can talk of an average baryon radius R_B and an average meson radius R_M . These are related to each other through¹⁰

$$R_B \approx R_M \left(\frac{3}{2}\right)^{1/3}. \quad (2.15)$$

Equation (2.14) can, then, be written as

$$M = \frac{4}{3} \left[\sum_i N_i w_i + \sum_{i<j} \lambda \vec{\sigma}_i \cdot \vec{\sigma}_j M_{ij} - E_c \right], \quad (2.16)$$

where the zero-point energy E_c is now a constant with one value for baryons and another for mesons.

In order to evaluate the model parameters, viz. the quark masses, the bag radii R_B and R_M , the gluon coupling constant α_c , and the zero-point energy E_c , the known values of the axial-vector coupling constant

$$g_A = \frac{5}{9} \left[\frac{2w^2 R^2 + 4mRwR - 3mR}{2wR(wR - 1) + mR} \right], \quad (2.17)$$

and the proton magnetic moment (in nuclear magnetons) are employed to solve the transcendental equation

$$\tan(w^2 R^2 - m^2 R^2)^{1/2} = -\frac{(w^2 R^2 - m^2 R^2)^{1/2}}{(wR + mR - 1)} \quad (2.18)$$

numerically, which yields⁷

TABLE I. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ for mesons as defined in Ref. 5.

Transition	q_1q_1	q_1q_2	q_2q_2
$\rho \rightarrow \pi\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{37}$
$\omega \rightarrow \pi\gamma$	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$
$\phi \rightarrow \eta\gamma$	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{3}{27}$
$K^{*+} \rightarrow K^+\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{27}$
$K^{0*} \rightarrow K^0\gamma$	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{3}{27}$

$$R_B = 8.88 \text{ GeV}^{-1},$$

$$m_n = 0.114 \text{ GeV},$$

$$w_n = 0.294 \text{ GeV}.$$

The values of w_s and m_s are then determined by using the mass separation¹¹ ($\Lambda - N$) and Eq. (2.18) to get

$$w_s = 0.427 \text{ GeV}, \quad m_s = 0.302 \text{ GeV}.$$

Finally, using the ground-state baryon masses, one gets $\alpha_c = 0.94$ and $E_c = 0.068 \text{ GeV}$. For mesons, assuming that the quark masses remain the same and that the meson radius is given by

$$R_M \approx R_B \left(\frac{2}{3}\right)^{1/3} = 7.75 \text{ GeV}^{-1},$$

the quark kinetic energies have been found to increase to⁷

$$w_n = 0.321 \text{ GeV},$$

$$w_s = 0.448 \text{ GeV}.$$

Employing the same value of α_c as for baryons, and fitting the experimental mass of the K meson, one gets $E_c = 0.178 \text{ GeV}$.

Note that since, in the present approach, the bag energy BV is not a constant, but varies from hadron to hadron, the model described here is not relativistically invariant. It is worth mentioning here that though the MIT bag model is relativistically invariant, this invariance has not been utilized in an essential manner in any of its applications.

III. AXIAL-VECTOR AND ELECTROMAGNETIC CHARGE RADII

It has been pointed out earlier⁸ that for any reasonable fixed-sphere bag model, the radius associated with the distribution of axial charge within a nucleon and its electromagnetic (em) charge radius should be equal, i.e.,

$$\langle r^2 \rangle_{\text{axial}}^{1/2} = \langle r^2 \rangle_{\text{em}}^{1/2}. \quad (3.1)$$

We make a check of this equation using the new bag phenomenology. We use the standard definitions⁸

$$\langle r^2 \rangle_{\text{axial}} = \langle p(s_z) | \int_{\text{bag}} d^3x \bar{x}^2 \psi^\dagger(\bar{x}) \times \tau_3 \sigma_z \psi(\bar{x}) | p(s_z) \rangle, \quad (3.2)$$

$$\langle r^2 \rangle_{\text{em}} = \langle p(s_z) | \int_{\text{bag}} d^3x | \bar{x} |^2 \psi^\dagger(x) \times Q \psi(x) | p(s_z) \rangle, \quad (3.3)$$

where p stands for a proton and $\psi(x)$ is a quark field operator. Equations (3.2) and (3.3) imply, on evaluation,

$$\langle r^2 \rangle_{\text{axial}} = \frac{10}{27} \frac{R^2}{(w^2 - m^2 R^2)} \frac{A(w, mR)}{(2w^2 - 2w + mR)} \quad (3.4)$$

with A , a quartic polynomial in w , given by

$$\begin{aligned} A = & w^4 + 2w^3(1 + mR) - w^2(4 + \frac{1}{2}mR + m^2R^2) \\ & + w(3 - mR - 2m^2R^2 - 2m^3R^3) \\ & + \frac{1}{2}mR(-\frac{3}{2} + mR + m^2R^2), \end{aligned} \quad (3.5)$$

and

$$\langle r^2 \rangle_{\text{em}} = \frac{R^2}{6} \frac{C}{(w^2 R^2 - m^2 R^2)(2w^2 R^2 - 2wR + mR)}, \quad (3.6)$$

where C is

$$\begin{aligned} C = & 4(wR)^4 - 4(wR)^3 + w^2 R^2(8 + 6mR - 4m^2 R^2) \\ & + wR(-6 - 8mR + 4m^2 R^2) \\ & + (9mR - 6m^2 R^2 - 6m^3 R^3). \end{aligned} \quad (3.7)$$

We evaluate these expressions using the values $mR = 1.012$, $wR = 2.61$, and $R = 8.88 \text{ GeV}^{-1}$ and find

TABLE II. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ for baryons as defined in Ref. 5.

Transition	q_1q_1	q_1q_2	q_1q_3	q_2q_2	q_2q_3	q_3q_3
$\Delta \rightarrow N\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$\Sigma^{*+} \rightarrow \Sigma^+\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$\Sigma^{0*} \rightarrow \Sigma^0\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$	$\frac{4}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$
$\Sigma^{*-} \rightarrow \Sigma^-\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{27}$
$\Sigma^{0*} \rightarrow \Lambda\gamma$	0	0	0	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$
$\Sigma^0 \rightarrow \Lambda\gamma$	0	0	0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
$\Xi^{0*} \rightarrow \Xi^0\gamma$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{8}{27}$	$\frac{1}{27}$	$\frac{8}{27}$	$\frac{16}{27}$
$\Xi^{*-} \rightarrow \Xi^-\gamma$	$\frac{1}{27}$	$\frac{2}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$	$-\frac{4}{27}$	$\frac{4}{27}$

TABLE III. Quark transition moments μ_α for mesons.

Transition	μ_{q_1}	μ_{q_2}
$\rho \rightarrow \pi\gamma$	0.4165	0.4165
$\omega \rightarrow \pi\gamma$	0.4094	0.4094
$\phi \rightarrow \eta\gamma$	0.3096	0.3096
$K^* \rightarrow K\gamma$	0.3567	0.5087

$$\begin{aligned} \langle r^2 \rangle_{\text{axial}}^{1/2} &= 1.21 \text{ fm} , \\ \langle r^2 \rangle_{\text{em}}^{1/2} &= 1.20 \text{ fm} . \end{aligned} \quad (3.8)$$

These values, though a little higher than the corresponding experimental numbers¹¹ available, agree very well with Eq. (3.1). This clearly indicates that the new bag phenomenology is inherently consistent.

IV. $M1$ RADIATIVE TRANSITIONS

As already known,⁵ the expression for $M1$ transition width is given by

$$\Gamma = \frac{e^2}{4\pi} k^3 \left(\frac{16}{3}\right) \sum_{\substack{\alpha, \beta \\ \alpha > \beta}} \mu_\alpha \mu_\beta C_{\alpha\beta}^{PQ} , \quad (4.1)$$

where α, β denote the quark flavors, k is the photon momentum, and the quark transition moment μ_α is given by

$$\begin{aligned} \mu_\alpha &= \frac{1}{2k} N_\alpha^2 \int_0^R dx x^2 j_1(kx) \\ &\quad \times 2 \left[j_0 \left[\frac{x_\alpha x}{R} \right] j_1 \left[\frac{x_\alpha x}{R} \right] \right. \\ &\quad \left. \times \left[\frac{w_\alpha + m_\alpha}{w_\alpha} \right]^{1/2} \left[\frac{w_\alpha - m_\alpha}{w_\alpha} \right]^{1/2} \right] . \end{aligned} \quad (4.2)$$

Here N_α , the quark normalization, is given by

$$\frac{1}{N_\alpha^2} = R^3 j_0^2(x_\alpha) \frac{2w_\alpha(w_\alpha - 1/R) + m_\alpha/R}{w_\alpha(w_\alpha - m_\alpha)} \quad (4.3)$$

with the frequency of the lowest mode given by Eq. (2.2). In Eq. (4.2), the j 's are spherical Bessel functions, and x_α satisfies the eigenvalue Eq. (2.11) which determines x_α for

TABLE V. $M1$ radiative decay widths (in keV) for mesons.

Transition	MIT (Hackman <i>et al.</i>)	Present analysis	Experiment (Ref. 11)
$\rho \rightarrow \pi\gamma$	34.4	39.6	63±8
$\omega \rightarrow \pi\gamma$	310.3	360.0	789±92
$\phi \rightarrow \eta\gamma$	43.7	78.6	65±15
$K^{*+} \rightarrow K^+\gamma$	8.69	5.35	< 80
			60±15
$K^{0*} \rightarrow K^0\gamma$	93.9	95.6	75±35

TABLE IV. Quark transition moments μ_α for baryons.

Transition	μ_{q_1}	μ_{q_2}	μ_{q_3}
$\Delta \rightarrow N\gamma$	0.6081	0.6081	0.6081
$\Sigma^{*+} \rightarrow \Sigma^+\gamma$	0.4792	0.6488	0.6488
$\Sigma^{*0} \rightarrow \Lambda\gamma$	0.4687	0.6253	0.6253
$\Sigma \rightarrow \Lambda\gamma$	0.5563	0.7535	0.7535
$\Xi^{*0} \rightarrow \Xi^0\gamma$	0.5160	0.5160	0.6951

each quark in each hadron. The explicit wave function for each quark in each hadron is

$$\psi_\alpha(r, t) = \frac{N_\alpha}{4\pi} \begin{bmatrix} \left[\frac{w_\alpha + m_\alpha}{w_\alpha} \right]^{1/2} ij_0 \left[\frac{x_\alpha r}{R} \right] U_\alpha \\ - \left[\frac{w_\alpha - m_\alpha}{w_\alpha} \right]^{1/2} j_1 \left[\frac{x_\alpha r}{R} \right] \vec{\sigma} \cdot \hat{r} U_\alpha \end{bmatrix} , \quad (4.4)$$

where $r = |\vec{r}|$ and U_α are two-component Pauli spinors. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ are given by

$$\begin{aligned} C_{\alpha\beta}^{PQ} &= \sum_{m, m', k, k'} \langle P | b_\alpha(m) Q_\alpha b_\alpha(m') | Q \rangle \\ &\quad \times \langle Q | b_\beta(k) Q_\beta b_\beta(k') | P \rangle U_m^\dagger \sigma_i U_{m'} U_k^\dagger \sigma_i U_{k'} , \end{aligned} \quad (4.5)$$

where $b_\alpha(m)$ is the destruction operator for a quark of type α with spin projection m , Q_α is the charge on the α th quark, and Q denotes an intermediate state contributing to the magnetic energy of a particle P . The various $C_{\alpha\beta}^{PQ}$ which are of interest for the present work are given in Tables I and II. In Tables III and IV, we present our values of the transition moments, using Eq. (4.2), for mesons and baryons. We then calculate the $M1$ radiative decay widths employing Eq. (4.1) and list our results in Table V for mesons, and Table VI for baryons. For simplicity, in our calculations involving isoscalar pseudoscalar meson η , we have neglected the dynamical mixing induced by transitions to pure gluon states.¹²

It is interesting to note that our results are similar to, and are somewhat nearer the experiment than the values

TABLE VI. $M1$ radiative decay widths (in keV) for baryons.

Transition	MIT (Hackman <i>et al.</i>)	Present analysis	Experiment (Ref. 11)
$\Delta \rightarrow N\gamma$	291.5	358.2	700±70
$\Sigma^{*+} \rightarrow \Sigma^+\gamma$	135.9	135.3	
$\Sigma^{0*} \rightarrow \Sigma^0\gamma$	26.4	31.5	< 1800
$\Sigma^{*-} \rightarrow \Sigma^-\gamma$	1.9	1.4	
$\Sigma^{0*} \rightarrow \Lambda\gamma$	197.7	232.2	< 2200
$\Sigma^0 \rightarrow \Lambda\gamma$	1.36	1.62	
$\Xi^{0*} \rightarrow \Xi^0\gamma$	145.0	183.3	
$\Xi^{*-} \rightarrow \Xi^-\gamma$	1.9	1.6	< 360

which Hackman *et al.*⁵ obtained by assuming an additional *ad hoc* term to the bag energy.

V. CONCLUSION

The MIT bag model with universal bag pressure, though a powerful tool in reproducing some of the salient features of hadrons, has not met with spectacular success when applied to nonspectroscopic calculations. There have been several attempts to resolve the discrepancies. However, such attempts are *ad hoc* in character and do not emerge logically from the theory. Callan *et al.*⁶ have presented a theory of hadronic structure leading to a bag-like picture of hadrons in which the bag pressure B is implicitly an energy-density dependent factor. Keeping this aspect in view, and following Ref. 7, we have discussed a new bag phenomenology with a state-dependent variable bag pressure as its essential ingredient. We have recalculated

the axial and em charge radii of proton, and $M1$ radiative transition rates for bosons and fermions. Our results for the radiative decay widths almost tally with the values obtained by Hackman *et al.*⁵ who, to lessen the disagreement between theory and experiment, proposed an additional term to the bag energy. Furthermore, in our approach, the ratio of the axial and em charge radii of proton comes out to be unity, which is quite consistent with an earlier prediction of Donoghue and Golowich.⁸

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*Present and permanent address: Department of Physics, Panjab University, Chandigarh 160014, India.

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