Testing the standard model by precise determinations of $W^{\pm}$ and Z masses

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We analyze the standard model's predictions for m_W, m_Z, and their interdependence. Simple formulas are given. Criteria for testing the theory at the tree and loop levels via precise mass measurements are proposed. Effects due to large Higgs-boson and top-quark masses as well as additional generations are illustrated. Constraints from neutral-current phenomenology are discussed. Present experimental values of m_W and m_Z are compared with theoretical expectations.

Recent experimental discoveries of the $W^{\pm}$ and Z bosons have generated considerable interest in the precise determination of their masses m_W and m_Z.[1-3] Two important reasons behind this interest are the following. (1) The standard $SU(3)_c \times SU(2)_L \times U(1)$ model's predictions for m_W, m_Z, and their interdependence is very constrained, so that a pronounced disagreement would indicate a need to modify the theory at the tree level. (2) Radiative corrections in the predictions for m_W and m_Z are quite large;[4-9] hence an accurate comparison of theory and experiment tests the standard model at the quantum loop level.

Motivated by such considerations, the aim of this paper is (a) to examine the standard model's predictions for m_W and m_Z, (b) to present a number of simple analytic expressions which describe the m_W,m_Z interdependence, (c) to establish criteria for deciding whether the theory needs to be modified at the tree level when accurate values of m_W and m_Z become available, (d) to illustrate the effect of exotic values for m_t and m_ϕ (the top-quark and Higgs-scalar masses) as well as additional fermion generations on the quantum corrections, and (e) to analyze already existing experimental constraints on possible modifications of the standard theory. Our discussion is based on the work in Refs. 5—7 which presented complete $O(\alpha)$ corrections to the m_W and m_Z predictions. Recent refinements of that analysis include the evaluation of two-loop $O(\alpha^2 \ln m)$ effects, where m is a generic fermion mass,[10] and detailed studies concerning QCD corrections to the electromagnetic contributions and potential effects arising from additional fermion contributions.[11]

We begin our discussion by recalling the standard model's prediction for the $W^{\pm}$ and Z masses:[5,10]

$$m_W = \left[\frac{\pi\alpha}{\sqrt{2} G_\mu \sin^2\theta_W (1-\Delta r)} \right]^{1/2} , \tag{1}$$

$$m_Z = m_W / \cos\theta_W , \tag{2}$$

where the weak mixing angle θ_W is defined[5] such that Eq. (2) is exact (i.e., $\cos\theta_W \equiv m_W/m_Z$) and Δr denotes the $O(\alpha)$ radiative corrections.[12] Inserting the precisely determined fine-structure constant[13]

$$\alpha = 1/137.035\,963(15) \tag{3a}$$

and muon decay constant[10]

$$G_\mu = 1.166\,34 \pm 0.000\,02 \times 10^{-5}\ \text{GeV}^{-2} \tag{3b}$$

in the above expressions yields

$$m_W = A/\sin\theta_W , \tag{4a}$$

$$m_Z = 2A/\sin 2\theta_W , \tag{4b}$$

$$A \equiv \left[\frac{\pi\alpha}{\sqrt{2}\, G_\mu} \right]^{1/2} \left[\frac{1}{1-\Delta r} \right]^{1/2} = \frac{37.2810 \pm 0.0003\ \text{GeV}}{(1-\Delta r)^{1/2}} . \tag{4c}$$

To study further the interdependence between m_W and m_Z, it is convenient to eliminate θ_W between Eqs. (2) and (4a). This leads to the following useful formulas:

$$m_Z = \frac{m_W}{(1 - A^2/m_W{}^2)^{1/2}} , \tag{5a}$$

$$m_W = m_Z \left[\frac{1 + (1 - 4A^2/m_Z{}^2)^{1/2}}{2} \right]^{1/2} , \tag{5b}$$

$$m_Z - m_W = m_W \left[\frac{1}{(1 - A^2/m_W{}^2)^{1/2}} - 1 \right] , \tag{5c}$$

$$m_Z - m_W = m_Z \left[1 - \left[\frac{1 + (1 - 4A^2/m_Z{}^2)^{1/2}}{2} \right]^{1/2} \right] , \tag{5d}$$

$$\Delta r = 1 - \frac{(37.281\ \text{GeV})^2}{m_W{}^2 (1 - m_W{}^2/m_Z{}^2)} , \tag{5e}$$

$$\sin^2\theta_W = 1 - m_W{}^2/m_Z{}^2 . \tag{5f}$$

In writing Eqs. (5b) and (5d) we have not considered the solution with a minus sign in front of $(1-4A^2/m_Z{}^2)^{1/2}$ because it corresponds to $\sin^2\theta_W \geq \frac{1}{2}$, which contradicts neutral-current phenomenology.

The complete $O(\alpha)$ corrections Δr were given in Refs. 5 and 6. They are quite large ($\Delta r \simeq 0.07$), primarily due to vacuum-polarization effects. Indeed, the dominant contributions to $\alpha/(1-\Delta r)$ in Eq. (1) can be interpreted as an effective electromagnetic coupling at a distance scale $1/m_W$ which has evolved to $\simeq 1/127.5$ as a result of fermion vacuum polarization.[4,14] A numerical evaluation of Δr employing $\sin^2\theta_W=0.217$ (the central value from deep-inelastic ν_μ scattering[7,15]), $m_\phi=m_Z$, $m_c=1.5$ GeV, $m_b=4.5$ GeV, $m_t=36$ GeV, Wetzel's analysis[16] of the low-frequency contribution to the dispersive integral in $e^+e^- \to$ hadrons, and QCD corrections[11] in the high-frequency part leads to

$$\Delta r = 0.0696 \pm 0.0020 \quad \text{for} \begin{cases} \sin^2\theta_W = 0.217\ , \\ m_\phi = m_Z\ , \\ m_t = 36\ \text{GeV}\ , \end{cases} \tag{6}$$

where an estimate of uncertainties in the hadronic contributions has been included. This numerical value is not sensitive to small shifts in $\sin^2\theta_W$, m_ϕ, or m_t. For this reason we will regard Eq. (6) as the standard value for Δr and we will take it to be constant throughout the discussion preceding Eq. (13). [The effect of large shifts in m_ϕ or m_t and higher generations is described in Eq. (13) and following.] Combining Eqs. (4c) and (6) leads to

$$A = 38.65 \pm 0.04\ \text{GeV}\ . \tag{7}$$

Insertion of Eq. (7) into Eqs. (4a) and (4b) is useful to predict m_W and m_Z from a separate determination of $\sin^2\theta_W$ or, inverting those equations, one can determine $\sin^2\theta_W$ by measuring m_W or m_Z.

The generally quoted predictions for m_W and m_Z are obtained by using the world-average value[7,15,17]

$$\sin^2\theta_W = 0.217 \pm 0.014 \tag{8}$$

which is currently obtained mainly from deep-inelastic ν_μ scattering and the e-D asymmetry, after correcting the experiments for radiative corrections. Equations (4a), (4b), (7), and (8) lead to

$$m_W = 83.0^{+2.9}_{-2.7}\ \text{GeV}\ , \tag{9a}$$

$$m_Z = 93.8^{+2.4}_{-2.2}\ \text{GeV}\ . \tag{9b}$$

The errors are of course correlated. Similarly, combining Eq. (7) with Eqs. (5a)—(5d) permits us to predict either mass in terms of the other, and the mass difference in terms of m_W or m_Z. For instance, over the range of m_Z values given in Eq. (9b), the predicted mass difference is

$$m_Z - m_W = 10.8 \pm 0.5\ \text{GeV} \tag{10}$$

which illustrates the interdependence of m_W and m_Z and the tight constraint on their values in the standard model. More generally we note that A, as defined by Eq. (4c), depends weakly on m_Z. Indeed, A departs from the value 38.65 GeV given in Eq. (7) by at most 0.15% for 88 GeV $< m_Z <$ 100 GeV; therefore, taking $A=38.65$ GeV to be constant is a good approximation. The implications of Eqs. (5d) and (7) are illustrated in Fig. 1 where we have plotted $m_Z - m_W$ vs m_Z. On the same graph, the values of $m_Z - m_W$ are given for $A=37.281$ GeV (i.e., no radiative corrections). Some numerical results are explicitly shown in Table I. The general features are apparent.

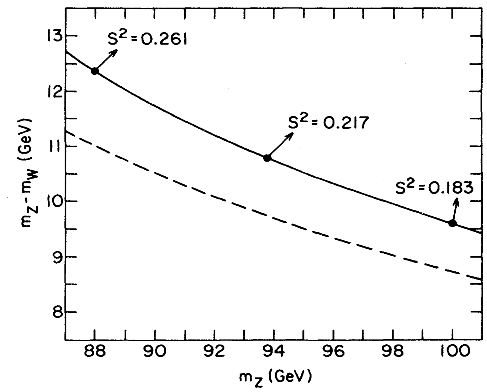

FIG. 1. Graph of $m_Z - m_W$ vs m_Z. The solid curve includes radiative corrections while the dashed curve is uncorrected.

(1) For a given m_Z, the radiative corrections increase the predicted mass difference by 10% to 12% over the tree-level result, a sizable effect.

(2) The mass difference decreases with increasing m_Z.

(3) Over the range 88 GeV $< m_Z <$ 100 GeV, $m_Z - m_W$ varies slowly from 12.4 to 9.6 GeV.

(4) For the central prediction $m_Z = 93.8$ GeV, one finds $m_Z - m_W = 10.8$ GeV [see Eq. (10)].

The formulas in Eqs. (5a)—(5d) will be particularly useful if both masses cannot be independently measured with high precision. We anticipate a very precise determination of m_Z, to within ± 0.1 GeV, at e^+e^- colliders before the end of the decade. The value of m_W may be measured with high precision by employing a transverse-mass-distribution plot[18] at hadron colliders. Using the known value of m_Z as a calibration, one might determine $m_Z - m_W$ to within ± 0.25 GeV in such a manner. A glance at Fig. 1 indicates that such a combination of precise measurements would clearly test the standard model at the level of its quantum corrections.

TABLE I. Predicted values for $m_Z - m_W$ as a function of m_Z [Eq. (5d)]. The second column contains the quantum corrections [Eqs. (4c), (6), and (7)]. The third column gives the tree-level predictions. The last column illustrates the fact that inclusion of quantum corrections leads to mass differences 10% to 12% larger than tree-level calculations.

m_Z (GeV)	$m_Z - m_W$ (GeV) for $A=38.65$ GeV	$m_Z - m_W$ (GeV) for $A=37.281$ GeV	Ratio of two last columns
100	9.60	8.72	1.101
98	9.95	9.02	1.103
96	10.32	9.34	1.105
93.8	10.79	9.73	1.109
92	11.21	10.08	1.112
90	11.74	10.51	1.117
88	12.35	11.00	1.123

When sufficiently accurate values of m_W and m_Z become available, one would like to have simple criteria for determining whether the theory needs to be modified either at the tree or loop levels. One possible indicator is the correction factor

$$\frac{1}{(1-\Delta r)^2}=\left[\frac{m_W}{37.281\text{ GeV}}\right]^4\left[1-\frac{m_W{}^2}{m_Z{}^2}\right]^2 . \tag{11}$$

That quantity describes radiative corrections to the muon decay rate when the latter is expressed in terms of α, m_W, and m_Z (the fundamental parameters of the theory). Recalling Eq. (6) we see that already in the standard model this factor deviates from 1 primarily because of large vacuum-polarization effects:

$$1/(1-\Delta r)^2=1.1552 . \tag{12}$$

They are, however, already summed by the renormalization group and will not appear in higher orders.[19] Any large departure from Eq. (12) would be difficult to explain as an additional $O(\alpha)$ correction and would presumably signal a need to change the theory at the tree level.

To illustrate this criterion, consider a hypothetical case in which $m_W=81.0$ GeV and $m_Z=95.0$ GeV with high precision. Then, from Eq. (11) we obtain $1/(1-\Delta r)^2=1.66$. Given such a large deviation from the prediction in Eq. (12), one could plausibly argue for a tree-level change in the theory.

If, on the other hand, a small deviation from the standard-model predictions is found, it could signal interesting new contributions to Δr rather than a failure of the standard model. Three ways to modify Δr are (1) to increase m_t significantly beyond 36 GeV, (2) to assume the existence of additional fermion generations, and (3) to increase significantly m_ϕ beyond m_Z. We discuss the three mechanisms in turn.

An important feature of the quantum corrections is that mass splittings between $T_3=\frac{1}{2}$ and $T_3=-\frac{1}{2}$ fermions of the order of a few times m_W can lead to large negative contributions to Δr (T_3 is the third component of weak isospin). For example, in the standard three generations case, for large $m_t{}^2/m_W{}^2$ one has asymptotically a contribution to Δr of the form[6]

$$\delta\simeq-\frac{3\alpha}{16\pi}\frac{\cos^2\theta_W}{\sin^4\theta_W}\frac{m_t{}^2}{m_W{}^2}+\cdots , \quad m_t{}^2/m_W{}^2\gg 1 . \tag{13}$$

In Eq. (13) and the following, δ denotes possible exotic contributions to Δr associated with large m_t or m_ϕ masses, additional fermion generations, etc., which are not reflected in the standard value of Eq. (6). Equation (13) gives only the leading term in the asymptotic expansion. The complete expressions for arbitrary m_t are given in Ref. 6 and have also been studied by other authors.[20] Using the exact formulas, the dependence of Δr, A, and the predicted value of m_Z-m_W on m_t is illustrated in Table II for $m_\phi=m_Z$, $\sin^2\theta_W=0.217$. One readily sees that the large-m_t effects are of opposite sign to the standard correction [Eq. (6)] and decrease the predicted values of m_Z-m_W in terms of m_W or m_Z [Eqs. (5c) and (5d)]. As an extreme example, $m_t=240$ GeV gives rise to a negative δ that slightly overcomes the standard correction of Eq. (6). This would lead, for given m_W or m_Z, to a mass difference m_Z-m_W slightly smaller than the tree-level prediction.

TABLE II. Dependence of the radiative correction Δr, the parameter A, and the predicted m_Z-m_W on the top-quark mass m_t. The table was evaluated for $\sin^2\theta_W=0.217$ and $m_\phi=m_Z$ using the exact formulas of Refs. 5 and 6.

m_t (GeV)	Δr	A (GeV)	Predicted m_Z-m_W (GeV) for $m_Z=93.8$ GeV
20	0.0699	38.66	10.79
36	0.0696	38.65	10.79
60	0.0714	38.69	10.82
83	0.0623	38.50	10.66
100	0.0558	38.37	10.56
150	0.0379	38.01	10.27
200	0.0167	37.60	9.96
240	−0.003 55	37.21	9.68
Tree approximation	0	37.28	9.73

Similarly, if there exist higher fermion generations, mass splittings between the $T_3=\frac{1}{2}$ and $T_3=-\frac{1}{2}$ fermions of the order of a few times m_W can also lead to large negative contributions to Δr. For example, if the ratio of the quark masses is close to 1, one obtains again contributions analogous to Eq. (13) with

$$m_t{}^2/m_W{}^2\rightarrow |U_{ij}|^2(m_i-m_j)^2/m_W{}^2 ,$$

where i and j refer to the $T_3=\frac{1}{2}$ and $T_3=-\frac{1}{2}$ quarks, respectively, and U_{ij} is the appropriate element of the generalized Kobayashi-Maskawa matrix. It should be pointed out, however, that large values of m_t or m_i,m_j of higher generations may give rise to additional theoretical problems: the Yukawa couplings of Higgs scalars and the massive fermions become large and perturbative expansions may lose their meaning.

Another important question is whether unseen fermions of higher generations can increase rather than decrease the value of Δr. Restricting the masses of the charged fermions to be >20 GeV, the answer is that the contribution to Δr of a single generation of quarks and leptons is bounded above by $\delta\lesssim 0.007$, the upper bound arising by optimizing the choice of the fermion masses.[11] This is at most a tenth of the standard value of Eq. (6). For given $\sin^2\theta_W$, this effect increases the predicted values for m_W and m_Z [Eqs. (4a) and (4b)] by $\lesssim 0.30$ GeV and $\lesssim 0.34$ GeV, respectively; for given m_W, one finds an increase of m_Z-m_W [Eq. (5c)] by $\lesssim 90$ MeV. An additional generation with degenerate masses can also give rise to small positive contributions to Δr. The conclusion is that unless there is a large number of additional generations with masses carefully contrived to give positive contributions, one does not expect a significant increase in Δr.

For large m_ϕ there are asymptotic contributions to Δr of the form[5,6]

$$\delta \simeq \frac{11}{48\pi}\frac{\alpha}{\sin^2\theta_W}\left[\frac{m_\phi{}^2}{m_Z{}^2}\right] + \cdots , \quad m_\phi{}^2/m_Z{}^2 \gg 1 . \tag{14}$$

Equation (14) increases Δr from the standard value of Eq. (6) and, therefore, it leads to larger $m_Z - m_W$ for given m_W or m_Z; however, for $m_\phi{}^2/m_Z{}^2 < 100$ suggested by considerations of perturbative unitarity,[21] the effects are small, namely, an increase <0.0088 in Δr (the bound is obtained by using the exact expressions of Refs. 5 and 6 rather than the asymptotic formula).

If the experimental results differ significantly from the predictions of the standard model, one possibility is to generalize Eq. (2) to read

$$\rho = m_W{}^2/(m_Z{}^2\cos^2\theta_W) . \tag{15}$$

Equation (15) describes a more general class of $SU(2)_L \times U(1)$ theories which would be appropriate, for instance, if Higgs multiplets not satisfying $T(T+1)=3T_3{}^2$ exist (T and T_3 stand for the neutral component's quantum numbers of weak isospin and its third component, respectively). Combining Eqs. (4a) and (15) one finds

$$\rho = \frac{m_W{}^2}{m_Z{}^2(1-A^2/m_W{}^2)} . \tag{16}$$

Clearly, the standard model corresponds to $\rho = 1$. The value of ρ is however constrained by neutral-current (NC) phenomenology. Indeed, the parameter $\rho_{\rm NC}^{(\nu;h)}$ which appears as a renormalization factor in neutral-current ν-hadron amplitudes when the latter are expressed in terms of G_μ (see Secs. III A and III D of Ref. 6) is related to ρ:

$$\rho_{\rm NC}^{(\nu;h)} = \rho(1+\epsilon_{\rm NC}^{(\nu;h)}) , \tag{17}$$

where $\epsilon_{\rm NC}^{(\nu;h)}$ denote $O(\alpha)$ corrections. As ρ is close to 1, a sensible approach is to evaluate approximately A in Eq. (16) and $\epsilon_{\rm NC}^{(\nu;h)}$ in Eq. (17) by using the radiative corrections of the standard model. In that case A is still given by Eq. (4c) and $\epsilon_{\rm NC}^{(\nu;h)}$ can be identified with the $O(\alpha)$ term on the right-hand side of Eq. (24a) of Ref. 6. Using the latter one finds that, for $m_\phi = m_Z$, $m_t = 36$ GeV, $\rho_{\rm NC}^{(\nu;h)}/\rho = 1.000\,52$. Thus, for such parameter values, ρ and $\rho_{\rm NC}^{(\nu;h)}$ are theoretically very close. After including radiative corrections to the experiments, a two-parameter fit to deep-inelastic ν_μ and $\bar{\nu}_\mu$ scattering data yields[22,23]

$$\rho_{\rm NC}^{(\nu;h)} = 1.02 \pm 0.02 , \tag{18a}$$

$$\sin^2\theta_W = 0.238 \pm 0.030 . \tag{18b}$$

Accepting Eq. (18a) and remembering the theoretical closeness of ρ and $\rho_{\rm NC}^{(\nu;h)}$ places a tight constraint on Eq. (16). It is important to note that similar constraints can be obtained even if $m_t \gg 36$ GeV. To understand this point, let us call $(\Delta r)_{\rm est}$ the standard value of Eq. (6) and $\rho_{\rm est}$ the corresponding quantity evaluated via Eqs. (7) and (16):[24]

$$\rho_{\rm est} \equiv \frac{m_W{}^2}{m_Z{}^2[1-(38.65\,{\rm GeV})^2/m_W{}^2]} . \tag{19a}$$

In the above "est" means "estimated." If the true radiative correction Δr differs from the standard value $(\Delta r)_{\rm est}$ by an amount δ due to large m_t or other exotic effects, so that $\Delta r = (\Delta r)_{\rm est} + \delta$, one finds on the basis of Eqs. (4a) and (4c) and Eqs. (15), (16), and (19a)

$$\frac{\rho_{\rm est}}{\rho} = \left[1+\frac{\tan^2\theta_W\delta}{1-(\Delta r)_{\rm est}}\right]^{-1} \approx 1 - \tan^2\theta_W\delta , \tag{19b}$$

where the last expression corresponds to the leading term in an expansion in powers of α. Combining with Eq. (17),

$$\frac{\rho_{\rm est}}{\rho_{\rm NC}^{(\nu;h)}} = \frac{1-\tan^2\theta_W\delta}{1+\epsilon_{\rm NC}^{(\nu;h)}} . \tag{19c}$$

Recalling Eq. (13) and the expression for $\epsilon_{\rm NC}^{(\nu;h)}$ given in Eq. (24a) of Ref. 6 reveals that $-\tan^2\theta_W\delta$ and $\epsilon_{\rm NC}^{(\nu;h)}$ have the same leading asymptotic behavior for large $m_t{}^2/m_W{}^2$, namely,

$$-\tan^2\theta_W\delta \approx \epsilon_{\rm NC}^{(\nu;h)} = \frac{3\alpha}{16\pi}\frac{1}{\sin^2\theta_W}\frac{m_t{}^2}{m_W{}^2} + \cdots , \quad m_t{}^2/m_W{}^2 \gg 1 . \tag{19d}$$

Indeed, Eq. (19d) is exactly the expression found by Veltman[25] in his studies of the radiative corrections to the ratio of neutral- and charged-current amplitudes. The analogy between $-\tan^2\theta_W\delta$ and $\epsilon_{\rm NC}^{(\nu;h)}$ is by no means perfect. For example, for large $m_t{}^2/m_W{}^2$ there are terms in $-\tan^2\theta_W\delta$ involving $\ln(m_t{}^2/m_W{}^2)$ which are not present in $\epsilon_{\rm NC}^{(\nu;h)}$. Also, for large $m_\phi{}^2/m_Z{}^2$, $-\tan^2\theta_W\delta$ contains an asymptotic term

$$-\tfrac{11}{48}(\alpha/\pi)(1/\cos^2\theta_W)\ln(m_\phi{}^2/m_Z{}^2)$$

[see Eq. (14)], while $\epsilon_{\rm NC}^{(\nu;h)}$ has a similar asymptotic behavior but with a slightly different coefficient: $\frac{9}{48}$ instead of $\frac{11}{48}$. Nonetheless, the fact that $\rho_{\rm est}$ and $\rho_{\rm NC}^{(\nu;h)}$ are nearly identical for $m_t = 36$ GeV, $m_\phi = m_Z$, and that $-\tan^2\theta_W\delta$ and $\epsilon_{\rm NC}^{(\nu;h)}$ have the same leading asymptotic behaviors for large fermion mass splittings (and nearly the same leading behavior for large $m_\phi{}^2/m_Z{}^2$) leads through Eq. (19c) to the theoretical expectation that the ratio $\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)}$ deviates from 1 only by a small amount over a large range of mass values. This is illustrated in Table III which lists calculated values for $\rho_{\rm est}/\rho$, $\rho_{\rm NC}^{(\nu;h)}/\rho$, and $\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)}$ as a function of m_t, for $m_\phi = m_Z$. The table employs the exact formulas for Δr and $(\Delta r)_{\rm est}$ rather than the asymptotic ones, the first equality in Eq. (19b) and the expression

$$\rho_{\rm NC}^{(\nu;h)}/\rho = 1 + \epsilon_{\rm NC}^{(\nu;h)}/(1-\Delta r - \epsilon_{\rm NC}^{(\nu;h)})$$

which incorporates leading $O(\alpha^2)$ effects and is therefore more accurate than Eq. (17). We see that in the range $m_t \le 240$ GeV, $\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)}$ is expected on theoretical grounds to deviate from unity by less than 0.5%, $\rho_{\rm est}$ being slightly larger than $\rho_{\rm NC}^{(\nu;h)}$ for large values of m_t.

We give two examples of the constraints implied by these considerations.

(1) Allowing a decrease by 2 standard deviations from the central value in Eq. (18a) would lead to

TABLE III. Dependence of $\rho_{\rm est}/\rho$, $\rho_{\rm NC}^{(\nu;h)}/\rho$, and $\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)}$ on m_t. The last column illustrates the fact that $\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)}$ remains close to 1 over a large range of m_t values.

m_t (GeV)	$(\rho_{\rm est}/\rho)-1$	$(\rho_{\rm NC}^{(\nu;h)}/\rho)-1$	$(\rho_{\rm est}/\rho_{\rm NC}^{(\nu;h)})-1$
20	-8.0×10^{-5}	2.8×10^{-4}	-3.6×10^{-4}
36	0	5.6×10^{-4}	-5.6×10^{-4}
60	-5.4×10^{-4}	1.3×10^{-3}	-1.8×10^{-3}
83	2.2×10^{-3}	2.3×10^{-3}	-1.1×10^{-4}
100	4.1×10^{-3}	3.2×10^{-3}	8.8×10^{-4}
150	9.5×10^{-3}	7.0×10^{-3}	2.5×10^{-3}
200	1.6×10^{-2}	1.2×10^{-2}	3.8×10^{-3}
240	2.2×10^{-2}	1.7×10^{-2}	5.0×10^{-3}

$\rho_{\rm est}\approx\rho_{\rm NC}^{(\nu;h)}\approx0.98$; for given m_W, this corresponds via Eq. (19a) to a value of m_Z-m_W larger by about 0.94 GeV than the standard-model prediction.

(2) The hypothetical mass values $m_W=81$ GeV, $m_Z=95$ GeV considered before correspond to $\rho_{\rm est}=0.94$ [Eq. (19a)], which is incompatible with Eq. (18a) and the theoretical closeness of $\rho_{\rm NC}^{(\nu;h)}$ and $\rho_{\rm est}$. Thus, by invoking neutral-current phenomenology one can rule out, in the context of the more general $SU(2)_L\times U(1)$ theories described by Eq. (15), such possibilities as (i) values of m_Z-m_W, for given m_W, more than about 1 GeV above the standard-model predictions, and (ii) pairs of mass values such as $m_W=81$ GeV, $m_Z=95$ GeV. To reconcile such hypothetical findings would probably require modifications in neutral-current amplitudes such as the introduction of additional Z^0's. This in turn means a generalization of the $SU(2)_L\times U(1)$ gauge group. In Fig. 2 we illustrate the allowed region in the m_W, m_Z plane, obtained by neglecting the difference between $\rho_{\rm est}$ and $\rho_{\rm NC}^{(\nu;h)}$ and employing the constraint in Eq. (18a).

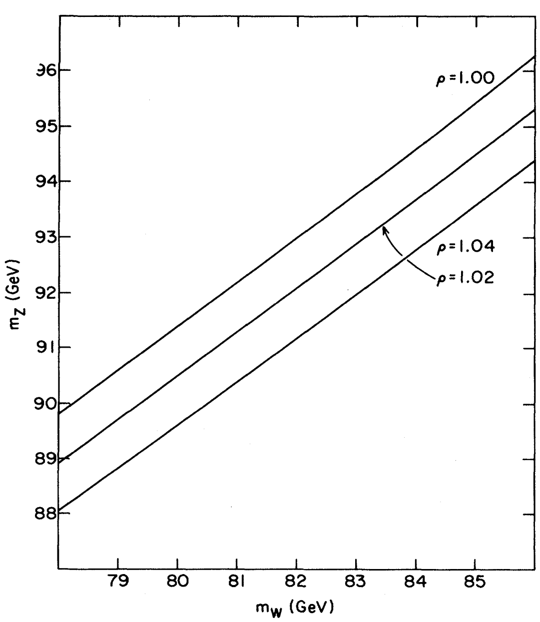

FIG. 2. Graph of m_Z vs m_W for $\rho=1.00$, 1.02, and 1.04. The constraint in Eq. (18a) allows only the region between these curves.

It is appropriate to conclude this paper by applying some of our formulas and general analysis to the $W^{\pm}$ and Z mass values recently obtained at CERN.[1-3] The UA1 Collaboration finds

$$m_W=80.9\pm1.5\pm2.4 \text{ GeV}\,,\qquad m_Z=95.6\pm1.5\pm2.9 \text{ GeV}\,, \tag{20}$$

where the second error represents a 3% systematic calibration uncertainty, while the UA2 Collaboration reports[3]

$$m_W=81.0\pm2.5\pm1.3 \text{ GeV}\,,\qquad m_Z=91.9\pm1.3\pm1.4 \text{ GeV}\,. \tag{21}$$

Within the rather large errors, both experiments agree with one another as well as with the standard model's prediction in Eq. (9).

Assuming 100% correlated systematic uncertainties in such a way that the calibration error is deemed to vanish in the ratio m_Z/m_W, the implied mass differences[26]

$$m_Z-m_W=14.7\pm2.1\pm0.4 \text{ GeV (UA1)}\,,\qquad m_Z-m_W=10.9\pm2.8\pm0.2 \text{ GeV (UA2)} \tag{22}$$

provide a nice test of the theory. UA2's central value is very close to the standard model's prediction [for $m_Z=91.9$ GeV, Eqs. (5d) and (7) predict $m_Z-m_W=11.2$ GeV] while the UA1 value is high (see, for example, Fig. 1).

To scrutinize these results further, we first determine $\sin^2\theta_W=1-m_W{}^2/m_Z{}^2$ and Δr as defined in Eq. (5e) using the above mass values. That procedure yields

$$\left.\begin{array}{l}\sin^2\theta_W=0.284\pm0.035\\ \Delta r=0.252\pm0.072\pm0.045\end{array}\right\}\text{UA1}\,, \tag{23}$$

$$\left.\begin{array}{l}\sin^2\theta_W=0.223\pm0.053\\ \Delta r=0.051\pm0.173\pm0.030\end{array}\right\}\text{UA2}\,. \tag{24}$$

The errors will have to be further reduced before the data tests the standard model at the level of its radiative corrections in Δr. (Remember, we found $\Delta r\simeq0.07$ from higher-order effects.) However, the UA1 value for Δr is noticeably on the high side.

A second procedure for comparing theory and experiment is to determine $\sin^2\theta_W$ from Eqs. (4a) and (7) using

TABLE IV. Comparison of the UA1 and UA2 results with theoretical expectations. Values given assume 100% correlation in the m_W and m_Z systematic uncertainties.

	UA1	UA2	Standard model with $\sin^2\theta_W = 0.217\pm0.014$
m_W (GeV)	$80.9\pm1.5\pm2.4$	$81.0\pm2.5\pm1.3$	$83.0^{+2.9}_{-2.7}$
m_Z (GeV)	$95.6\pm1.5\pm2.9$	$91.9\pm1.3\pm1.4$	$93.8^{+2.4}_{-2.2}$
$m_Z - m_W$ (GeV)	$14.7\pm2.1\pm0.4$	$10.9\pm2.8\pm0.2$	10.8 ± 0.5
$\sin^2\theta_W = 1 - m_W{}^2/m_Z{}^2$	0.284 ± 0.035	0.223 ± 0.053	0.217 ± 0.014
Δr	$0.252\pm0.072\pm0.045$	$0.051\pm0.173\pm0.030$	0.0696 ± 0.0020
$\sin^2\theta_W = \left[\dfrac{38.65\text{ GeV}}{m_W}\right]^2$	$0.228\pm0.008\pm0.014$	$0.228\pm0.014\pm0.007$	0.217 ± 0.014
ρ	$0.928\pm0.038\pm0.016$	$1.006\pm0.052\pm0.010$	1

m_W alone as input and then calculate ρ via Eq. (19a). In that way we find

$$\left.\begin{aligned} \sin^2\theta_W &= 0.228\pm0.008\pm0.014 \\ \rho &= 0.928\pm0.038\pm0.016 \end{aligned}\right\}\text{UA1}\,, \tag{25}$$

$$\left.\begin{aligned} \sin^2\theta_W &= 0.228\pm0.014\pm0.007 \\ \rho &= 1.006\pm0.052\pm0.010 \end{aligned}\right\}\text{UA2}\,. \tag{26}$$

Note that ρ is the ratio of the values of $\cos^2\theta_W$ determined by these two distinct methods and its deviation from 1 in the UA1 results reflects the difference in the $\sin^2\theta_W$ values of Eqs. (23) and (25). The ρ parameter provides a particularly good test of the standard model at the tree level. Also, given the limited Z statistics (four events for each group), using m_W alone presumably yields a more reliable determination of $\sin^2\theta_W$.

The results of our above comparison are summarized in Table IV. Agreement between theory and experiment is quite good. In particular, UA2's central values are in impressive agreement with theory. On the other hand UA1's values for $m_Z - m_W$, ρ, and Δr deviate somewhat (at the 1σ level) from expectations. One should keep an eye on their $m_Z - m_W$ mass difference which is presumably not very sensitive to calibration uncertainties. The standard model cannot accommodate a 14-GeV mass difference in any sensible way.

It will be very interesting to watch the experimental uncertainties diminish and the confrontation between theory and experiment grow more exciting. Surprises in the properties of the $W^\pm$ and Z may yet await us.

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$$(\Delta F)_{\rm cal} = \left.\frac{\partial F}{\partial m_W}\right|_{m_Z/m_W = \rm const} (\Delta m_W)_{\rm cal} \, .$$