Light-meson spectroscopy

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We study the mass spectrum and radiative decays of the light pseudoscalar and vector mesons in a nonrelativistic quark model. The quark content of the ground state and first two radially excited states is discussed as well as possible mixing with a glueball state in the isoscalar sector. The radiative decays of the first radially excited states are predicted and would appear to give tests for the existence of gluonium.

I. INTRODUCTION

The nonrelativistic quark model has developed over the years as a reliable dynamical scheme for describing hadrons. Starting out with guesses for interquark forces it has developed quantitatively with the formulation of QCD, the theory of strong interactions. While QCD is a strongly interacting field theory at large distances, at short distances it becomes asymptotically free and perturbative techniques can be applied.

In this paper we propose to investigate the spectra and radiative decays of the low-lying (non-charm-carrying) mesons based on the quark model. We view the radiative decays as a check on the reasonableness of the eigenvectors obtained in the analysis of the pseudoscalar and vector meson spectrum.

Our paper is divided as follows: In Sec. II we define our model and then analyze the isovector and isospinor pseudoscalar and vector meson spectrum, i.e., the π, ρ, K, and K^* systems in terms of a ground state and the first two radially excited states. With masses and expectation value of the potential fixed we investigate the isoscalar sector in Sec. III. We find mixing close to conventional values. In Sec. IV we then study possible mixing of a glueball state at around 1.44 GeV with the pseudoscalars. In Sec. V we compute the known radiative decays of ground-state mesons as well as predict the radiative decays of the first radially excited (2S) states. Our conclusions are drawn in Sec. VI.

II. ISOVECTOR AND ISOSPINOR SECTORS

We investigate the spectroscopy of light mesons—bound states of u, d, and s quarks only—in the ground state and first two radially excited states.[1] We restrict ourselves to S waves only. Our model is a nonrelativistic quark model with a harmonic-oscillator potential. The bare (unperturbed) states are therefore harmonic-oscillator basis states and the perturbation that mixes them is the hyperfine (Fermi-Breit) interaction which we take to be

$$V_S(r)=\text{const}\times\delta^3(r)\vec{S}_1\cdot\vec{S}_2\ ,$$

where S_i refers to quark spin. The constituent-quark Hamiltonian therefore has the form

$$H=H_0+V_C+V_S\ ,$$

where H_0 contains quark kinetic-energy and mass terms, V_C is the confinement potential (which we approximate by a harmonic-oscillator potential), and V_S the spin-spin splitting. Since we are only dealing with S waves possible spin-orbit and tensor potentials will not enter into the calculation. The spectra consist of the masses of the following particles:

	$S=0$ (pseudoscalars)			$S=1$ (vectors)		
$I=1$ (isovectors)	π_1	π_2	π_3	ρ_1	ρ_2	ρ_3
$I=\frac{1}{2}$ (isospinors)	K_1	K_2	K_3	K_1^*	K_2^*	K_3^*

Our notation here means that, for example, π_1 is the ground-state π meson with π_2 and π_3 being the $2\,^1S_0$ and $3\,^1S_0$ radially excited mesons. Within each sector the mass difference between the pseudoscalar and the vector mesons arises from the splitting generated by the term $V_S(r)$ in the Hamiltonian. We attempt to fit the masses in these sectors at the same time as the experimentally determined radiative widths with parameters M=mass of the u,d quark, M_S=mass of the strange quark, ω=the harmonic-oscillator strength, A=hyperfine-splitting parameter for the nonstrange sector (π,ρ). A_S, the hyperfine splitting for the strange sector (K,K^*) is related to A by $A_S=(M/M_S)A$. As we discuss later, use of the radiative widths imposes restrictions on the eigenvectors; without such data being included many possible "solutions" can be obtained.

Our model consists of diagonalizing the 3×3 matrix in each sector. For example, the form of the mass matrix in $I=1, S=1$ (ρ_1,ρ_2,ρ_3) is

TABLE I. Predictions of masses in GeV in parentheses with experimental values from Ref. 12.

$I=1$ sector			
$\pi_1(0.135)$	0.140	$\rho_1(0.766)$	0.770
$\pi_2(1.10)$	1.3	$\rho_2(1.54)$	1.6
$\pi_3(1.92)$	1.6?	$\rho_3(2.36)$	?
$I=\frac{1}{2}$ sector			
$K_1(0.513)$	0.497	$K_1^*(0.902)$	0.89
$K_2(1.27)$	1.4?	$K_2^*(1.6)$	1.4
$K_3(2.0)$	2.0?	$K_3^*(2.3)$	?
$I=0$ sector		Without gluonium	
Pseudoscalars			
$\eta_1(0.530)$	0.549	$\eta_1'(1.07)$	0.959
$\eta_2(1.39)$	1.275?	$\eta_2'(2.50)$	?
$\eta_3(2.1)$	?	$\eta_3'(3.2)$	?
Vectors			
$\omega_1(0.768)$	0.783	$\phi_1(1.05)$	1.02
$\omega_2(1.56)$	1.67	$\phi_2(1.69)$	1.68
$\omega_3(2.42)$	?	$\phi_3(2.33)$	?
$I=0$ sector		With gluonium	
Pseudoscalars			
$\eta_1(0.570)$	0.549	$\eta_1'(1.05)$	0.959
$\eta_2(1.39)$	1.275?	$\eta_2'(2.1)$	?
$\eta_3(2.8)$	?	$\eta_3'(3.6)$	?
$G(1.50)$	1.44		

$$\begin{bmatrix} 2M+A & -3\left[\frac{3}{2}\right]^{1/2}AZ_VA & 3\left[\frac{15}{8}\right]^{1/2}AZ_VA \\ -3\left[\frac{3}{2}\right]^{1/2}AZ_VA & 2M+2\omega-1.5A & -3\left[\frac{45}{16}\right]^{1/2}AZ_VA \\ 3\left[\frac{15}{8}\right]^{1/2}AZ_VA & -3\left[\frac{45}{16}\right]^{1/2}AZ_VA & 2M+4\omega+1.875A \end{bmatrix},$$

where $M_{ij}=\langle \rho_i \mid H \mid (q_1q_2)_j\rangle$ and

$$\tfrac{3}{4}AZ_V=(\vec{S}_1\cdot\vec{S}_2)_{S=1}=\tfrac{1}{4}$$

is the relative hyperfine splitting within the vector system.

The form of the mass matrix in $I=\frac{1}{2}$, $S=0$ sector (K_1,K_2,K_3) is

$$\begin{bmatrix} M+M_s-3A_s & -3\left[\frac{3}{2}\right]^{1/2}AZ_PA_s & 3\left[\frac{15}{8}\right]^{1/2}AZ_PA_s \\ -3\left[\frac{3}{2}\right]^{1/2}AZ_PA_s & M+M_s+2\omega-4.5A_s & -3\left[\frac{45}{16}\right]^{1/2}AZ_PA_s \\ 3\left[\frac{15}{8}\right]^{1/2}AZ_PA_s & -3\left[\frac{45}{16}\right]^{1/2}AZ_PA_s & M+M_s+4\omega-5.625A_s \end{bmatrix},$$

where again $M_{ij}=\langle K_i \mid H \mid (q_1q_2)_j\rangle$ and

$$\tfrac{3}{4}AZ_P=(\vec{S}_1\cdot\vec{S}_2)_{S=0}=-\tfrac{3}{4}$$

is the relative hyperfine splitting within the pseudoscalar system.

The results obtained are given in Table I. We fit the masses of the ground states and first radially excited states. For this solution the values of the parameters are found to be

$$M=0.350\text{ GeV},\quad M_s=0.503\text{ GeV},$$
$$\omega=0.365\text{ GeV},\quad A=0.0865\text{ GeV},$$
$$A_s=(M/M_s)A=0.0619\text{ GeV}.$$

Note the similarity of the quark masses with the results of baryon spectroscopy. Using this procedure we obtain the physical states as a mixture of harmonic-oscillator basis states and also the mixing between $n=1,2,3$ states.

The result is

$$|\pi_1\rangle=0.836\,|N\rangle_0-0.446\,|N\rangle_1+0.319\,|N\rangle_2\,,$$
$$|\pi_2\rangle=0.533\,|N\rangle_0+0.800\,|N\rangle_1-0.277\,|N\rangle_2\,,$$
$$|\pi_3\rangle=0.131\,|N\rangle_0+0.402\,|N\rangle_0+0.906\,|N\rangle_2\,,$$
$$|\rho_1\rangle=0.991\,|N\rangle_0+0.120\,|N\rangle_1-0.064\,|N\rangle_2\,,$$
$$|\rho_2\rangle=0.106\,|N\rangle_0-0.975\,|N\rangle_1-0.197\,|N\rangle_2\,,$$
$$|\rho_3\rangle=0.086\,|N\rangle_0-0.188\,|N\rangle_1+0.978\,|N\rangle_2\,,$$
$$|K_1\rangle=0.888\,|N\rangle_0-0.381\,|N\rangle_1+0.258\,|N\rangle_2\,,$$
$$|K_2\rangle=0.445\,|N\rangle_0+0.855\,|N\rangle_1-0.267\,|N\rangle_2\,,$$
$$|K_3\rangle=-0.119\,|N\rangle_0+0.353\,|N\rangle_1+0.928\,|N\rangle_2\,,$$
$$|K_1\rangle=0.994\,|N\rangle_0+0.099\,|N\rangle_1-0.053\,|N\rangle_2\,,$$
$$|K_2\rangle=0.09\,|N\rangle_0-0.983\,|N\rangle_1-0.157\,|N\rangle_2\,,$$
$$|K_3\rangle=0.068\,|N\rangle_0-0.152\,|N\rangle_1+0.986\,|N\rangle_2\,,$$

where $|N\rangle$ is

for π^+,ρ^+ , $-|u\bar{d}\rangle$,

for π^0,ρ^0 , $\frac{1}{\sqrt{2}}|u\bar{u}-d\bar{d}\rangle$,

for π^-,ρ^- , $|d\bar{u}\rangle$,

for K^+,K^{*+} , $|u\bar{s}\rangle$,

for K^0,K^{*0} , $|d\bar{s}\rangle$,

for K^{-0},K^{-*0} , $-|s\bar{d}\rangle$,

for K^-,K^{*-} , $|s\bar{u}\rangle$.

III. ISOSCALARS WITHOUT GLUONIUM

We have performed a similar calculation for the $I=0$ sector but everything is more complicated here due to the presence of octet-singlet mixing. The masses to be determined are the pseudoscalars

$$\eta_1,\eta_2,\eta_3\,,\quad \eta_1',\eta_2',\eta_3'$$

and the vectors

$$\omega_1,\omega_2,\omega_3\,,\quad \phi_1,\phi_2,\phi_3\,.$$

Since the isoscalar vector system is almost ideally mixed we expect it to be well fitted by the set of parameters used to fit the isovector and isospinor system. The only new parameter we introduce is the singlet-octet mixing Δ_V. However, as in most attempts dealing with the isoscalar-pseudoscalar system, we need to use a different parametrization. Instead of introducing a new parameter we fit the hyperfine-splitting parameter, again fixing $A_{ss}=(M/M_s)^2A$, the hyperfine splitting for the $s\bar{s}$ states.

The (6×6) matrix for the pseudoscalar case is given in Table II (by omitting the seventh row and column). The

TABLE II. Isoscalar mass matrix. $A\equiv A_\eta$ is the hyperfine-splitting parameter for systems containing only nonstrange quarks and antiquarks. A_{ss} is the hyperfine-splitting parameter for systems containing only strange quarks and antiquarks. Δ is the singlet-octet mixing (flavor mixing) between 1S states. Δ' is the singlet-octet mixing (flavor mixing) between 2S states. Δ'' is the singlet-octet mixing (flavor mixing) between 3S states. M is the nonstrange-quark mass fitted from $\pi\rho KK^*$ system $=0.350$ GeV. $M_s=0.503$ GeV from same fitting. $\omega=0.365$ GeV=harmonic-oscillator strength for two nonstrange q and $\bar{q}$. ω_{ss} is the harmonic-oscillator strength for two strange q and $\bar{q}$, $\omega_{ss}=\omega\sqrt{M/M_s}$. ϵ is the coupling of gluonium to quarkonium states. M_G is the glueball mass = 1.44 GeV.

$2M-3A+2\Delta$	$3.674\,22A$	$-4.107\,92A$	$\sqrt{2}\Delta$	0.0	0.0	$\sqrt{2}\epsilon$
$3.674\,22A$	$2M+2\omega-4.5A+2\Delta'$	$5.031\,15A$	0	$\sqrt{2}\Delta'$	0.0	$\sqrt{2}\epsilon$
$-4.107\,92A$	$5.031\,15A$	$2M+4\omega-5.625A+2\Delta''$	0	0	$\sqrt{2}\Delta''$	$\sqrt{2}\epsilon$
$\sqrt{2}\Delta$	0	0	$2M_s-3A_{ss}+\Delta$	$3.674\,22A_{ss}$	$-4.107\,92A_{ss}$	ϵ
0.0	$\sqrt{2}\Delta'$	0	$3.674\,22A_{ss}$	$2M_s+2\omega_{ss}-4.5A_{ss}+\Delta'$	$5.031\,15A_{ss}$	ϵ
0.0	0.0	$\sqrt{2}\Delta''$	$-4.107\,92A_{ss}$	$5.031\,15A_{ss}$	$2M_s+4\omega_{ss}-5.625A_{ss}+\Delta''$	ϵ
$\sqrt{2}\epsilon$	$\sqrt{2}\epsilon$	$\sqrt{2}\epsilon$	ϵ	ϵ	ϵ	M_G

only free parameters in it are as follows: A = hyperfine splitting for systems containing nonstrange quark and antiquark, Δ = the singlet-octet mixing between $1S$ states, and Δ' = the singlet-octet mixing between $2S$ states; Δ'' = the singlet-octet mixing between $3S$ states is set equal to Δ'.

The parameters corresponding to the solution in Table I are

$$A = 0.0539 \text{ GeV} ,$$

$$A_{ss} = (M/M_s)^2 A = 0.0261 \text{ GeV},$$

$$\Delta = 0.117 \text{ GeV} ,$$

$$\Delta' = 0.392 \text{ GeV} ,$$

$$\Delta'' = \Delta' .$$

The masses obtained are shown in Table I. As in the case of isovectors and isospinors the eigenvectors of the mixing matrix give the mixing between the physical states and the harmonic-oscillator basis states $(1/\sqrt{2})\,|u\bar{u}+d\bar{d}\rangle$ and $|s\bar{s}\rangle$ for the ground state and first two radially excited states. The mixings we obtain are close to the conventional values and are given below:

$$|\eta_1(0.530)\rangle = 0.863\,|N\rangle_0 - 0.211\,|N\rangle_1 + 0.245\,|N\rangle_2 - 0.349\,|S\rangle_0 + 0.122\,|S\rangle_1 - 0.118\,|S\rangle_2 ,$$

$$|\eta_1'(1.07)\rangle = 0.370\,|N\rangle_0 - 0.042\,|N\rangle_1 + 0.071\,|N\rangle_2 + 0.912\,|S\rangle_0 - 0.099\,|S\rangle_1 + 0.126\,|S\rangle_2 ,$$

$$|\eta_2(1.39)\rangle = -0.237\,|N\rangle_0 - 0.636\,|N\rangle_1 + 0.098\,|N\rangle_2 + 0.150\,|S\rangle_0 + 0.704\,|S\rangle_1 - 0.105\,|S\rangle_2 ,$$

$$|\eta_3(2.1)\rangle = -0.176\,|N\rangle_0 + 0.079\,|N\rangle_1 + 0.588\,|N\rangle_2 + 0.111\,|S\rangle_0 - 0.205\,|S\rangle_1 - 0.751\,|S\rangle_2 ,$$

$$|\eta_2'(2.45)\rangle = 0.123\,|N\rangle_0 + 0.708\,|N\rangle_1 - 0.116\,|N\rangle_2 + 0.090\,|S\rangle_0 + 0.647\,|S\rangle_1 - 0.209\,|S\rangle_2 ,$$

$$|\eta_3'(3.23)\rangle = -0.128\,|N\rangle_0 + 0.203\,|N\rangle_1 + 0.753\,|N\rangle_2 - 0.064\,|S\rangle_0 + 0.138\,|S\rangle_1 + 0.593\,|S\rangle_2 ,$$

where $|N\rangle = (1/\sqrt{2})\,|u\bar{u}+d\bar{d}\rangle$ and $|S\rangle = |s\bar{s}\rangle$. The analysis for the vector mesons in the isoscalar channel is shown in Table I for the same parameters as in the isovector-isospinor case and the flavor-mixing parameters

$$\Delta_V = \Delta_V' = \Delta_V'' = 0.015 \text{ GeV} .$$

As expected, the flavor mixing (octet-singlet) is a lot smaller for the vector case (again in agreement with conventional values), and the light-quark content for vector mesons is obtained as follows:

$$|\omega_1(0.768)\rangle = +0.982\,|N\rangle_0 + 0.101\,|N\rangle_1 - 0.144\,|N\rangle_2 - 0.075\,|S\rangle_0 - 0.006\,|S\rangle_1 + 0.007\,|S\rangle_2 ,$$

$$|\phi_1(1.05)\rangle = -0.074\,|N\rangle_0 - 0.009\,|N\rangle_1 + 0.012\,|N\rangle_2 - 0.991\,|S\rangle_0 - 0.070\,|S\rangle_1 + 0.089\,|S\rangle_2 ,$$

$$|\omega_2(1.56)\rangle = +0.070\,|N\rangle_0 - 0.963\,|N\rangle_1 - 0.200\,|N\rangle_2 - 0.009\,|S\rangle_0 + 0.167\,|S\rangle_1 + 0.023\,|S\rangle_2 ,$$

$$|\phi_2(1.69)\rangle = -0.011\,|N\rangle_0 + 0.164\,|N\rangle_1 + 0.036\,|N\rangle_2 - 0.058\,|S\rangle_0 + 0.978\,|S\rangle_1 + 0.119\,|S\rangle_2 ,$$

$$|\omega_3(2.33)\rangle = -0.038\,|N\rangle_0 + 0.046\,|N\rangle_1 - 0.222\,|N\rangle_2 + 0.094\,|S\rangle_0 - 0.112\,|S\rangle_1 + 0.962\,|S\rangle_2 ,$$

$$|\phi_3(2.42)\rangle = -0.157\,|N\rangle_0 + 0.184\,|N\rangle_1 - 0.943\,|N\rangle_2 - 0.023\,|S\rangle_0 + 0.029\,|S\rangle_1 - 0.227\,|S\rangle_2 .$$

IV. ISOSCALARS WITH GLUONIUM

From the previous analysis we see that we can accommodate all well-known states well from just studying mixing of radially excited states of quarkonium. However, the eigenvectors show that the physical states in the isoscalar region have a composition in terms of $q\bar{q}$ which is approximately in agreement with traditional ideas. As a consequence, the arguments which have been given against the ι being a radial excitation probably hold. Consequently, we include the mixing with a possible gluonium candidate in a separate calculation.

The discovery of the $\iota(1440)$ (Refs. 2 and 3) has raised the question whether it is a glueball or a radially excited $q\bar{q}$ state. There have been many arguments in favor of the interpretation of $\iota(1440)$ as a gluonium state and about an equal number dismissing this interpretation.[5,4] Early calculations of meson masses predicted in 1978 (Ref. 6) and 1979 (Ref. 7) the existence of a radially excited state at 1.4 GeV (before the discovery of ι). The new upper limit[4] for $B(J\to\iota\gamma)B(\iota\to\eta\pi\pi)$ of 2×10^{-3} is in mild conflict with the assumed dominance $\iota\to KK\pi$. Furthermore many of the guiding principles for distinguishing the gluonium signal do not appear to be valid.

We incorporate the gluonium state as a seventh state in the Hamiltonian and include a new parameter ϵ describing the mixing of the glueball with the quarkonium candidates. (We assume ϵ is independent of the quarkonium states the gluonium mixes with in order to avoid introducing too many parameters.) We diagonalize the 7×7 mass matrix shown in Table II keeping M, M_s, and ω constant. The parameters corresponding to our solutions are

$$A = 0.0500 \text{ GeV} ,$$

$$A_{ss} = (M/M_s)^2 A = 0.0242 \text{ GeV} ,$$

$$\Delta = 0.138 \text{ GeV} ,$$

$$\Delta' = \Delta'' = 0.500 \text{ GeV} ,$$

$$\epsilon = 0.112 \text{ GeV} .$$

It is interesting to see that $\epsilon = 112$ MeV is close to what is obtained in a very different bag-model calculation,[9] where $\epsilon \sim 0.4\alpha_S{}^2/R$ is about 120 MeV for $\alpha_S = 1.1$.

The analysis gives the masses shown in Table I. Again we obtain the composition of the physical states in terms of $(1/\sqrt{2})\,|u\bar{u}+d\bar{d}\rangle$, $|s\bar{s}\rangle$, and gluonium states:

$$|\eta_1(0.570)\rangle = +0.856|N\rangle_0 - 0.175|N\rangle_1 + 0.220|N\rangle_2 - 0.380|S\rangle_0 + 0.129|S\rangle_1 - 0.117|S\rangle_2 - 0.117|G\rangle ,$$

$$|\eta_1'(1.05)\rangle = -0.327|N\rangle_0 - 0.035|N\rangle_1 - 0.062|N\rangle_2 - 0.828|S\rangle_0 + 0.068|S\rangle_1 - 0.119|S\rangle_2 + 0.429|G\rangle ,$$

$$|\eta_2(1.39)\rangle = -0.119|N\rangle_0 - 0.630|N\rangle_1 + 0.096|N\rangle_2 + 0.260|S\rangle_0 + 0.666|S\rangle_1 - 0.102|S\rangle_2 + 0.204|G\rangle ,$$

$$|\eta_3(2.10)\rangle = -0.165|N\rangle_0 + 0.097|N\rangle_1 + 0.582|N\rangle_2 + 0.103|S\rangle_0 - 0.173|S\rangle_1 - 0.764|S\rangle_2 - 0.018|G\rangle ,$$

$$|\eta_2'(2.82)\rangle = -0.118|N\rangle_0 - 0.717|N\rangle_1 + 0.167|N\rangle_2 - 0.076|S\rangle_0 - 0.621|S\rangle_1 + 0.195|S\rangle_2 - 0.118|G\rangle ,$$

$$|\eta_3'(3.57)\rangle = 0.100|N\rangle_0 - 0.213|N\rangle_1 - 0.756|N\rangle_2 + 0.046|S\rangle_0 - 0.146|S\rangle_1 - 0.583|S\rangle_2 - 0.100|G\rangle .$$

The gluonium state is

$$|G(1.50)\rangle = -0.308|N\rangle_0 - 0.049|N\rangle_1 + 0.019|N\rangle_2 - 0.290|S\rangle_0 + 0.312|S\rangle_1 - 0.015|S\rangle_2 - 0.849|G\rangle ,$$

where $|N\rangle = (1/\sqrt{2})\,|u\bar{u}+d\bar{d}\rangle$, $|S\rangle = |s\bar{s}\rangle$, $|G\rangle = |\text{gluonium}\rangle$.

This analysis shows the importance of the mixing with radially excited states. For example, the mixing of both $\eta_1(0.570)$ and $\eta_1'(1.05)$ with gluonium differs somewhat from simpler mixing models.[8,10] Also, as we intuitively expected, the $\iota(1440)$ mixes strongest with η_1' and η_2, the radially excited states closest in mass to its own mass.

V. RADIATIVE DECAYS

An important feature of our calculation is the inclusion of radiative decays. We have found that fits to the masses of the particles can be made relatively easily but the determination of a solution requires the inclusion of radiative decays which depend on the eigenvectors of the mass matrix. We have therefore fitted the known radiative decays of physical states at the same time as we fitted masses (and with no additional parameters).

The general form of the widths for the radiative decays will have a phase-space factor, kinematic factors, and the transition matrix element where the physics lies. We can express it for the decay $V\to P\gamma$, e.g., as[11]

$$\Gamma(V\to P\gamma) = k^3\phi\,|M_{VP\gamma}|^2 ,$$

where $k = (M_V{}^2 - M_P{}^2)/2M_V$, $k^3\phi$ is the phase-space factor, and $M_{VP\gamma}$ is the transition matrix element.

For decays of the form $P\to V\gamma$ we only interchange V and P in the formula and note that the phase-space factor will be multiplied by 3 since in $\Gamma(V\to P\gamma)$ we take an average over initial spin states. The transition matrix element may be written as

$$M_{VP\gamma} = \sum_{f=u,s} P_f \left[\sum_{m=1}^{3} \sum_{n=1}^{3} O_{mn} F_{Vm} F_{Pn} \right] ,$$

where P_f is the relative weight of the decay (depending on the quark content and charge of both pseudoscalar and vector mesons) and

$$O_{mn} = \langle \psi_m | e^{i\vec{K}\cdot\vec{r}/2} | \psi_n \rangle$$

is the $M1$ transition matrix element, with ψ_m being the harmonic-oscillator wave function for the mth radially excited state. Here $F_{Vm} = \langle m\,{}^3S_1 | V\rangle$ represents the composition of the vector physical state in terms of basis states. Similarly $F_{Pn} = \langle n\,{}^1S_0 | P\rangle$ represents the composition of the pseudoscalar physical state in terms of basis states. These represent the mixings given in the above equations. It is in these last two expressions that we include the eigenvectors of the mass matrix. For example, for the π system, $F_{\pi n}$ will be the first column in the eigenvector matrix. The relative weights P_f are given in Table III. Note that for decays which involve two particles from the isoscalar sector where flavor mixing occurs the relative weights are different for $V(s\bar{s})\to P(s\bar{s})\gamma$ compared to $V(u\bar{u})\to P(u\bar{u})\gamma$ and a sum over flavors in the transition-matrix expression is required. Table III also shows the predicted radiative decay widths for the ground states in comparison to the experimental values. The results shown are without gluonium; introducing gluonium shifts these values by too small an amount to be shown.

From our solution, we can predict the radiative decays for first excited states. There are as yet no experimental values against which these could be tested. (Note that we give predictions only for radiative decays of possible experimental candidates for the radially excited states.) We give the predictions of these widths with and without inclusion of gluonium. The most interesting feature of these predictions is the possibility of some significant shifts in the radiative decays if there is indeed a glueball state mixing with the pseudoscalar system. A measurement of these widths will be an indication of whether there is a glueball state at 1.44 GeV or whether the experiments have seen only a radial excitation. The results are given in Table IV.

VI. CONCLUSION

Based on a simple nonrelativistic quark model we have discussed the spectrum of the low-lying mesons including hyperfine splitting and octet-singlet mixing where applicable.

An overall fit such as ours has, of course, an error at-

TABLE III. Radiative decays of ground states. Widths in keV.

Decay	Relative weight P_f^2 $P_{(u\bar{u})}^2$	$P_{(s\bar{s})}^2$	Predicted values	Experimental values (Ref. 12)
$\rho\to\pi\gamma$	$\frac{1}{9}$	0	71	67 ±7
$\omega\to\pi\gamma$	1	0	762	789 ±92
$K^{*+}\to K^{+}\gamma$	$(2-M/M_s)^2/9$	0	76.3	62 ±14
$K^{*0}\to K^{0}\gamma$	$(1+M/M_s)^2/9$	0	129.1	75 ±35
$\rho\to\eta\gamma$	1	0	97.1	52.5±13.7
$\phi\to\pi\gamma$	1	0	11.0	6.5±1.9
$\eta'\to\rho\gamma$	$\frac{1}{9}$	0	37.4	93.1±25.1
$\phi\to\eta\gamma$	$\frac{1}{9}$	$(2M/M_s)^2/9$	30.1	67.7±9.5
$\omega\to\eta\gamma$	$\frac{1}{9}$	$(3M/M_s)^2/9$	10.9	3.2±2.6
$\eta'\to\omega\gamma$	$1/\sqrt{3}$	$(2M/M_s)^2/\sqrt{3}$	5.6	8.4±2.7

tached to it which is difficult to estimate other than by trial and error. Different attempts lead us to an estimate of 10—15%, which taken with the remarks of the Particle Data Group for most of the meson data makes our results seem reasonable. In an earlier paper on radial excitations[6] the lowest ($n=1$) and first excited ($n=2$) radial states were analyzed. Despite the nonrelativistic nature of the calculation many of the predicted excited states appear to have been seen experimentally[12] and with the correct properties. It is on this basis that we have extended the calculation to include the second $n=2$ radial excitations. We feel that examining three states will give a check on the validity of the model. Our $n=2$ model gave a simple account of the spectra even for sparse data. We expect that indeed in a simple model with δ-function interaction the mixings will tend to increase with increasing n. However, so long as the mixings remain small and within our estimated errors we do not consider convergence to be an important problem.

In this calculation we have fitted 15 masses (all the

TABLE IV. Predicted radiative decays for radially excited states.

Decay mode	Predicted width (keV) Without gluonium	With gluonium
$\rho_2\to\pi_1\gamma$	871	871
$\rho_2\to\pi_2\gamma$	6.67	6.67
$\rho_2\to\eta_1\gamma$	3718.5	3229.6
$\rho_2\to\eta_2\gamma$	127.7	134.7
$\rho_2\to\eta_1'\gamma$	98.6	50.4
$\pi_2\to\rho_1\gamma$	67.2	67.2
$\pi_2\to\omega_1\gamma$	318.2	318.2
$\pi_2\to\phi_1\gamma$	2.37	2.37
$\omega_2\to\pi_1\gamma$	8534.9	8534.9
$\omega_2\to\pi_2\gamma$	137.3	137.3
$\omega_2\to\eta_1\gamma$	402.7	342.7
$\omega_2\to\eta_2\gamma$	10.6	12.1
$\omega_2\to\eta_1'\gamma$	29.7	18.8
$\phi_2\to\pi_1\gamma$	27.2	271.7
$\phi_2\to\pi_2\gamma$	4.5	4.5
$\phi_1\to\eta_1'\gamma$	0.71	0.59
$\phi_2\to\eta_1'\gamma$	77.2	62.7
$\phi_2\to\eta_1\gamma$	70.9	72.4
$\phi_2\to\eta_2\gamma$	73.1	63.1
$\eta_2\to\rho_1\gamma$	121.3	269.3
$\eta_2\to\omega_1\gamma$	9.2	0.6
$\eta_2\to\phi_1\gamma$	5.4	12.9
$K_2^{*+}\to K_1^{+}\gamma$	387.4	387.4
$K_2^{+}\to K_1^{*+}\gamma$	855.1	855.1
$K_2^{*0}\to K_1^{0}\gamma$	655.1	655.1
$K_2^{0}\to K_1^{*}\gamma$	144.6	144.6

ground and the first radially excited states, except η_2 and η_2') and 10 radiative widths using 10 parameters (M, M_s, ω, A, A_η, Δ, Δ', Δ_V, ϵ, and M_G). The composition of the physical states which we obtain in terms of the unmixed states and our $q\bar{q}$ states are approximately what we would expect from traditional ideas. Our procedure results in definite mixings, which lie within Rosner's weaker bounds.[13]

We have investigated the isoscalar-meson spectroscopy with and without gluonium mixing. That is, we have investigated what happens if $\iota(1440)$ is a glueball or if it is a radial excitation. From our results we cannot interpret the existing states in the 1—2-GeV region as pure quarkonium or pure gluonium. Clearly the existence or nonexistence of a gluonium state at 1.44 GeV mainly affects the neighboring $\eta+\eta'$ mass spectrum. We include radiative decays in our model so the mass spectrum and decay widths are consistent and find a fair agreement with the experiment. The prediction of radiative decay widths shows that their measurements would be a good way of testing whether there is a glueball at 1.44 GeV. Unfortunately, for all particles in question these partial decay widths are small compared to the total widths.

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