PHYSICAL REVIEW D VOLUME 29, NUMBER 5 1 MARCH 1984

Constituent-quark and baryon loops in pseudoscalar-meson transitions

Ll. Ametller, C. Ayala, and A. Bramon
Departament de Física Teórica, Universitat Autónoma de Barcelona, Bellaterra, Barcelona, Spain
(Received 15 September 1983)

Predictions for the $\pi\gamma\gamma$ coupling, the pion charge radius $\langle r_{\pi^+}{}^2\rangle$, the q^2 dependence of the pion decay constant near $q^2=0$ $[f'_\pi(0)]$, and the ratio γ describing radiative pion decay are obtained in the context of two related models: the constituent-quark model and a baryon-loop model with $n_f=2$, 3, or 4 flavors. All these models lead to two relations among the four above-mentioned parameters, which are satisfied by the data. A sort of equivalence or duality for the two models is observed, particularly for $n_f=4$.

I. INTRODUCTION

The $\pi^0\to\gamma\gamma$ transition has played a relevant role in our present understanding of hadron dynamics. It represents one of the cleanest tests of the existence of three colors, $n_c=3$, for each current quark appearing in the QCD (quantum chromodynamics) Lagrangian. Indeed, in this theory the coupling constant (see below for notation) is given by $g_{\pi\gamma\gamma}=-n_c e^2/12\pi^2 f_\pi$, which is very close to the experimental result for $n_c=3$ and for the pion decay constant $f_\pi=93$ MeV. In the soft limit, the above result comes exclusively from the lowest-order (quark-triangle) diagram and is unaltered by radiative corrections.[1] At the phenomenological level of the constituent-quark model one can assume that the effects of the higher-order gluonic corrections consist of converting the low-mass current quarks into constituent (or dressed) quarks and that in this context the lowest-order calculation makes sense.[2] One obtains $g_{\pi\gamma\gamma}=-n_c e^2 g_{\pi qq}/12\pi^2 m_q$, which coincides with the previous successful result if one assumes the validity of the Goldberger-Treiman (GT) relation at the quark level, $f_\pi=m_q/g_{\pi qq}$, and a constituent quark mass $m_q\sim\frac{1}{3}M_N\gg m_\pi$. Even more phenomenologically, Steinberger[3] in 1949 obtained a similar result working with a single proton loop, i.e., with the simplest fermionic system coupled to pions and photons consisting of three dressed and bound quarks. He found

$$g_{\pi\gamma\gamma}=-e^2 g_{\pi NN}/4\pi^2 M_N=-e^2 g_A/4\pi^2 f_\pi\ ,$$

which coincides with the preceding expressions except for the factor $g_A=1.25$ introduced in the last step through the GT relation for nucleons. Finally, and completely in the opposite direction, the $\pi\gamma\gamma$ anomaly could also be relevant in the context of composite models for quarks and leptons.[4]

Leaving aside this last and more speculative question, the $\pi\to\gamma\gamma$ transition suggests the possibility that besides the genuinely QCD analysis (the QCD phase,[5] involving current quarks and gluons) two other phenomenological contexts could lead at least in some cases to equivalent and satisfactory results. The purpose of this paper is to discuss several physically relevant processes which are reasonably well described by both a simple loop of constituent quarks (the quark-model phase, involving quarks dressed by gluon clouds) and a baryon loop [the hadronic or PCAC (partially conserved axial-vector current) phase, involving bound systems of quarks[5]]. As follows from the preceding paragraph and due to the constituent-quark and baryon mass relation $m_q\simeq\frac{1}{3}M_N$, the equivalence between the two alternative descriptions is possible only if the mass parameters appear divided by the coupling constant, $m_q/g_{\pi qq}$ or $M_N/g_{\pi NN}$, and the GT relation introducing the PCAC constant f_π or $g_A f_\pi$ can be used. For this reason we restrict ourselves to the analysis of the processes which share this feature. (Therefore we do not discuss here the form factors of pseudoscalar mesons P for $P\to 1^+1^-\gamma$ and $P\to 1^+1^-$ decays, which are sensitive to the internal mass circulating in the loop; this mass cannot be eliminated by using the GT relation.)[6] In addition to the $\pi\to\gamma\gamma$ decay (which we briefly review in Sec. II), there are three other relevant cases for which essentially equivalent descriptions are obtained: the pseudoscalar-meson charge radii $\langle r_P{}^2\rangle$ (Sec. III), the difference $f_\pi(m_\pi{}^2)-f_\pi(0)$ where f_π is the PCAC constant (Sec. IV), and the ratio γ of the axial-vector to the vector form factor in the structure-dependent part of the $\pi\to e\nu\gamma$ decay (Sec. V). Our conclusions will be summarized in Sec. VI.

The external particles of the above processes are pseudoscalar mesons ϕ (in most cases pions), real or virtual photons A_μ, and virtual W bosons (or, equivalently, lepton pairs $l\nu$). The dynamics of these processes will be described with lowest-order loops of quarks or baryons. The basic interaction Lagrangians and coupling constants are defined as[7]

$$\mathscr{L}_{\rm strong}=-ig_P\bar\psi\gamma_5\psi\phi,\quad \mathscr{L}_{\rm EM}=-e\bar\psi\gamma_\mu\psi A^\mu\ ,$$
$$\mathscr{L}_{\rm weak}=\frac{G}{\sqrt{2}}[\bar\psi\gamma_\mu(1-g_A\gamma_5)\psi][\bar l\gamma^\mu(1-\gamma_5)\nu]\ , \tag{1}$$

where the internal-symmetry indices and the Cabibbo angle have been ignored, $g_A=1$ ($g_A=1.25$) for quarks (baryons), and g_P, e, and G are usual Yukawa and electroweak coupling constants. Notice that quarks and baryons are treated as elementary objects with the simplest interactions (e.g., without anomalous magnetic moments).

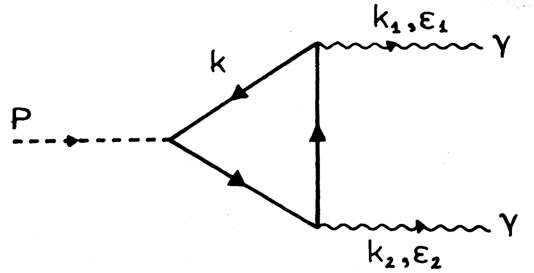

FIG. 1. Diagram for $P \to \gamma\gamma$. Continuous lines are always quark (baryon) lines. A diagram with the two photons crossed should also be included.

The only exception is g_A for baryons, for which we adopt the physical value $g_A = 1.25$ as one usually does in this type of model.[2,8]

II. THE $\pi\gamma\gamma$ VERTEX

For completeness and also for further reference we will briefly consider the well-known $\pi^0 \to \gamma\gamma$ transition. Defining the corresponding amplitude as

$$A(\pi^0 \to \gamma\gamma) = -i g_{\pi\gamma\gamma} \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma \epsilon_1^\mu \epsilon_2^\nu , \tag{2}$$

where $\epsilon_{1,2}$ and $k_{1,2}$ are the polarizations and four-momenta of the photons, the $\pi^0 \to \gamma\gamma$ decay width is given by $\Gamma(\pi^0 \to \gamma\gamma) = (g_{\pi\gamma\gamma})^2 m_\pi{}^3/64\pi$. In our framework the dynamics of the $\pi^0 \to \gamma\gamma$ vertex, i.e., the $g_{\pi\gamma\gamma}$ coupling constant, is simply given by the fermion loop of Fig. 1 and its crossed version. For quarks $q = u,d$ of mass m_q and charge eQ_q one has

$$A(\pi^0 \to \gamma\gamma) = 2n_c \epsilon_1^\mu \epsilon_2^\nu \sum_q g_{\pi qq}(eQ_q)^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma_5 \frac{i}{\not k - m_q} \gamma_\mu \frac{i}{\not k + \not k_1 - m_q} \gamma_\nu \frac{i}{\not k + \not k_1 + \not k_2 - m_q}\right] \tag{3}$$

and, through Eq. (2) and the GT relation $f_\pi = m_q/g_q$,

$$g_{\pi\gamma\gamma} = -n_c \frac{e^2}{4\pi^2}(Q_u{}^2 - Q_d{}^2)\frac{g_q}{m_q} \equiv -\frac{e^2}{4\pi^2}\frac{g_q}{m_q} = -\frac{e^2}{4\pi^2}\frac{1}{f_\pi} , \tag{4}$$

where $g_q = g_{\pi uu} = -g_{\pi dd}$ and $m_q \simeq m_u \simeq m_d$. Taking $f_\pi = 93$ MeV, Eq. (4) leads to $\Gamma(\pi \to \gamma\gamma) = 7.64$ eV in good agreement with the experimental value[9] $\Gamma(\pi \to \gamma\gamma) = 7.95 \pm 0.55$ eV.

For the nucleon-loop model[3] one similarly obtains

$$g_{\pi\gamma\gamma} = -\frac{e^2}{4\pi^2}\frac{g_{\pi NN}}{M_N} = -\frac{e^2}{4\pi^2}\frac{g_A}{f_\pi} , \tag{5}$$

where the similarity (except for the additional factor $g_A = 1.25$) with Eq. (4) is a consequence of the cancellation of the color factor $n_c = 3$ with $Q_u{}^2 - Q_d{}^2 = \frac{1}{3}$ in Eq. (4). From Eq. (5), which only contains the proton-loop contribution, one predicts[3] a $g_{\pi\gamma\gamma}$ coupling constant which is only some 20% larger than the experimental one. The situation is not improved when taking the contributions of strange baryons into account, i.e., when including the whole SU(3) baryon octet. One finds[10]

$$g_{\pi\gamma\gamma} = -\frac{e^2}{4\pi^2}\frac{g_A}{f_\pi} F_3(\alpha) \equiv -\frac{e^2}{4\pi^2}\frac{g_A}{f_\pi}\left|1 + (2\alpha - 1)\frac{M_N}{M_\Xi}\right| , \tag{6}$$

which for reasonable values of $\alpha \equiv d/(d+f)$, α around $\frac{2}{3}$, tends to increase slightly the previous, small discrepancy. The cancellation between the Σ^+ and Σ^- contributions in Eq. (6) at the SU(3) level is no longer true at the SU(4) level for the Σ_c isotriplet. These particles, together with the Λ_c^+ isosinglet and the Ξ_c isodoublet, should give additional contributions—with a so-far-unknown sign—to $g_{\pi\gamma\gamma}$ [other, double-charmed baryons belonging to the 20 representations of SU(4), as well as baryons containing quarks of heavier flavors, are expected to give negligible contributions to $g_{\pi\gamma\gamma}$ due to their large mass]. As a result, the predictions for $g_{\pi\gamma\gamma}$ in a baryon-loop model are not very different from those coming from the quark model and the experimental data. In particular, it seems quite possible that at the level of SU(4) one could recover the successful quark-model prediction. Unfortunately, the present knowledge of the coupling of charmed baryons to the pion does not allow confirmation of this attractive possibility.

III. PSEUDOSCALAR-MESON CHARGE RADII

In our quark-model framework, the electromagnetic (EM) form factor of a pseudoscalar meson P is given by the triangle diagram of Fig. 2 (and its crossed one) to which we refer for kinematics and notation. For a given P, containing a quark l and an antiquark j with charges $eQ_{l,j}$ ($e > 0$) and masses $m_{l,j}$, one has[11]

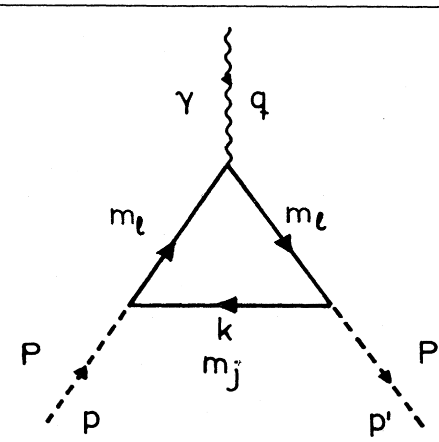

FIG. 2. Diagram contributing to the pion (kaon) electromagnetic form factor. The crossed diagram should be included.

$$-iF_P(q^2)(p+p')^\mu = 2n_c g^2 i \int \frac{d^2k}{(2\pi)^4} \mathrm{Tr}\left[Q_l \gamma^\mu \frac{i}{\not p + \not k - m_l} \gamma_5 \frac{i}{\not k - m_j} \gamma_5 \frac{i}{\not p' + \not k - m_l} + \begin{bmatrix} l \leftrightarrow j \\ p \leftrightarrow -p' \end{bmatrix} \right], \tag{7}$$

where $q = p' - p$. In the soft limit and defining the EM charge radius as

$$\langle r_P{}^2 \rangle \equiv 6 dF_P(q^2)/dq^2 \,|_{q^2=0}$$

one immediately obtains

$$\langle r_{\pi^+}{}^2 \rangle = \frac{n_c g^2}{4\pi^2 m_q{}^2} = 0.34 \text{ fm}^2 ,$$
$$\langle r_{K^+}{}^2 \rangle = \frac{n_c g^2}{4\pi^2 m_q{}^2} \left[1 - \frac{5}{6} \frac{\delta m}{m_q} + \cdots \right] = 0.25 \text{ fm}^2 , \tag{8}$$

where $m_q = m_u \simeq m_d$, $\delta m = m_s - m_q \simeq m_q/3$, $g = g_{\pi u \bar{u}}$, and the dots refer to (negligible) terms of order $(\delta m/m_q)^2$, or higher. These results were first obtained by Gerasimov[12] and by Tarrach,[13] and are discussed in Ref. 11. They compare rather favorably with the experimental data[14]

$$\langle r_{\pi^+}{}^2 \rangle_{\text{expt}} = 0.46 \pm 0.01,\ 0.439 \pm 0.030 \text{ fm}^2 ,$$
$$\langle r_{K^+}{}^2 \rangle_{\text{expt}} = 0.28 \pm 0.05 \text{ fm}^2 . \tag{9}$$

When turning to the alternative baryon-loop approach one has simply to reinterpret our main Eq. (7). In a two-flavored world, one would have

$$\langle r_{\pi^+}{}^2 \rangle = \frac{(g_{\pi NN})^2}{4\pi^2} \frac{1}{M_N{}^2} = \frac{g_A{}^2}{4\pi^2} \frac{1}{f_\pi{}^2} \simeq 0.20 \text{ fm}^2 , \tag{10}$$

while the inclusion of the third, strange flavor allows us to write

$$\langle r_{\pi^+}{}^2 \rangle = \frac{g_A{}^2}{4\pi^2} \frac{1}{f_\pi{}^2} G_3(\alpha) , \tag{11}$$

$$G_3(\alpha) \equiv 1 + 4(1-\alpha)^2 \frac{M_N{}^2}{M_\Sigma{}^2} + \tfrac{4}{3}\alpha^2 \frac{M_N{}^2}{M_{\Sigma\Lambda}{}^2} + (2\alpha - 1)^2 \frac{M_N{}^2}{M_\Xi{}^2} , \tag{12}$$

and similar expressions for $\langle r_{K^+}{}^2 \rangle$. The values predicted for $\langle r_{\pi^+}{}^2 \rangle$ and $\langle r_{K^+}{}^2 \rangle$ at the SU(3) level, and for reasonable values[15] of $\alpha \equiv d/(d+f)$, $0.5 < \alpha < 0.8$, have been plotted in Fig. 3. One observes a striking stability $\langle r_{\pi^+}{}^2 \rangle \sim 0.34$ fm^2 and $\langle r_{K^+}{}^2 \rangle \sim 0.28$ fm^2 very close to the quark-model predictions, Eqs. (8). The inclusion of the charmed (and heavier) quark(s) has the effect of increasing moderately these predictions. Indeed, any additional baryonic contribution to Eq. (12) brings a positive sign which comes from the appearance of the square of the coupling constant to the pseudoscalar meson and the fact that only positive baryons couple to the photon in the diagram of Fig. 2, while the negative ones couple in the crossed diagram whose sign must be reversed. (Note that this is not the case for the K^0 charge radius. For this reason we do not consider $\langle r_{K^0}{}^2 \rangle$ in this paper.) Therefore, our results turn out to be somewhat larger than those predicted by the quark model, Eq. (8), and quite close to the experimental values, Eq. (9).

IV. q^2 DEPENDENCE OF f_π

Our alternative approaches can be similarly used to calculate the slight q^2 variation from $q^2 = m_\pi{}^2$ (where f_π takes its physical, on-shell value) to $q^2 = 0$ (where f_π can be deduced from the GT relation). This variation is linked to the well-known discrepancy of the GT relation,[2]

$$\Delta_{\text{expt}} = 1 - (f_\pi g_{\pi NN}/M_N g_A) = 0.06 \pm 0.01 ,$$

and represents a substantial fraction of this 6%. Indeed, the traditional attempts to explain this discrepancy are based on the variation in q^2 of the πNN form factor and predict[2,16] $\Delta \simeq 0.01 - 0.03$. Therefore, there seems to be room for a second effect, associated with the q^2 dependence of f_π, in such a way that

$$1 - f_\pi(0)/f_\pi(m_\pi{}^2) \simeq 0.04 . \tag{13}$$

In our context, this effect has to be attributed to the q^2 dependence of the diagram in Fig. 4.

At the constituent-quark level a simple evaluation leads immediately to[2]

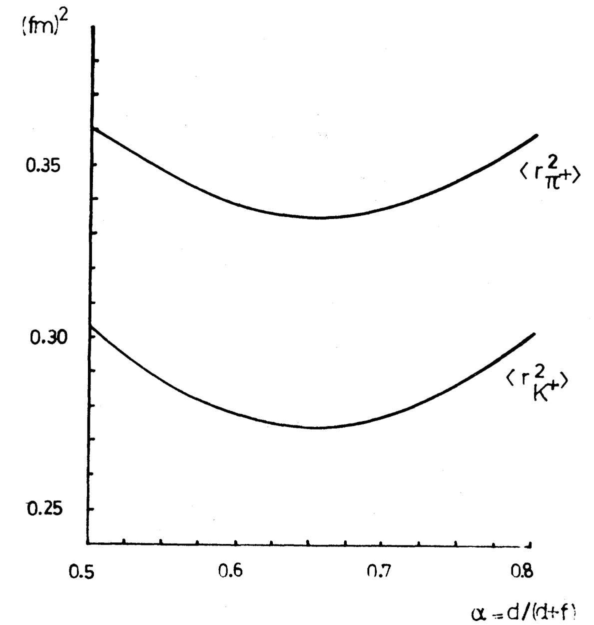

FIG. 3. Predictions for $\langle r_{\pi^+}{}^2 \rangle$ and $\langle r_{K^+}{}^2 \rangle$ as a function of $\alpha = d/(d+f)$ in an SU(3) baryon-loop model.

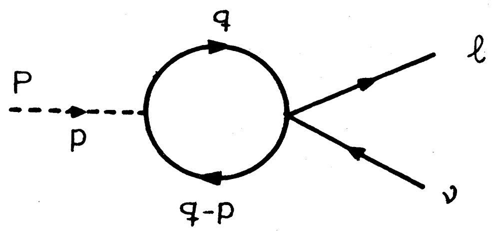

FIG. 4. Diagram contributing to $\pi \rightarrow l\nu$.

$$1-f_\pi(0)/f_\pi(m_\pi{}^2)=\frac{n_c m_\pi{}^2}{24\pi^2 f_\pi{}^2}\simeq 0.029 \quad (14)$$

for $m_q \gg m_\pi$, $n_c=3$, and the usual values of m_π and f_π. This result compares rather well with the value quoted in Eq. (13) and represents a success for the three-colored-quark model. A similar evaluation and result can be obtained for the variation of the K decay constant f_K. But the large K mass and the uncertainties associated with the GT relation for kaons do not allow for a meaningful comparison with the data.

Turning now to the nucleon-loop or SU(2) model, one readily obtains

$$1-f_\pi(0)/f_\pi(m_\pi{}^2)=\frac{g_A{}^2 m_\pi{}^2}{24\pi^2 f_\pi{}^2}\simeq 0.014 \ , \quad (15)$$

which is only a fraction of the required value, Eq. (13). The effects of strange baryons are exactly the same as in the case of $\langle r_{\pi^+}{}^2\rangle$ i.e., one has to multiply Eq. (15) by $G_3(\alpha)$ given in Eq. (12). As a result, our prediction has the same α dependence shown in Fig. 3 and, in particular, is stable around the physical value of $\alpha\simeq\frac{2}{3}$, with

$$1-f_\pi(0)/f_\pi(m_\pi{}^2)\simeq 0.024 \ . \quad (16)$$

One can easily convince oneself that this value is increased when adding a fourth (and other) flavor precisely by the same amount that increases $\langle r_{\pi^+}{}^2\rangle$, in spite of the extra coupling to the photon appearing only in the latter case. Therefore, in our real world with four relevant flavors the variation of f_π predicted by a simple baryon-loop model is again rather consistent with the numbers quoted in Eqs. (13) and (14).

Comparing the results of this section and Sec. III one easily deduces

$$1-f_\pi(0)/f_\pi(m_\pi{}^2)=\tfrac{1}{6}m_\pi{}^2\langle r_{\pi^+}{}^2\rangle \ , \quad (17)$$

which is a universal relation for all the cases discussed in this paper (the quark model and the baryon model with any number of flavors). It is very well verified by the data quoted in Eqs. (9) and (13).

V. γ RATIO IN RADIATIVE PION DECAY

The ratio $\gamma\equiv F_A(0)/F_V(0)$ between the axial-vector and vector form factors describing the $\pi\rightarrow e\nu\gamma$ decay at $t=(p_\pi-q_\gamma)^2=0$ has been measured by several groups[17] and analyzed in the context of different models.[15,17–19] The experimental situation is not accurate but all the results give quite consistently

$$\gamma^-\simeq -0.3 \text{ or } \gamma^+\simeq +2 \ . \quad (18)$$

The theoretical situation is much more confused and the different predictions[17,19] cover a very wide range of values.

In our context the axial-vector form factor F_A is closely related to our previous evaluation of the pion charge radius. Similarly, the vector form factor is deducible via an isospin rotation from our result for $g_{\pi\gamma\gamma}$. Therefore, we can readily obtain a relation between γ, $\langle r_\pi{}^2\rangle$, and $g_{\pi\gamma\gamma}$ which is valid in the soft-pion limit and for all the models so far discussed. This simple relation is

$$\gamma=e^2 f_\pi\langle r_{\pi^+}{}^2\rangle/3\,|g_{\pi\gamma\gamma}| \quad (19)$$

and predicts

$$\gamma\simeq 1.35 \quad (20)$$

when the experimental values of f_π, $\langle r_\pi{}^2\rangle$ [Eq. (9)], and $\Gamma(\pi^0\rightarrow\gamma\gamma)$ are introduced in the right-hand side (RHS) of Eq. (19). Since, according to Secs. II and III, the experimental values for $\Gamma(\pi^0\rightarrow\gamma\gamma)$ and $\langle r_\pi{}^2\rangle$ are obtainable in the context of the baryon-loop model with $n_f=4$, the value quoted in Eq. (20), $\gamma=1.35$, is also the prediction for γ in this model. Similarly one recovers the quark-model prediction,[18] $\gamma=1$, when the quark-model values of $\langle r_{\pi^+}{}^2\rangle$ and $g_{\pi\gamma\gamma}$, Eqs. (4) and (8), are used, and the nucleon-model prediction,[17] $\gamma=g_A/3=0.42$, when the corresponding values (5) and (10) are introduced in the RHS of Eq. (19). The agreement between our generic prediction (20) and the positive experimental solution (18) seems again rather satisfactory. Note also that Eq. (20) shares some similarity and seems to be a simplified version of the relations that one predicts in the context of different schemes inspired in current algebra.[17]

VI. CONCLUSIONS

We have discussed a set of four experimentally known physical parameters describing some transitions of pseudoscalar mesons (mainly pions) in the context of two parallel approaches. One is essentially the constituent-quark model and the other a baryon-loop model where the fundamental multiplet of baryons plays the role played by the quarks in the previous one. Both approaches are clearly phenomenological and are expected to be of interest for our set of low-energy processes where QCD (which is possibly related to our models) cannot be applied.

For all these models we obtain two simple relations, Eqs. (17) and (19), connecting the four parameters considered here: the $\pi^0\gamma\gamma$ coupling, the pion charge radius $\langle r_{\pi^+}{}^2\rangle$, the q^2 dependence near $q^2=0$ of the pion decay constant $[f'_\pi(0)]$, and the ratio $\gamma\equiv F_A(0)/F_V(0)$ describing the structure-dependent part of $\pi\rightarrow e\nu\gamma$. These two generic relations are fully compatible with the available experimental information.

The constituent-quark-model approach, apart from the well-known result for the $\pi\gamma\gamma$ anomaly, predicts values for $\langle r_{\pi^+}{}^2 \rangle$ and $\langle r_{K^+}{}^2 \rangle$ only slightly smaller than the experimental ones. Because of the fulfillment of our generic Eqs. (17) and (19) this represents a seemingly small prediction for $f'_\pi(0)$ and the known result $\gamma = 1$ which is smaller than $\gamma^+_{\rm expt} \sim +2$ (or larger than $\gamma^-_{\rm expt} \sim -0.3$). However, this set of predictions can hardly be considered as unsatisfactory due to the simplicity of the model and the absence of adjustable parameters.

On the other hand, the predictions of the baryon-loop model are sensitive to the number of flavors, $n_f = 2$, 3, or 4, carried by the baryons with a not too large mass. The $\pi\gamma\gamma$ coupling turns out to be somewhat larger than the experimental value for $n_f = 2$ or 3, but the model can be easily adjusted to give the right answer for $n_f = 4$. The π^+ and K^+ charge radii are predicted to be too small for $n_f = 2$, very similar to the quark-model values (i.e., only slightly smaller than the experimental ones) for $n_f = 3$ and reasonable values of α, and are somewhat increased (and hence close to the experiment) for $n_f = 4$. Consequently, Eqs. (17) and (19), lead, for $n_f = 4$, to larger predictions for $f'_\pi(0)$ and γ thus improving the quark-model results.

It is quite remarkable that our two parallel approaches lead to reasonably similar predictions, in particular, for the most reasonable number of relevant flavors, $n_f = 4$. Numerically this is a consequence of the cancellation of the factor $n_c = 3$ and that coming from the fractional quark charges with other factors related to g_A and SU(n_f) parameters, and also of having restricted ourselves to the set of processes where the use of the GT relation at the quark or baryon level allows for the suppression of (quark and baryon) couplings and masses. It is interesting to observe the rough equivalence or dual character of the two phenomenological approaches, which suggests that they could (hopefully) be derived from a unique theory such as QCD.

---

[1]S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969).

[2]M. D. Scadron, Rep. Prog. Phys. 44, 213 (1981).

[3]J. Steinberger, Phys. Rev. 76, 1180 (1949).

[4]R. Barbieri, in *Proceedings of the XIIIth Gift International Seminar on Theoretical Physics and Xth International Winter Meeting on Fundamental Physics, Marsella, Spain, 1982* (World Scientific, Singapore, 1983).

[5]N. S. Craigie, ICTP, Trieste, Report No. IC/80/62 (unpublished).

[6]L. Bergström and H. Snellman, Z. Phys. C 8, 363 (1981); Ll. Ametller *et al.*, Nucl. Phys. B228, 301 (1983); A. Bramon and E. Massó, Phys. Lett. 104B, 311 (1981).

[7]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[8]V. N. Pervushin and M. K. Volkov, Phys. Lett. 58B, 74 (1975); 51B, 357 (1974).

[9]Particle Data Group, Phys. Lett. 111B, 1 (1982).

[10]B. Lautrup and P. Olesen, Phys. Lett. 22, 342 (1966).

[11]Ll. Ametller, C. Ayala, and A. Bramon, Phys. Rev. D 24, 233 (1981).

[12]S. B. Gerasimov, Yad. Phys. 29, 513 (1979) [Sov. J. Nucl. Phys. 29, 259 (1979)].

[13]R. Tarrach, Z. Phys. C 2, 221 (1979).

[14]A. Quenzer *et al.*, Phys. Lett. 76B, 512 (1977); E. B. Dally *et al.*, Phys. Rev. Lett. 48, 375 (1982); E. B. Dally *et al.*, *ibid.* 45, 232 (1980).

[15]Our results coincide with those obtained in Ref. 8 in a similar model and for the specific value $\alpha = \frac{2}{3}$.

[16]C. A. Dominguez, Phys. Rev. D 27, 1572 (1983).

[17]See D. A. Bryman *et al.*, Phys. Rep. 88, 152 (1982).

[18]M. Moreno and J. Pestieau, Phys. Rev. D 13, 175 (1976).

[19]P. Pascual and R. Tarrach, Nucl. Phys. B146, 509 (1978).