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Neutrino-oscillation search with cosmic-ray neutrinos

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A sensitive search for neutrino oscillations involving ν_e, ν_μ, and ν_τ may be provided by measurements of the ratio of the total interaction rates of upward- and downward-going cosmic-ray neutrinos within a massive (~ 10 kton) detector. Assuming mixing between all pairs of ν_e, ν_μ, and ν_τ, the experiment is capable of observing time-averaged probabilities $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ of magnitude set by mixing strengths corresponding to, e.g., the d- to s-quark mixing strength, and of reaching the limit $\Delta m_{ij}^2 \equiv |m_i{}^2 - m_j{}^2| \approx 10^{-4}$ eV^2, where m_i, and m_j are neutrino mass eigenstates, and $P_{e\tau}$ and $P_{\mu\tau}$ are the probabilities for ν_e and ν_μ, respectively, to oscillate into ν_τ after traversing a distance $L \approx$ diameter of the Earth. Possible ambiguities may be resolved through comparison of the ratios N_e/N_μ for the upward- and downward-going neutrinos.

I. INTRODUCTION

Neutrino-oscillation experiments fall into two classes: (i) "appearance" experiments in which a search is made for the appearance of a given neutrino flavor in an incident flux which initially did not contain that flavor (except possibly as a small contamination) and (ii) "disappearance" experiments in which a suitably normalized measurement is made of the flavor content of a neutrino beam after it has traversed a given distance to provide a search for the disappearance of a fraction of a given neutrino flavor originally present at zero distance. In the latter class, experiments sensitive to small values of Δm^2 are done with $\bar{\nu}_e$ from reactors at the level $\Delta m^2 > 10^{-2}$ eV^2 (see Fig. 1), and may be done with ν_e from the Sun at the level $\Delta m^2 > 10^{-11}$ eV^2. The strength of neutrino mixing that is accessible in such disappearance experiments is of magnitude $\sin^2 2\alpha > 0.1$.

Use of detectors in deep mines and under the sea to search for neutrino-flavor oscillations[1] employing the atmospheric, i.e., cosmic-ray, neutrino flux has been discussed extensively in recent years.[2] The idea is attractive because of the small value of neutrino mass difference that may be explored by upward-going neutrinos which traverse the Earth (see Fig. 2) after their production in the atmosphere. Thus, for a full-wavelength oscillation

$$\Delta m^2(\text{eV}^2) \equiv |m_1{}^2 - m_2{}^2| \cong 2.5\langle E_\nu(\text{MeV})\rangle / L\ (\text{m})\ ,$$
$$\Delta m^2 \cong 2.5\times 600/1.3\times 10^7\ \text{eV}^2 \cong 10^{-4}\ \text{eV}^2\ , \tag{1}$$

where m_1 and m_2 represent neutrino mass eigenstates, $\langle E\rangle$ is the average incident neutrino energy, and $L = 1.3\times 10^7$ m is the difference in distance ($\sim$ diameter of the Earth) traversed by the upward- and downward-going neutrinos.

There is an additional contribution to the relative phases between different neutrino mass eigenstates besides that due to the neutrino masses and mixing. This is due to the different forward scattering amplitudes and resultant different indices of refraction of different neutrino flavors.[3] This arises because the upward-going neutrinos traverse a large amount of matter of substantial density. The characteristic wavelength describing these matter-induced oscillations is independent of the neutrino energy E_ν and is given by $l_0 = \sqrt{2}\pi/G_F N_e = (9.7\times 10^{32}\ \text{cm}^{-2})/N_e$, where $N_e = \rho_E (Z/A) N_{\text{Avogadro}}$ denotes the number density of electrons. For the Earth, ρ_E varies from ~ 2 g/cm^3 in the mantle to ~ 5 g/cm^3 in the core; taking average values, $\rho_E = 3$ g/cm^3 and $Z/A = 0.5$, we obtain $l_0 = 1.1\times 10^7$ m $\approx D_E$, where D_E is the diameter of the Earth. Oscillations due to neutrino masses and mixing alone ("vacuum oscillations") with wavelengths

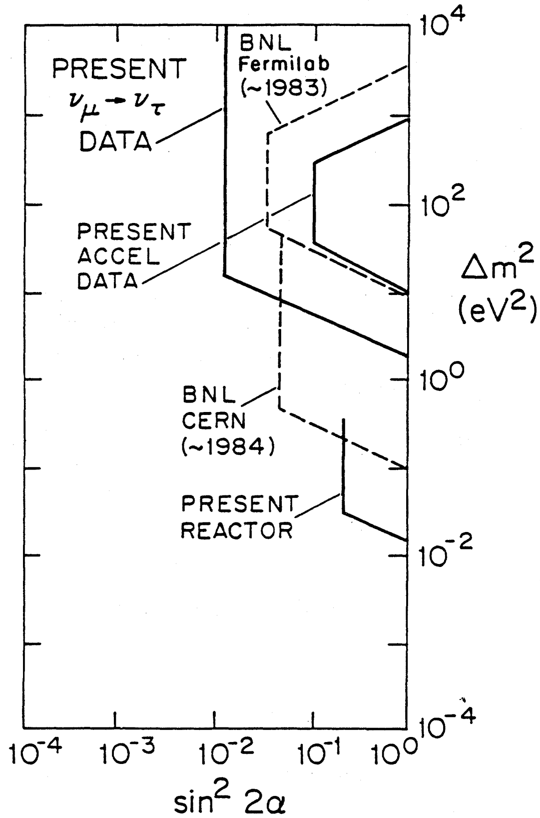

FIG. 1. Plot of the generalized mass parameter Δm^2 (eV^2) vs the generalized mixing parameter $\sin^2 2\alpha$ showing approximate limits of present and expected neutrino-oscillation data from experiments that measure the disappearance of neutrinos of a given flavor from an incident beam. Also shown is the present limit on ν_μ-ν_τ oscillations. [Taken from a paper by Lanou (Ref. 15).]

$l_{vac} = 4\pi E_\nu / |\Delta m^2| \gg l_0$ would therefore be suppressed. However, this condition does not reduce the sensitivity to small Δm^2 of the experiment described here. Since $l_0 \approx D_E$, it follows that if $l_{vac} \gg l_0$, then also $l_{vac} \gg D_E$, so that the experiment would not have been able to detect the vacuum oscillations anyway. If $l_{vac} \approx l_0$, the matter-induced oscillations can even enhance the effect for either ν or $\bar{\nu}$. Thus, we find that a neutrino-oscillation experiment using cosmic-ray neutrinos is indeed sensitive to $\Delta m^2 \geq 10^{-4}$ eV^2.

For comparison we show in Fig. 1 a summary of the limits set by present and anticipated neutrino-oscillation data using accelerator and reactor neutrinos. One sees that the value of Δm^2 in Eq. (1) is roughly two orders of magnitude smaller than the lowest limit on Δm^2 obtained from the present neutrino-oscillation experiments in which a search is made with reactor-produced $\bar{\nu}_e$ for the "disappearance" of a fraction of the incident $\bar{\nu}_e$ beam, i.e., $\bar{\nu}_e \to \bar{\nu}_\mu, \bar{\nu}_\tau, \ldots$. In addition, as will be seen, the cosmic-ray neutrino-oscillation search is capable in certain circumstances of measuring strengths of mixing between neutrino flavors about equal to that obtainable in reactor experiments, viz., similar in magnitude to the mixing between d and s quarks. It is of particular importance that the cosmic-ray oscillation search is done primarily with ν_μ and $\bar{\nu}_\mu$ (see below) because, to our knowledge, no other neutrino-oscillation experiment searching for the disappearance of ν_μ or $\bar{\nu}_\mu$ is capable of reaching such a low value of Δm^2.

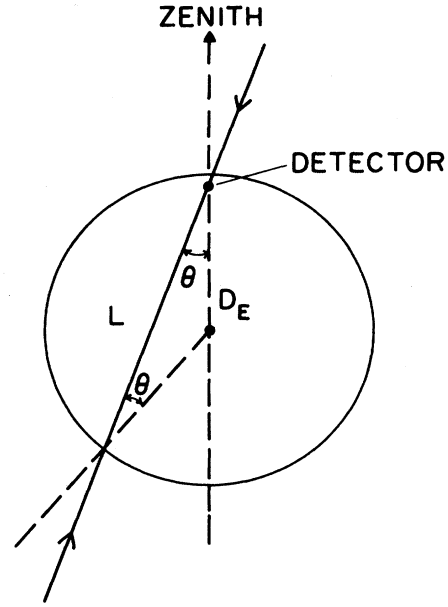

FIG. 2. Sketch of the experimental method. The neutrino detector is located $\sim$2 km below the Earth's surface. Neutrinos originate in the 10—20-km-thick atmospheric shell surrounding the Earth. Neutrinos from near the zenith that intersect little of the Earth's matter before interacting in the detector are called down-going, N(dn). Neutrinos that have traversed a large fraction of the Earth's diameter ($D_E = 1.3 \times 10^7$ m) and are observed to produced upward-going interactions in the detector are called up-going, N(up). Present limits on neutrino oscillations suggest that such oscillations have a negligible effect on the down-going atmospheric neutrino flux.

The primary obstacle to realizing the sensitivity represented in Eq. (1) is the low flux of atmospheric neutrinos,[4–6] for which the calculated spectra are shown in Fig. 3. One type of experiment attempts to overcome the low-flux obstacle by looking at upward-going muons produced by ν_μ in the large volume of rock within muon range of a buried detector. The differential neutrino flux is slightly steeper than $E_\nu{}^{-3}$. The muon range R_μ is proportional to $E_\mu = E_\nu(1-y)$, where $y = E_H/E_\nu$ and E_H is the energy of the hadrons produced in charged-current interactions. Moreover, σ_ν is approximately proportional to E_ν. Hence, the differential spectrum of parent neutrinos of the observed upward-going muons is nearly proportional to $E_\nu{}^{-1}$ up to the TeV range where both R_μ and σ_ν cut off. This means that a very large interval of neutrino energies contributes to the muons actually observed. Consequently, the relevant neutrino energies (1 to 100 GeV) in such an experiment are significantly higher than that in Eq. (1) ($\sim$1 GeV). Preliminary results of this type of experiment have been reported recently which are consistent with no oscillation.[7] This conclusion depends, however,

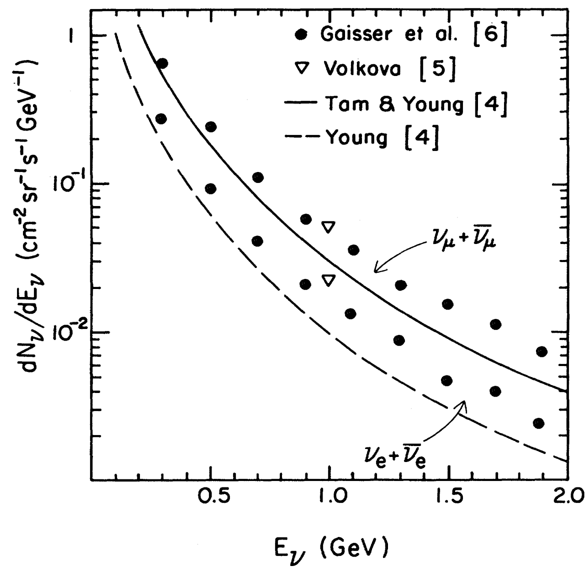

FIG. 3. Calculated cosmic-ray neutrino spectra.

on a calculation that produces an absolute value of the expected muon rate which is compared with the observed rate. It folds the primary cosmic-ray spectrum into meson-production cross sections, meson and muon decay distributions, neutrino-interaction cross sections, the muon range-energy relation, and the detector acceptance. A deficiency of observed ν_μ interactions would signal neutrino oscillations.

Because of the large systematic uncertainties inherent in this calculation, observation of a flux of upward-going muons within a factor of about 2 of the expected flux, as

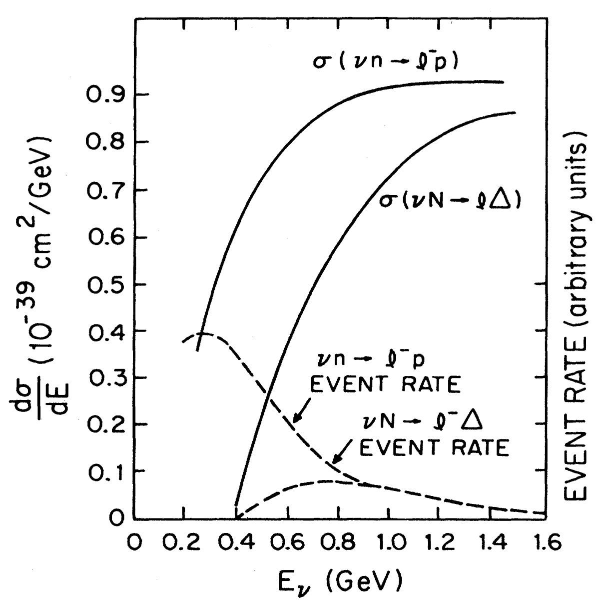

FIG. 4. Neutrino cross sections and event rates versus energy. The left-hand ordinate is the cross section per neutron for quasi-elastic interactions and two times the cross section per nucleon for single-pion production.

has already been done, is exploiting this technique as fully as possible. The uncertainties of normalization mentioned above would be largely removed by a measurement of the angular dependence of muons arriving from below the horizon. Because of the much higher neutrino energies, however, the Δm^2 values probed would be significantly larger than in Eq. (1) and correspondingly less interesting.

To obtain a statistically significant result on neutrino oscillations using cosmic-ray neutrinos, it is desirable to observe for an appreciable time interval interactions that are contained in a sufficiently massive detector. The detector must be capable of event reconstruction from which the upward- and downward-neutrino fluxes can be extracted without ambiguity. This measurement now appears to be within reach given the new generation of detectors aimed principally at searching for nucleon decay,[8] and, indeed, it significantly increases the physics potential of such detectors. In this paper we discuss the nature, possible outcomes, and limitations of a class of searches for neutrino oscillations using cosmic-ray neutrinos in which statistical and systematic errors are small enough to allow the sensitivity indicated in Eq. (1) to be realized.

II. DISCUSSION

A. Geometrical considerations

A straightforward geometrical construction (see Fig. 2) suffices to show that the upward and downward geometries are symmetric if the detector itself is up-down symmetric. For every downward-going trajectory of zenith angle in the interval θ to $\theta+d\theta$ there is a corresponding upward-going trajectory in the same solid angle. For each neutrino type, there is an effective angular-dependent height of production $h(\theta)$ above the level of the detector measured along the extension of the chord of length L. The acceptance a of the detector of area A is, for downward-going neutrinos,

$$a_{\rm dn}=2\pi h^2\sin\theta\, d\theta(A/h^2)$$

and for upward-going neutrinos

$$a_{\rm up}=2\pi(h+L)^2\sin\theta\, d\theta[A/(h+L)^2]\ .$$

Since $a_{\rm up}=a_{\rm dn}$ for each θ the geometry is completely symmetric, even if the zenith angle and energy dependences of ν_e and ν_μ are significantly different.

Even though the geometries are symmetric, geomagnetic effects on the neutrino spectra are not.[9] For a detector at a high-latitude site, upward-going neutrinos are suppressed relative to downward-going neutrinos by some 25% because the mean geomagnetic cutoff is significantly higher for the primary cosmic rays that give rise to the upward-going neutrinos.[6] The uncertainty in the calculation of this effect may limit the ultimate sensitivity of a neutrino-oscillation search based on measurements of $N_{\rm tot}(\rm up)$ and $N_{\rm tot}(\rm dn)$. However, it is in principle possible to check the calculation empirically by measuring the geomagnetic effects in detectors located at very different latitudes. This limitation is numerically less important

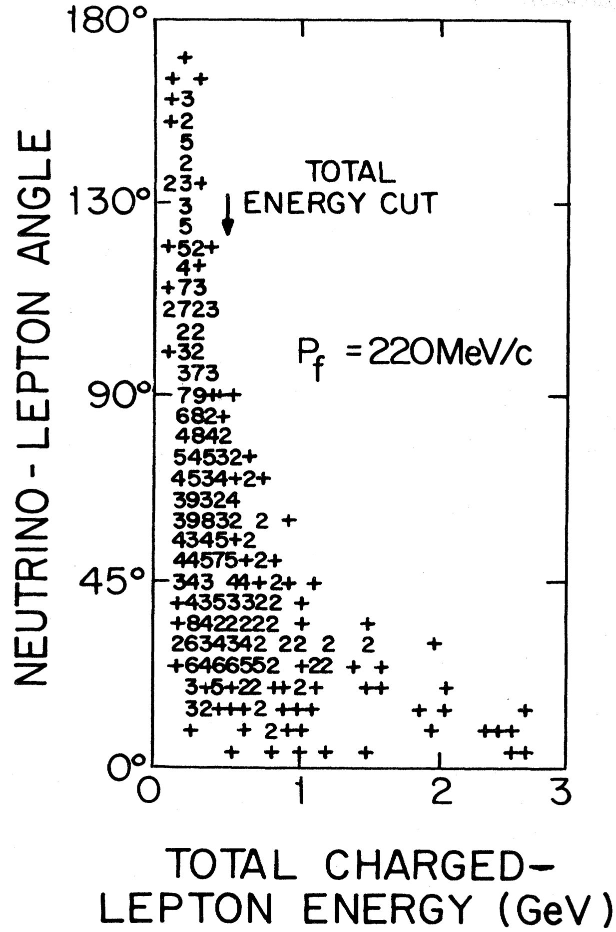

FIG. 5. The correlation between the angle of the charged lepton relative to the incoming neutrino and the energy of the charged lepton in $\nu n \rightarrow \mu^- p$.

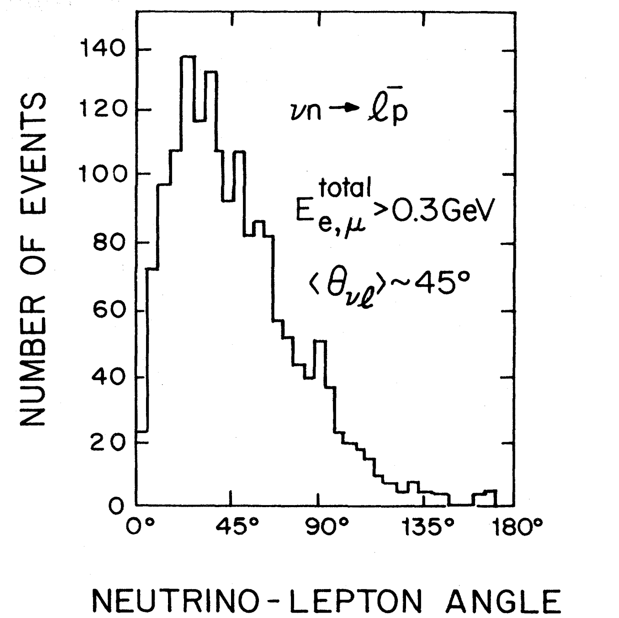

FIG. 6. The distribution of the angle between the charged lepton and the incident neutrino after a total energy cut of 0.3 GeV on the charged lepton in elastic scattering.

for the ratios $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{dn}$. Thus, study of those ratios may have the advantage of achieving a superior self-consistent oscillation limit.

B. Comparison of $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{dn}$

We have investigated the characteristics of neutrino-induced events in a 10-kton detector to determine the event rates and the resolution relative to the direction of the incident neutrinos. (See Cortez and Sulak, Ref. 2.) To simulate events in the detector, several ingredients are necessary, including the neutrino flux and the charged-current cross sections. The atmospheric neutrino spectra for ν_e and ν_μ are given in Fig. 3. It is also assumed that $\nu/\bar{\nu}=1.2$. The zenith-angle distribution of the fluxes was taken from Ref. 4; roughly, the flux decreases by 25% from $\theta_Z=90°$ to $\theta_Z=0°$. The ν_μ charged-current cross sections near 1 GeV are well known.[10] The energy dependence for elastic scattering and $\Delta(\frac{3}{2},\frac{3}{2})$ production are shown in Fig. 4, as well as the product of flux and cross section. Clearly, the dominant signal for the oscillation experiment is expected to be $\nu_\mu+n\rightarrow\mu^-+p$ and $\nu_e+n\rightarrow e^-+p$.

The correlation between the direction of the charged lepton and the incident neutrino is shown as a function of the lepton total energy in Fig. 5. In this calculation, the Fermi motion of the nucleons is assumed to be that characteristic of carbon or oxygen, with $P_f=220$ MeV/c. To ensure that the angular correlation between the outgoing charged lepton and the incoming neutrino is sufficient to sense an up-down asymmetry, a cut at $\sim$0.3 GeV in total lepton energy is necessary if the recoil nucleon is not measured. Events with this cut have the angular distribution of Fig. 6 and mean correlation angle of 45°. Although this energy cut would eliminate half of the events from the data sample, the rejected events would have low energies and would therefore be more susceptible to background contamination and to misidentification. After the charged-lepton-energy cut, the mean neutrino energy of the accepted events is $\sim$0.7 GeV. The apparatus resolution in angle for the charged lepton must be sufficiently sharp ($<20°$) to preserve any up-down asymmetry. Also, the energy resolution of the detector must be good enough ($<15\%/\sqrt{E}$) so as not to smear the energy cut significantly.

If one neglects any asymmetry due to the geomagnetic field,[9] the ratios N_e/N_μ expected under the simplest oscillation hypothesis are as follows. For no oscillations N_e/N_μ should be about $\frac{1}{2}$ whether initiated by upward- or downward-going neutrinos. Assuming maximum mixing, both upward and downward N_e/N_μ ratios will be unity if $\nu_e\leftrightarrow\nu_\mu$ oscillations exist with $l_{vac}\ll 100$ km. If $100<l_{vac}<13\,000$ km with maximum mixing then $N_e/N_\mu=\frac{1}{2}$ for downward events but $N_e/N_\mu=1$ for upward events. Thus, for $\nu_e\leftrightarrow\nu_\mu$ oscillations with maximum mixing the statistical power of a 20-kton yr exposure with the cuts mentioned above would yield a 4σ difference between $(N_e/N_\mu)_{\rm up}$ and its expected value for $10^{-4}<\Delta m^2<10^{-2}$ eV^2, where up is defined by $\theta_Z>90°$. Other special cases, e.g., ν_e do not mix but ν_μ and ν_τ do, are discussed in Sec. II C.

C. Comparison of $N_{\rm tot}(\rm up)$ and $N_{\rm tot}(\rm dn)$

Although it is desirable to measure the interaction rates of both neutrino types ν_μ and ν_e (and $\bar\nu_\mu$ and $\bar\nu_e$) over all solid angles in the detector, and to compare the ratios $(N_e/N_\mu)_{\rm dn}$ and $(N_e/N_\mu)_{\rm up}$, this procedure for atmospheric neutrinos is statistically limited. Furthermore, systematic problems may exist in measurements of $(N_e/N_\mu)_{\rm up}$ and $(N_e/N_\mu)_{\rm dn}$ that cancel for the sums $(N_e+N_\mu)_{\rm up}$ and $(N_e+N_\mu)_{\rm dn}$. In some fraction of the events, identification of the incident-neutrino type may be confounded by the π's in ν_μ-induced neutral-current channels, e.g., $\nu_\mu+n\to\nu_\mu+n+\pi^0$, $\nu_\mu+p\to\nu_\mu+p+\pi^0$, as well as in charged-current channels, $\nu_\mu+n\to\mu^-+p+\pi^0$, in which the muon range is very short. Even in a sophisticated massive detector the distinction between electromagnetic showers produced by an electron and a photon can be accomplished only with limited efficiency (roughly one photon in five is likely to be misidentified as an electron). For these reasons we now explore experiments based on a simple comparison of total upward versus downward fluxes.

Measurement and comparison of the total rates, $N_{\rm tot}(\rm up)=N_e(\rm up)+N_\mu(\rm up)$ and $N_{\rm tot}(\rm dn)=N_e(\rm dn)+N_\mu(\rm dn)$, has the following advantages: (i) The comparison can be made with greater statistical significance to allow relatively small oscillation probabilities to be observed, and (ii) the systematic uncertainties should be inherently smaller and more tractable. Thus, using the fluxes shown in Fig. 3 in the energy region above 600 MeV, and assuming a 10-kton detector exposed for one year, we find the total number of observed downward-going-neutrino interactions within the angular interval $\theta_Z\le 60°$ [$N_{\rm tot}(\rm dn)$] to be about 300 per year, assuming a solid angle of π sr.[11] In the absence of any geomagnetic effect the flux of upward and downward neutrinos (without neutrino oscillations) would be identical, i.e., $1-N_{\rm tot}(\rm up)/N_{\rm tot}(\rm dn)=0$. For simplicity, we make this assumption in the following discussion. Later, we illustrate in the tabulation of examples for oscillations among three flavors how the geomagnetic correction will affect the results at a typical high-lattitude site.

We consider several illustrative cases of possible neutrino oscillations that might be observed in an experiment measuring $N_{\rm tot}$. The examples are organized as follows: (1) two-flavor oscillations, (2) oscillations among three flavors, and (3) oscillations involving more than three flavors. We devote most attention to the case of three flavors in Sec. II C 2.

1. $\nu_\mu(\bar\nu_\mu)\leftrightarrows\nu_e(\bar\nu_e)$ only

Assuming the flux of downward-going neutrinos is unaffected by oscillations, we can write immediately

$$N_e(\rm up)=N_\mu(\rm dn)P_{\mu e}+N_e(\rm dn)(1-P_{e\mu})\ , \tag{2}$$

where $N_e(\rm up)$ and $N_\mu(\rm up)$ are the numbers of upward-going $(\nu_e+\bar\nu_e)$- and $(\nu_\mu+\bar\nu_\mu)$-induced events observed in the detector; $N_e(\rm dn)$ and $N_\mu(\rm dn)$ are the corresponding numbers of downward-going events. $P_{e\mu}\equiv P(\nu_\mu,L\mid\nu_e,0)$ is the probability that a ν_μ appears at a distance L for a ν_e present at the flux origin. Assuming *CP* invariance, Eqs. (2) yield directly

$$N_{\rm tot}(\rm up)=N_{\rm tot}(\rm dn)\ . \tag{3}$$

It follows then that there are two possible explanations for the result in Eq. (3). If it is observed, either (a) there are no oscillations between ν_e and ν_μ within experimental error, i.e., $P(\nu_\mu,L\mid\nu_e,0)=P(\nu_e,L\mid\nu_\mu,0)\ll 1$, or (b) oscillations do occur, but only between ν_e and ν_μ. In this case only the difference between $(N_e/N_\mu)_{\rm up}$ and $(N_e/N_\mu)_{\rm dn}$ can demonstrate the existence of oscillations.

It is instructive to note, however, that in alternative (b), if time averaging of $P_{e\mu}=\sin^2 2\alpha\sin^2 1.27L\Delta m^2/E_\nu$ is appropriate, then Eqs. (2) become

$$\begin{aligned}N_e(\rm up)&=[N_\mu(\rm dn)-N_e(\rm dn)]\tfrac{1}{2}\sin^2 2\alpha+N_e(\rm dn)\ ,\\ N_\mu(\rm up)&=[N_e(\rm dn)-N_\mu(\rm dn)]\tfrac{1}{2}\sin^2 2\alpha+N_\mu(\rm dn)\ ,\end{aligned}\tag{4}$$

where $\sin^2 2\alpha$ is the strength of mixing between ν_e and ν_μ. If $\sin^2 2\alpha\lesssim 0.1$ (as suggested by the reactor data in the region of $\Delta m^2>10^{-2}$ in Fig. 1) then using $\sin^2 2\alpha=0.1$ and the expected values of $N_e(\rm dn)$ and $N_\mu(\rm dn)$ for a 10-kton detector,[11] one finds $N_e(\rm up)/N_\mu(\rm up)=0.60$. Compare this with $N_e(\rm dn)N_\mu(\rm dn)=0.56$. The difference is less than the statistical error on either of the ratios, which reflects the difficulty of reaching a definitive conclusion from measurements of the ratios N_e/N_μ.

2. $\nu_e(\bar\nu_e)\leftrightarrows\nu_\mu(\bar\nu_\mu),\nu_\tau(\bar\nu_\tau)$; $\nu_\mu(\bar\nu_\mu)\leftrightarrows\nu_e(\bar\nu_e),\nu_\tau(\bar\nu_\tau)$

Again, if the flux of downward-going neutrinos is unaffected by oscillations, we may write directly

$$\begin{aligned}N_e(\rm up)&=N_\mu(\rm dn)\langle P_{\mu e}\rangle_t+N_e(\rm dn)(1-\langle P_{e\mu}\rangle_t-\langle P_{e\tau}\rangle_t)\ ,\\ N_\mu(\rm up)&=N_e(\rm dn)\langle P_{e\mu}\rangle_t+N_\mu(\rm dn)(1-\langle P_{\mu e}\rangle_t-\langle P_{\mu\tau}\rangle_t)\ ,\end{aligned}\tag{5}$$

or

$$N_{\rm tot}(\rm up)=N_{\rm tot}(\rm dn)-N_e(\rm dn)\langle P_{e\tau}\rangle_t-N_\mu(\rm dn)\langle P_{\mu\tau}\rangle_t\ . \tag{6}$$

Here $\langle P_{e\tau}\rangle_t\equiv\langle P(\nu_\tau,L\mid\nu_e,0)\rangle$ time average, $\langle P_{\mu e}\rangle_t\equiv\langle P(\nu_e,L\mid\nu_\mu,0)\rangle$ time average, etc. Equation (6) shows, as expected, that the only oscillations which contribute to the diminution of the sum of the incident ν_e $(\bar\nu_e)$ and ν_μ $(\bar\nu_\mu)$ fluxes are those to ν_τ $(\bar\nu_\tau)$. Consequently comparison of $N_{\rm tot}(\rm up)$ and $N_{\rm tot}(\rm dn)$ in a cosmic-ray neutrino-oscillation experiment leads to a measurement of the linear combination $N_e(\rm dn)\langle P_{e\tau}\rangle_t+N_\mu(\rm dn)\langle P_{\mu\tau}\rangle_t$, if there are only three oscillating neutrino flavors.

To evaluate $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ requires knowledge of the lepton mixing matrix U, which it is the aim of the experiment to determine. Here we estimate possible outcomes of the experiment for certain possible mixing ma-

TABLE I. Values of time-averaged probabilities $\langle P_{e\tau}\rangle_t$, $\langle P_{\mu\tau}\rangle_t$, and $\langle P_{e\mu}\rangle_t$ calculated for the various three-flavor mixing schemes discussed in the text. The angle δ in Eq. (13) is taken to conserve *CP*. Statistics are for a 10-kton yr exposure (Ref. 11). $B=0$ means geomagnetic field off and $B\neq 0$ assumes that there is a 25% geomagnetic suppression of upward fluxes. In both cases, Δ is defined so that $\Delta\neq 0$ indicates neutrino oscillations: $\Delta_{\text{tot}}\equiv[N_{\text{tot}}(\text{up})/N_{\text{tot}}(\text{dn})]_0-[N_{\text{tot}}(\text{up})/N_{\text{tot}}(\text{dn})]_{\text{expt}}$ and $\Delta_{e/\mu}\equiv[N_e(\text{up})/N_\mu(\text{up})]_{\text{expt}}-[N_e(\text{up})/N_\mu(\text{up})]_0$ where the subscript 0 denotes the values expected in the absence of oscillations.

Model	$\langle P_{e\tau}\rangle_t$	$\langle P_{\mu\tau}\rangle_t$	$\langle P_{e\mu}\rangle_t$	Δ_{tot} $B=0$	Δ_{tot} $B\neq 0$	$\Delta_{e/\mu}$ $B=0$	$\Delta_{e/\mu}$ $B\neq 0$
Mann-Primakoff (Ref. 1)	0.17	0.33	0.33	0.27±0.07	0.20±0.06	0.62±0.16	0.62±0.19
Wolfenstein (Ref. 13)	0.33	0.33	0.33	0.33±0.06	0.25±0.07	0.44±0.14	0.44±0.16
Kobayashi-Maskawa (Ref. 14):							
$\theta_1=11°$, $\theta_2=13°$, $\theta_3=15°$, $\delta=0°$	0.0043	0.0024	0.006	0 ±0.08	0 ±0.08	0.05±0.07	0.05±0.08
$\theta_1=11°$, $\theta_2=13°$, $\theta_3=15°$, $\delta=180°$	0.011	0.33	0.059	0.21±0.09	0.16±0.08	0.34±0.11	0.34±0.14
$\theta_1=\theta_2=\theta_3=13°$, $\delta=0°$	0.005	0.000 18	0.091	0 ±0.08	0 ±0.08	0.07±0.07	0.07±0.09
$\theta_1=\theta_2=\theta_3=13°$, $\theta=180°$	0.014	0.30	0.083	0.20±0.09	0.15±0.08	0.33±0.12	0.33±0.13

trices in the case that there are $n=3$ generations of leptons (the case $n>3$ is treated in Sec. II C 3). We list the results for each case in Table I with statistics appropriate for a 10-kton yr exposure.[11] We tabulate $\Delta_{\text{tot}}\equiv[N_{\text{tot}}(\text{up})/N_{\text{tot}}(\text{dn})]_0-[N_{\text{tot}}(\text{up})/N_{\text{tot}}(\text{dn})]$ and $\Delta_{e/\mu}\equiv[N_e(\text{up})/N_\mu(\text{up})]-[N_e(\text{up})/N_\mu(\text{up})]_0$, where the subscript 0 denotes the result expected in the absence of any oscillation. Significant deviations of Δ from zero would indicate significant oscillation effects.

Existing neutrino-oscillation searches and experiments attempting to detect the effects of neutrino masses and mixing in weak nuclear and particle decay[12] place constraints on the form of the matrix U. These depend, however, on the values of the relevant neutrino masses (or differences of masses squared). For sufficiently small masses, commensurately large mixings are allowed. As illustrations, consider mixing matrices with large off-diagonal elements (which thus implicitly require appropriately small $|\Delta m^2|$). First,[1] we look at

$$U\equiv\begin{bmatrix}\langle\nu_e|\nu_1\rangle & \langle\nu_e|\nu_2\rangle & \langle\nu_e|\nu_3\rangle\\ \langle\nu_\mu|\nu_1\rangle & \langle\nu_\mu|\nu_2\rangle & \langle\nu_\mu|\nu_3\rangle\\ \langle\nu_\tau|\nu_1\rangle & \langle\nu_\tau|\nu_2\rangle & \langle\nu_\tau|\nu_3\rangle\end{bmatrix} =\begin{bmatrix}-1/\sqrt{2} & 0 & 1/\sqrt{2}\\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}\\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6}\end{bmatrix} \tag{7}$$

from which, using for the time-averaged case of interest here,

$$\langle P_{\eta\eta}\rangle_t=\sum_j|\langle\nu_\eta|\nu_j\rangle|^4\,,\qquad \langle P_{\eta\xi}\rangle_t=\sum_j|\langle\nu_\eta|\nu_j\rangle|^2|\langle\nu_j|\nu_\xi\rangle|^2\,, \tag{8}$$

where $\eta,\xi=e,\mu,\tau$ and $j=1,2,3$. We find

$$\langle P_{e\tau}\rangle_t=\tfrac{1}{6}\text{ and }\langle P_{\mu\tau}\rangle_t=\tfrac{1}{3}\,. \tag{9}$$

Alternatively, choosing the matrix[13] (which violates *CP* invariance, unlike other choices here)

$$\langle\nu_\eta|\nu_\kappa\rangle=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & w & w^*\\ 1 & w^* & w\end{bmatrix}\,,\quad w=\exp(2\pi i/3) \tag{10}$$

yields

$$\langle P_{e\tau}\rangle_t=\langle P_{\mu\tau}\rangle_t=\tfrac{1}{3}\,. \tag{11}$$

Finally, we consider the general lepton mixing matrix, parametrized in a standard manner:[14]

$$\langle\nu_\eta|\nu_k\rangle=\begin{bmatrix}\cos\theta_1 & \sin\theta_1\cos\theta_3 & \sin\theta_1\sin\theta_3\\ -\sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3+\sin\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3-\sin\theta_2\cos\theta_3e^{i\delta}\\ -\sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3-\cos\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3+\cos\theta_2\cos\theta_3e^{i\delta}\end{bmatrix}\,. \tag{12}$$

The values $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ resulting from four sets of angle choices which are *CP* conserving and involve rotations all of magnitude of the Cabibbo angle are given in Table I.

For the *CP*-violating mixing matrix of Wolfenstein,[13] one finds

$$1-\frac{N_{\text{tot}}(\text{up})}{N_{\text{tot}}(\text{dn})}=\frac{1}{3}\,. \tag{13}$$

For the general Kobayashi-Maskawa matrix with Cabibbo-size rotation angles[14] this quantity (Table I) can be as large as

$$1-\frac{N_{\rm tot}({\rm up})}{N_{\rm tot}({\rm dn})}=\frac{1}{5} \tag{14}$$

for $\delta=180°$. Equations (13) and (14) suggest that, if either of these cases is a reasonable approximation to the actual neutrino mixing matrix, the cosmic-ray neutrino-oscillation search described here (10-kton yr exposure) should yield a positive result with statistical significance of about 3 standard deviations.

Assuming $\theta_1=\theta_2=\theta_3=13°$ and $\delta=\pi$, we note that the Kobayashi-Maskawa matrix also yields $\langle P_{e\mu}\rangle_t=0.083$, which, in conjunction with $\langle P_{e\tau}\rangle_t=0.014$, tends to keep $N_e({\rm up})\approx N_e({\rm dn})$. On the other hand, the relatively large value of $\langle P_{\mu\tau}\rangle_t$ diminishes N_μ(up), so that in this particular example the ratio $(N_e/N_\mu)_{\rm up}$ is statistically different from $(N_e/N_\mu)_{\rm dn}$, as is $N_{\rm tot}$(up) from $N_{\rm tot}$(dn).

In the presence of a 25% geomagnetic suppression of upward neutrinos (as is roughly appropriate for the northern U.S.A.) the ratio $N_i({\rm up})/N_i({\rm dn})=0.75$ for $i=\nu_e$, ν_μ, or $\nu_e+\nu_\mu$ in the absence of any neutrino oscillations. The uncertainty in the calculation of $N_i({\rm up})/N_i({\rm dn})$ will have two components: (a) the uncertainty in the calculation of the neutrino flux itself in the presence of the geomagnetic field and (b) the further uncertainty that arises from folding the energy-dependent, modulated neutrino flux with the energy-dependent acceptance of a particular detector. Detailed estimates of the resulting uncertainty in calculation of the geomagnetic effect for particular detectors have yet to be made. For purposes of illustration we arbitrarily assume $\pm 20\%$ uncertainty in the effect, so that for the examples in Table I the up-down ratios in the presence of the geomagnetic field are taken to be 0.75 ± 0.05.

The geomagnetic field will dilute any effect of neutrino oscillations differently for $(N_e/N_\mu)_{\rm up}$ vs $(N_e/N_\mu)_{\rm dn}$ and for $N_{\rm tot}$(up) vs $N_{\rm tot}$(dn). For the former, since both N_e(up) and N_μ(up) are affected similarly, the geomagnetic suppression of upward fluxes simply increases the relative statistical uncertainty as indicated in Table I. On the other hand, the geomagnetic field causes $N_{\rm tot}({\rm up})/N_{\rm tot}({\rm dn})\neq 1$ even in the absence of oscillations. Thus, the field affects the central value of this ratio as well as the uncertainty. Moreover, the error in the measurement of $\Delta_{\rm tot}$ has a contribution from the uncertainty in the magnitude of the geomagnetic correction which needs to be carefully determined.

3. Oscillations among four neutrino flavors, each capable of oscillating into the others

A general result applicable to this case (but with *CP* conservation assumed) has been obtained implicitly in a paper by Frampton and Glashow.[2] They form, for atmospheric neutrinos, the two-dimensional plot (see Fig. 7) N_μ(up) vs N_e(up), assuming $N_\mu({\rm dn})=2N_e({\rm dn})=1$.[16] They identify the following quantities in that plot: (a) a given line as the locus of values of N_μ(up) and N_e(up) corresponding to two-flavor ν_e-ν_μ mixing, and (b) a given area as containing all paired values of N_μ(up) and N_e(up) corresponding to three-flavor ν_e-ν_μ-ν_τ mixing. Paired values of N_μ(up) and N_e(up) that lie outside the three-flavor area indicate the existence of a fourth neutrino flavor. The region outside the three-flavor area can, in turn, be subdivided into two parts: one part ($\sim 20\%$) in which the values of N_μ(up) and N_e(up) are correlated such that $\frac{2}{3}<N_{\rm tot}({\rm up})/N_{\rm tot}({\rm dn})<1.0$ and a second part ($\sim 80\%$), shown shaded in Fig. 7, in which the correlation between N_e(up) and N_μ(up) satisfies $N_{\rm tot}({\rm up})/N_{\rm tot}({\rm dn})<\frac{2}{3}$.

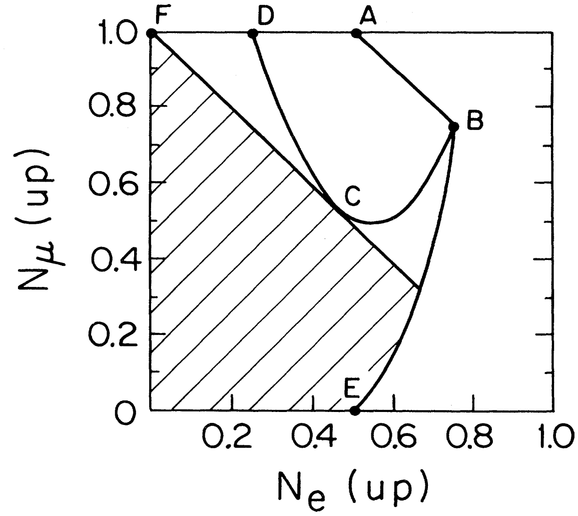

FIG. 7. Plot of N_μ(up) vs N_e(up) after Frampton and Glashow (Ref. 2). The ratio $N_\mu({\rm dn})/N_e({\rm dn})$ is taken to be 2:1, and $N_{\rm tot}$(dn) to be 1.5. The four allowed regions of interest are as follows. (I) The point *A* corresponds to no mixing of ν_μ or ν_e. Departure from this point requires a nonzero neutrino mass. (II) The line *AB* corresponds to $N=2$ flavor ν_μ-ν_e mixing. Departure from this line signals a third neutrino flavor. (III) The region *ABCDA* corresponds to $N=3$ flavor ν_μ-ν_e-ν_τ mixing. Departure from this region reveals the existence of a fourth neutrino flavor. (IV) The allowed domain *ABEOFA* corresponds to arbitrary mixing of any number ($N\rightarrow\infty$) of neutrino flavors. This limit follows from the Schwarz inequality and unitarity. The region outside the three-flavor area may be subdivided into two parts, one of which (shown crosshatched) contains paired values of N_μ(up) and N_e(up) that all satisfy the condition $N_{\rm tot}({\rm up})/N_{\rm tot}({\rm dn})<\frac{2}{3}$. At a high-latitude site the upward fluxes will be scaled down somewhat by geomagnetic effects (see Ref. 6).

It follows then that four-flavor mixing, if it exists at all, might (4 to 1 odds) yield a statistically significant, detectable reduction in the ratio $N_{\rm tot}({\rm up})/N_{\rm tot}({\rm dn})$ of magnitude equal to or greater than that possible in the three-flavor case.

III. SUMMARY

In this paper we have explored complementary disappearance experiments that may be carried out with $\nu_e+\bar{\nu}_e$ and $\nu_\mu+\bar{\nu}_\mu$ from cosmic-ray sources at the level of $\Delta m^2>10^{-4}$ eV^2 and $\sin^2 2\alpha>0.1$. The experiments have the following advantages: (1) they are the only experiments to our knowledge that are capable of searching for the disappearance of ν_μ and $\bar{\nu}_\mu$ at the limiting value $\Delta m^2>10^{-4}$ eV^2, (2) because they measure the quantities

N_{tot}(up) and N_{tot}(dn) as well as N_e/N_μ(up) and N_e/N_μ(down) they are relatively insensitive to systematic errors, (3) the experiments are capable of observing time-averaged probabilities $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ of magnitude set by mixing strengths corresponding to, e.g., the *d*- and *s*-quark mixing strength, and (4) although the experiments rely on the upward-going neutrinos (see Fig. 1) traversing a substantial fraction of the Earth's diameter, their sensitivity is not limited by matter-induced oscillations. The principal disadvantage of the experiments is that they require a very massive ($\sim$10-kton) detector in which the neutrino interactions must occur and be contained. The detector must be well enough instrumented to distinguish clearly upward-going from downward-going neutrinos. Also the detector must be located underground ($\sim$2000 m of water equivalent) so that the number of interactions initiated by cosmic-ray muons is reduced to a value substantially less than that on the Earth's surface. To obtain sufficient statistical precision, the data-taking period must be a minimum of one year, and a careful correction for asymmetry induced by the geomagnetic field must be made.

The very massive detectors intended to search for proton and bound-neutron decay, which are now and soon will be in operation deep underground, have limitations in carrying out these experiments. Namely their sensitivity to low-energy leptons and their ability to distinguish electrons from muons may not be sufficient. Nevertheless, if these limitations can be overcome, the results could place an upper bound on Δm^2 about three orders of magnitude lower than the upper bound that can be obtained from any corresponding experiment using accelerator-produced neutrinos.

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