Koba-Nielsen-Olesen scaling and rapidity distribution in nondiffractive hadronic reactions

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The relationship between Koba-Nielsen-Olesen (KNO) scaling and rapidity distributions in nondiffractive hadron-hadron reactions at high bombarding energies is discussed. The following is shown: The striking properties of rapidity distributions, in particular the existence of a central plateau as well as its energy and multiplicity dependence, are natural consequences of the statistical model which has been proposed to describe the KNO scaling behaviors in such reactions. Pseudorapidity density distributions at the CERN ISR and $p\bar{p}$ collider energies for total charge multiplicity and for different multiplicity intervals are calculated. The results are in good agreement with the data.

I. INTRODUCTION

An attempt has been made[1] to understand the most striking features of multiplicity distributions observed in the recent $p\bar{p}$ collider experiments.[2–5] It is shown in particular that the observed scaling behaviors with respect to the variables $z_{\rm nd}=n_{\rm nd}/\langle n_{\rm nd}\rangle$ and $z_c=n_c/\langle n_c\rangle$ (here $n_{\rm nd}$ is the multiplicity of charged hadrons in nondiffractive reactions, n_c is that seen in the central rapidity region, and $\langle n_{\rm nd}\rangle$ and $\langle n_c\rangle$ are the mean values of $n_{\rm nd}$ and n_c, respectively) are consequences of a three-fireball model (TFM)[1] which is based on the following assumptions:

(i) In most of the high-energy nondiffractive hadron-hadron collision events, the colliding objects (which have many degrees of freedom) hit each other gently such that they "go through" each other. During this process, a considerable amount of kinetic energy of the colliding objects is converted into excitation energy which is randomly distributed into three systems P^*, T^*, and C^*. We call them fireballs. The three fireballs are located, in general, in three distinct regions in rapidity space.

(ii) The excitation energies E_i^* $(i=C^*,P^*,T^*)$ of the three systems hadronize independently. The multiplicity n_i of charged hadrons produced in the system i is proportional to the excitation energy E_i^* of that system. That is,

$$n_i=E_i^*/\epsilon;\quad i=C^*,P^*,T^*\ , \tag{1}$$

where ϵ depends only on the total center-of-mass-system (c.m.s.) energy ($\sqrt{s}$). (The average energy per particle, whether neutral or charged, is taken to be the same.)

Evidently, while the three systems P^*, T^*, and C^* are associated with the observed projectile-fragmentation, target-fragmentation, and pionization products, respectively, Eq. (1) simply reflects the empirical fact that the overwhelming part of the produced particles are pions with approximately the same transverse momentum.

It is shown[6] that the randomness for the system i $(i=C^*,P^*,T^*)$ to obtain a given amount of energy E_i^* from two energy sources P and T (the moving projectile P and the moving target T, viewed from the rest frame of the fireball i) leads to the probability density

$$P(E_i^*)=CE_i^*\exp(-BE_i^*)\ . \tag{2}$$

That is, Eq. (2) is valid provided that the system i, formed by P and T, does not "remember" how much of its energy E_i^* was originally taken from P and how much of it from T.

The constants B and C in Eq. (2) are determined by the normalization conditions

$$\int dE_i^*P(E_i^*)=1\ , \tag{3}$$

$$\int dE_i^*E_i^*P(E_i^*)=\langle E_i^*\rangle\ . \tag{4}$$

Equations (1)—(4) immediately give the probability density for n_i:

$$P(n_i)=\frac{4n_i}{\langle n_i\rangle^2}\exp\left[-\frac{2n_i}{\langle n_i\rangle}\right]\quad i=C^*,P^*,T^*\ . \tag{5}$$

We approximate the observed multiplicity n_c of charged hadron in the central rapidity region[2,7] by n_{C^*} (the charge multiplicity of the excited central fireball). Hence, $\langle n_{C^*}\rangle P(n_{C^*})$ is the Koba-Nielsen-Olesen (KNO) scaling function for multiplicity of charged hadrons observed in the central rapidity region.

The corresponding probability density $P(n_{\rm nd})$ in the entire rapidity space ($n_{\rm nd}$ stands for the observed multiplicity $n_{\rm nd}=n_{C^*}+n_{P^*}+n_{T^*}$ of all charged hadrons in nondiffractive hadron-hadron collisions) is[1]

$$P(n_{\rm nd})=\int 2\delta(n_{C^*}+n_{P^*}+n_{T^*}-n_{\rm nd})\times\prod_{i=C^*,P^*,T^*}P(n_i)dn_i\ , \tag{6}$$

where

$$\langle n_{C^*}\rangle=\alpha\langle n_{\rm nd}\rangle\ , \tag{7}$$

$$\langle n_{P^*}\rangle=\langle n_{T^*}\rangle=\frac{1-\alpha}{2}\langle n_{\rm nd}\rangle\ . \tag{8}$$

Here, α is a parameter the value of which is between 0 and 1. It characterizes the relative "average size" of the three

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fireballs, and is in general s dependent. Numerical calculations show that the integral is rather insensitive to α. In fact, up to $\sqrt{s}=28$ GeV good fits can be obtained[1,8] for $0.15<\alpha<0.5$; and at $\sqrt{s}=540$ GeV, the best value for α is 0.75.[8] [See Figs. 1(a) and 1(b). Note that the factor 2 in Eq. (6) is introduced because the experimental plots for $P(n_{\text{inel}})$ and $P(n_{\text{nd}})$ are normalized to 2 instead of 1. See, e.g., Refs. 7 and 9.] Furthermore, it has been shown[1] that also other striking properties, such as the existence of forward-backward long-range correlations[10,11] can be understood in terms of the proposed model.

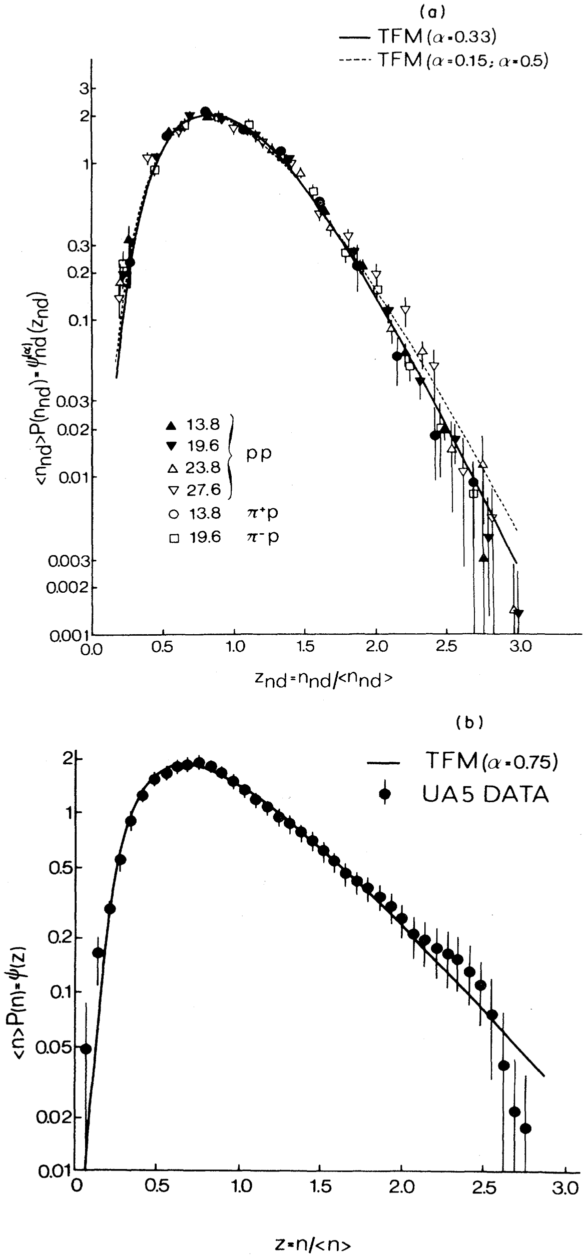

FIG. 1. Multiplicity (n_{nd}) distributions of nondiffractive collisions plotted in terms of $n_{\text{nd}}/\langle n_{\text{nd}}\rangle$ where $\langle n_{\text{nd}}\rangle$ is the average value of n_{nd} and $P(n_{\text{nd}})$ denote the probability for n_{nd}. Data in (a) is taken from Ref. 14 and those in (b) are taken from Ref. 3.

It is therefore natural to ask: What does the model say about rapidity distribution in inclusive and in semi-inclusive experiments? Can we understand the following features observed in the recent experiments?[2–5,7]

(a) There is always a central plateau in nondiffractive hadron-hadron collisions at sufficiently high incident energies.

(b) The height of the plateau raises with increasing incident energy. There is approximately an 80% increase in going from the top CERN ISR energy to the $p\bar{p}$ collider energy.

(c) The width of the plateau is much narrower than expected from a simple extrapolation of ISR data. It has grown by only approximately 2 units in the above-mentioned energy range, whereas the separation in rapidity of the two beam particles has increased by 4.6 units over this energy.

(d) The shape of rapidity distributions depends on multiplicity. There is a prominent central dip at low multiplicity which disappears in going to high multiplicities and a shrinking of the distribution with increasing multiplicity.

(e) The dependence on multiplicity mentioned in (d) is qualitatively the same at ISR and at collider energies.

II. EXISTENCE OF CENTRAL PLATEAU

We show in this section that the observed central rapidity plateau can be readily obtained by taking into account the energy and momentum conservation in the three-fireball model (TFM). To be more specific, here we only discuss collision processes at a fixed bombarding energy. The energy dependence will be discussed in Sec. III.

We recall that the TFM is based on two assumptions which can be expressed as follows:

(i) Nondiffractive hadron-hadron collisions at sufficiently high bombarding energies takes place in two stages. In the first stage, three fireballs (C^*, P^*, and T^*) are formed. In the second stage, final-state hadrons (mostly pions) are produced by the fireballs.

(ii) The multiplicity n_i of charged hadrons produced in the system i ($i=C^*,P^*,T^*$) is proportional to the excitation energy E_i^* of that fireball. [See Eq. (1).] E_i^* is nothing else but the mass M_i of the fireball i. The distribution of M_i is [see Eqs. (2)–(4)]

$$P(M_i)=\frac{4M_i}{\langle M_i\rangle^2}\exp\left[-\frac{2M_i}{\langle M_i\rangle}\right],\quad i=C^*,P^*,T^* . \tag{9}$$

Although not explicitly used in the calculation of multiplicity distributions, it is clear that also the basic conservation laws should be satsified. In particular, the energy and momentum conservations in the first stage of the process demands

$$(p_{\|C^*}{}^2+M_{C^*}{}^2)^{1/2}+(p_{\|P^*}{}^2+M_{P^*}{}^2)^{1/2}+(p_{\|T^*}{}^2+M_{T^*}{}^2)^{1/2}=E , \tag{10}$$

$$p_{\|C^*}+p_{\|P^*}+p_{\|T^*}=0 , \tag{11}$$

where $p_{\|i}$ ($i=C^*$, P^*, and T^*) are the corresponding momenta along the beam axis in the total c.m.s. frame. The momenta of the fireballs in transverse directions have been neglected for the sake of simplicity. E is the total c.m.s. energy of the three fireballs. It is in general a fraction (f) of the total c.m.s. energy $\sqrt{s}$. The rest is carried away by the leading particles. Hereafter we shall use the standard empirical value $f=\frac{1}{2}$.

Furthermore, the following working hypothesis concerning fireball decay is assumed to be valid in first-order approximation.

The transverse momenta of the observed particles are those due to fireball decay. If the decay is isotropic with respect to the fireball rest frame, the rapidity (y) distribution of a produced particle from the fireball i ($i=C^*,P^*,T^*$) is approximately[12]

$$\frac{dn_i}{dy}=\frac{n_i}{2\cosh^2(y-y_i)}\ , \tag{12}$$

where

$$y_i=\ln\frac{(p_{\parallel i}{}^2+M_i{}^2)^{1/2}+p_{\parallel i}}{M_i} \tag{13}$$

is the rapidity of the fireball i. Both y and y_i are measured in the total c.m.s. frame.

We now show that the characteristic features of rapidity distributions in nondiffractive hadron-hadron collisions are determined by the assumptions (i) and (ii) [see Eqs. (1) and (9)], energy-momentum conservation [see Eqs. (10) and (11)], and phase-space considerations (see appendix). To be more precise, for a given value of total c.m.s. energy $\sqrt{s}$, we obtain for each fireball a rapidity distribution $F(y_i)$ where y_i, $p_{\parallel i}$, and M_i are related to one another according to Eq. (13).

As an illustrative example, we consider the rapidity distribution of the three fireballs at $\sqrt{s}=540$ GeV before they decay. [That is, the fireball i ($i=C^*,P^*,T^*$) is considered as a system of n_i charged hadrons at rest in this fireball rest frame.] The result, as given in Fig. 2, shows that the distribution in the central rapidity region is indeed rather broad and flat. It shows explicitly that a central rapidity plateau can be readily obtained by taking energy-momentum conservation into account in the proposed model.

It is clear that the distribution will become broader and flatter when the distributions due to fireball decay are taken into account. In fact, this is a rather general feature, and does not depend on the details of the fireball decay. Hereafter, we shall assume, for the sake of simplicity and concreteness, isotropic decay in the rest frame of each fireball, as we have given in Eq. (12). The resulting distribution[13] for $\sqrt{s}=540$ GeV is shown in Fig. 3, together with the data of the UA1 collaboration[2,4] and that of the UA5 collaboration.[3,5] It should be mentioned in this connection that the only free parameter α in this calculation has been determined[8] by the multiplicity distribution [see Fig. 1(b)].

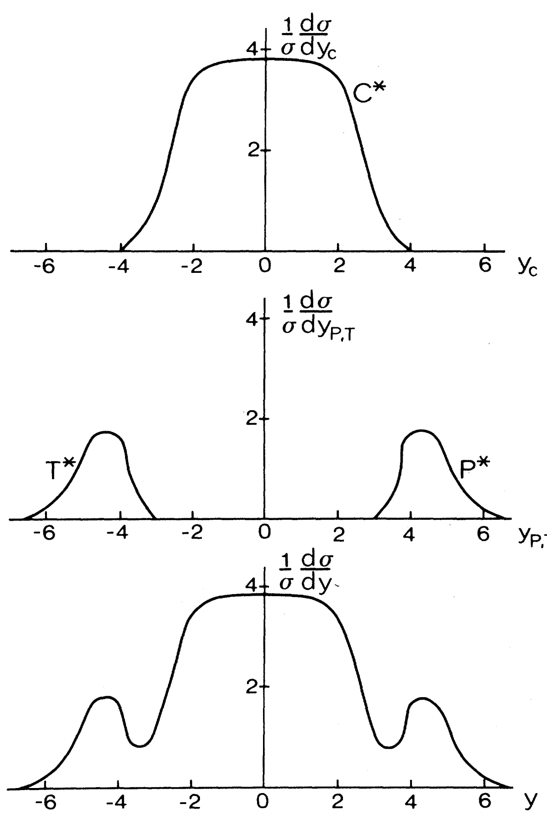

FIG. 2. Rapidity distribution for $\sqrt{s}=540$ GeV calculated from the three-fireball model, where the distributions due to fireball decay have not been taken into account.

III. ENERGY AND MULTIPLICITY DEPENDENCE OF RAPIDITY DISTRIBUTIONS

"How does the rapidity distribution depend on the total c.m.s. energy $\sqrt{s}$?" This is one of the questions which have received particular attention for many years. According to most of the current theories, the height of the central rapidity plateau should remain constant, and the width should increase like $\ln s$ for increasing $\sqrt{s}$. For quite a long time, these properties have been considered as

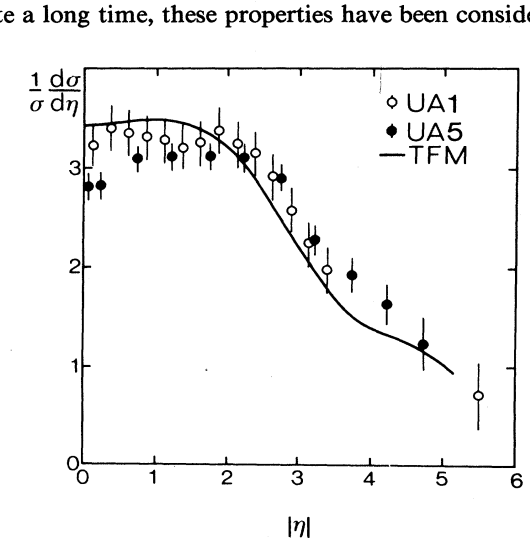

FIG. 3. Comparison between data and the calculated rapidity distribution at $\sqrt{s}=540$ GeV. Distributions due to fireball decay are taken into account. Data are taken from Refs. 2—5.

established—at least approximately (see the review articles given in Ref. 14 and the papers cited therein for details about the experimental and theoretical developments in the past). The experimental observations[2–5] at $\sqrt{s}=540$ GeV (see Sec. I), which clearly contradict the conventional belief, are therefore of fundamental importance for the on-going problem of understanding the reaction mechanism of high-energy hadron-hadron collision processes.

The observed s dependence of the rapidity distributions can be understood in terms of the proposed model.[1] First, since in this model the multiplicity, $n_{\rm nd}$, in nondiffractive collision processes is $n_{\rm nd}=n_{C^*}+n_{P^*}+n_{T^*}$ [see Eq. (6)], the empirical fact that the average multiplicity $\langle n_{\rm nd}\rangle$ increases with increasing s implies that the sum of the average multiplicity $\langle n_i\rangle$ increases with increasing s. Now, since large $\langle n_{C^*}\rangle$ as well as large $\langle n_{P^*}\rangle$ and/or $\langle n_{T^*}\rangle$ contribute predominantly to the central rapidity region (recall that the momentum transfer in gentle collisions remains to be small, hence "larger fireballs move slower"), the growth of the average multiplicities of the fireballs implies the increase in height of the central rapidity plateau. Second, note that the rapidity distribution dn/dy in nondiffractive collision at a given $\sqrt{s}$ can be calculated from the multiplicity distributions $P(n_i)$ of the fireballs $(i=C^*,P^*,T^*)$ by taking the energy-momentum conservation into account. Also note that $P(n_i)$ $(i=C^*,P^*,T^*)$ are determined by $\langle n_{\rm nd}\rangle$ and the corresponding α [see Eqs. (5), (7), and (8)]. Hence, the s dependence of dn/dy is determined by that of $\langle n_{\rm nd}\rangle$ and that of α. Here, the rapidity distributions at different values of $\sqrt{s}$ are calculated by using the experimental values of $\langle n_{\rm nd}\rangle$ (note that the three-fireball model does not dictate the functional dependence between $\langle n_{\rm nd}\rangle$ and s) and the corresponding α values which are determined[1,8] from the corresponding multiplicity distribution data.[2–5,7,9] The results[13] are shown in Figs. 3 and 4.

We next consider the rapidity distribution in nondiffractive events of different charge multiplicity $n_{\rm nd}$. We recall that according to energy and momentum conservation, as given in Eqs. (10) and (11), the masses M_i $(\equiv E_i^*)$ and the rapidities y_i of the fireballs $(i=C^*,P^*,T^*)$ are closely connected to one another. Hence the relationship between E_i^* and n_i [see Eq. (1)], and that between $n_{\rm nd}$ and n_i $(n_{\rm nd}=n_{C^*}+n_{P^*}+n_{T^*})$ imply that the rapidity distribution depends on $n_{\rm nd}$. To be more specific, since large $n_{\rm nd}$ means large M_{C^*} and/or large $M_{P^*}(M_{T^*})$, the corresponding values for $|y_i|$ are relatively small. Therefore, high-$n_{\rm nd}$ events are associated with more centrally $(y=0)$ peaked rapidity distributions, while low $n_{\rm nd}$ events correspond to those in which $|y_{P^*}|$ and $|y_{T^*}|$ are relatively large. In the latter case, all n_i and hence all the masses of the fireballs $M_i(i=C^*,P^*,T^*)$ are comparatively small. Furthermore, since such general kinematical properties are valid not only at special values of incident energies, the dependence of the shape of rapidity distributions on multiplicity is expected not to change much at different values of $\sqrt{s}$.

We divided the rapidity distributions calculated in this and the preceding section into multiplicity intervals in which the experimental data[2–5,7] are available. The results are shown in Figs. 5 and 6.

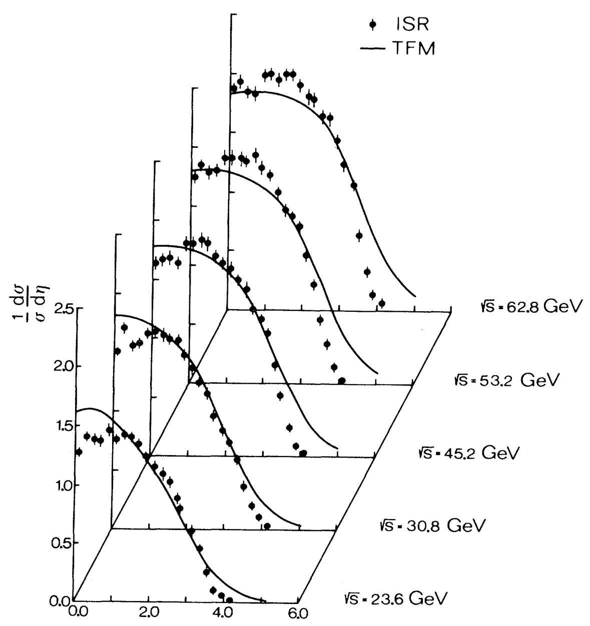

FIG. 4. Comparison between data and calculated rapidity distributions at the standard CERN ISR energies. Data are taken from Ref. 7.

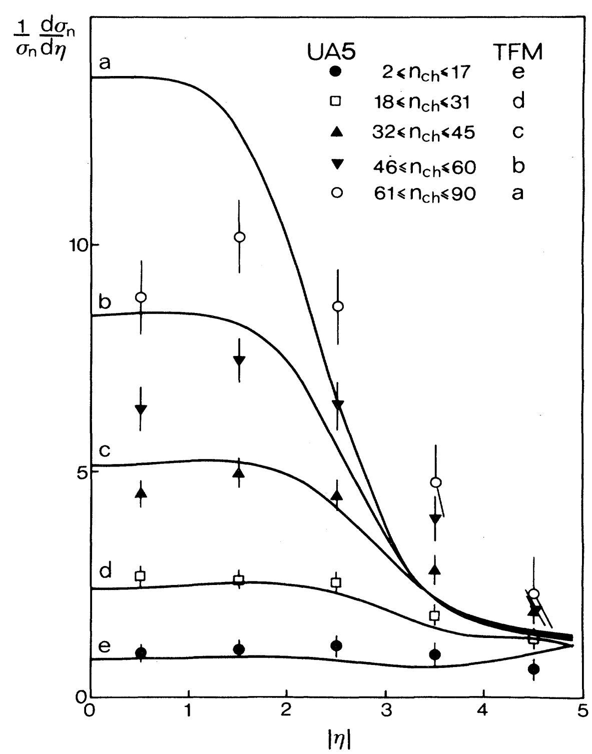

FIG. 5. Comparison between data and the calculated multiplicity dependence of the shape of rapidity distributions at $\sqrt{s}=540$ GeV. Data are taken from Refs. 3 and 5.

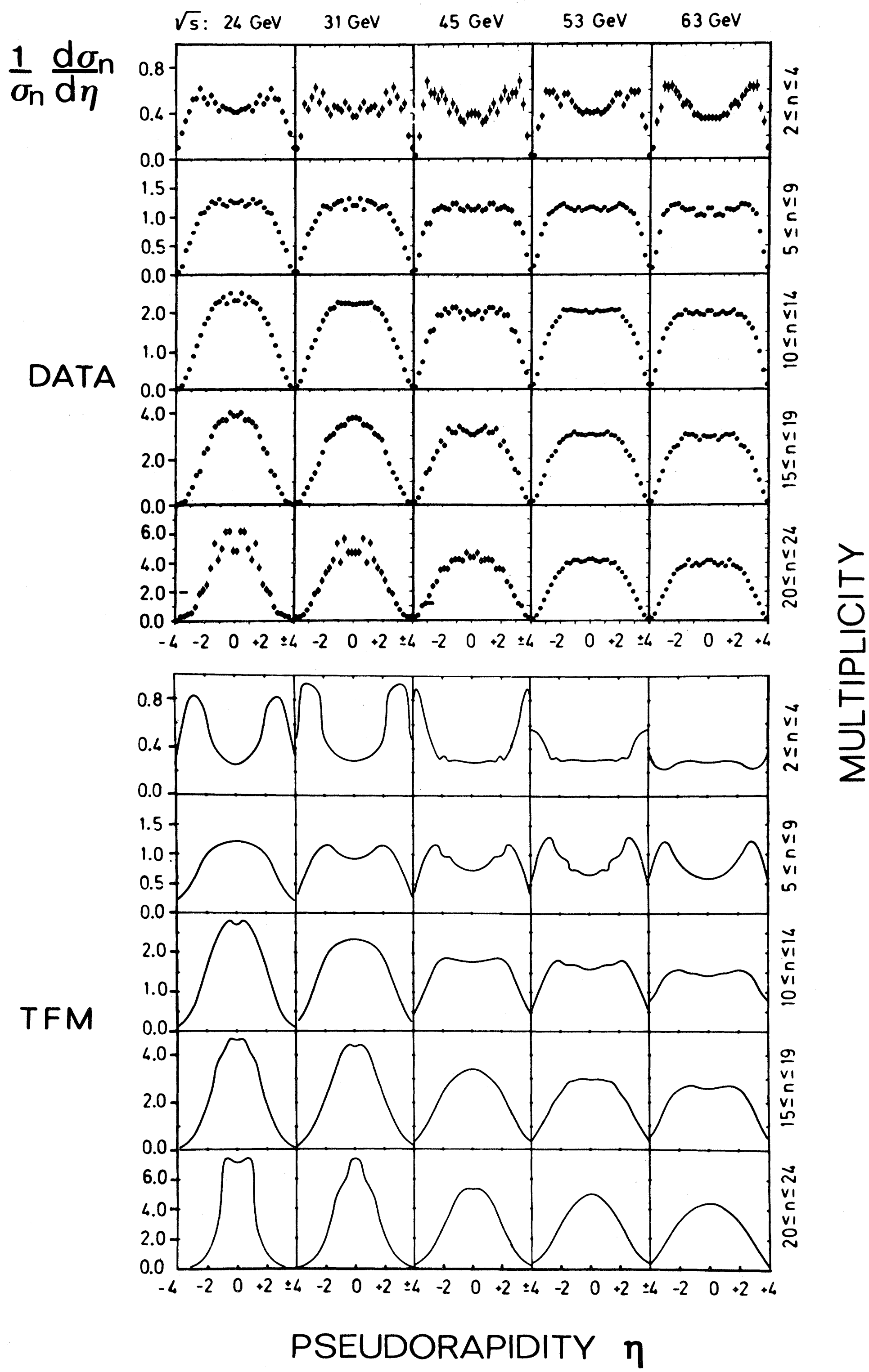

FIG. 6. Comparison between the data and the calculated multiplicity dependence of the shape of rapidity distributions at the standard CERN ISR energies. Data are taken from Ref. 7.

IV. DISCUSSION

(1) We have seen in Secs. II and III that the most striking features of the rapidity distributions observed in CERN ISR and $p\bar{p}$ collider experiments can be readily reproduced in terms of the three-fireball model, provided that energy and momentum conservation are taken into account. These distributions show, in particular, not only the existence of the central rapidity plateau, but also the

energy and multiplicity dependences of rapidity distributions are natural consequences of the basic assumptions of this model. We note, because of the dominating role played by the multiplicity distribution in this calculation, the agreement between model and data in rapidity distributions provides further evidence for the conjecture made in previous papers[1,6] that the KNO scaling function of a given process reflects its reaction mechanism.

(2) Careful comparison between the calculated result and the data shows that there is a discrepancy in rapidity distributions at high multiplicities (see Fig. 5). This is probably due to the fact that the contributions of *violent* collision events have been neglected. We recall[15] that in such collision events, the colliding objects hit each other so violently that they give their entire amount of kinetic energy to a common system. The new system formed by the two colliding objects decays after expansion. This kind of collision event corresponds to those of very small impact parameters. Because of the large momentum transfer, such processes are associated with extremely high multiplicity and/or average transverse momentum. Details about the effect of such events will be given elsewhere.

(3) The difference between a *gentle* and a *violent* collision event in nondiffraction hadron-hadron processes[15] can also be expressed in the quark-gluon language: If we adapt the picture proposed by Van Hove and Pokorski,[16] in which it is assumed that every colliding hadron is a system of colored valence quarks (antiquarks) surrounded by a large number of colored gluons, the two kinds of processes can be described as follows: In *gentle* collisions, the valence quarks (antiquarks) of the two colliding hadrons pass one another during the process. They build the main parts of the leading particles. Gluons (maybe also sea quarks) of the colliding hadrons interact and form the (color singlet) fireballs which subsequently decay. In *violent* collisions, the valence quarks of the two colliding hadrons first form a new (color singlet) system which decays subsequently. Hence, the processes discussed by Van Hove and Pokorski[16] are, in our terms, *gentle* collisions.[15]

It should be pointed out that although the details of the Van Hove—Pokorski model and that of ours are very much different, the spirits of the two models are quite similar. In order to explore the connections between hadron structure and hadron-hadron collisions in general, and the relationship between these two approaches in particular, further studies along this line would be helpful.

(4) Encouraged by the successful description of high-energy hadron-hadron collision processes, the three-fireball model will now be applied to hadron-nucleus and nucleus-nucleus collisions. Preliminary studies seem to show that this approach is rather promising.

ACKNOWLEDGMENT

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APPENDIX

According to the proposed model, three fireballs (C^*,P^*,T^*) are formed in the first stage of every nondiffractive event in high-energy hadron-hadron collisions. The masses M_i and the rapidities y_i $(i=C^*,P^*,T^*)$ of the fireballs (transverse momenta neglected) satisfy the following conditions (hereafter we shall omit the asterisks on C^*,P^*,T^*, when they are written as indices):

$$M_C\cosh y_C+M_P\cosh y_P+M_T\cosh y_T=E\ , \tag{A1}$$

$$M_C\sinh y_C+M_P\sinh y_P+M_T\sinh y_T=0\ , \tag{A2}$$

$$y_T<y_C<y_P\ , \tag{A3}$$

where E is the c.m.s. energy of the three-fireball system. Equations (A1) and (A2) show nothing else but conservation of energy and longitudinal momentum, respectively; (A3) is a constraint required by the dynamics of the model. We eliminate two (y_P,y_T,say) of the six variables M_i,y_i $(i=C^*,P^*,T^*)$ by using Eqs. (A1) and (A2), and denote (M_C,M_P,M_T) by $\vec{M}$. Thus, there is now no functional relationship between the four variables $\vec{M},y_C$, but because of the constraints (A1)—(A3), the allowed values of $\vec{M}$ and y_C are correlated so that they are not stochastically independent.

Let $f(\vec{M},y_C)$ be the joint distribution, from which the marginal distributions $P(\vec{M})$ and $Q(y_C)$,

$$P(\vec{M})=\int dy_C f(\vec{M},y_C),\ \ y_C\in\Omega(y_C;\vec{M})\ , \tag{A4}$$

$$Q(y_C)=\int d\vec{M} f(\vec{M},y_C),\ \ \vec{M}\in\Omega(\vec{M};y_C)\ , \tag{A5}$$

can be obtained. Here $\Omega(y_C;\vec{M})$ denotes the set of all y_C satisfying the conditions given in (A1), (A2), and (A3) for given $\vec{M}$. $\Omega(\vec{M};y_C)$ is defined correspondingly.

The marginal distribution $P(\vec{M})$ is a known function in the three-fireball model,

$$P(\vec{M})=\prod_{i=C,P,T} P(M_i)\ . \tag{A6}$$

In order to find $Q(y_C)$, we note that the corresponding joint distribution $f(\vec{M},y_C)$ can be written in the following way:

$$f(\vec{M},y_C)=P(\vec{M})q(y_C\mid\vec{M})\ , \tag{A7}$$

where

$$q(y_C\mid\vec{M})=\begin{cases}1/\Gamma(y_C;\vec{M}), & \text{for } y_C\in\Omega(y_C;\vec{M})\\ 0, & \text{otherwise}\ .\end{cases} \tag{A8}$$

$\Gamma(y_C;\vec{M})$ is the "size" of $\Omega(y_C;\vec{M})$:

$$\Gamma(y_C;\vec{M})=\int dy_C,\ \ y_C\in\Omega(y_C;\vec{M})\ . \tag{A9}$$

This is because, for given $\vec{M}$, y_C is not restricted by any conditions other than the energy-momentum conservation laws given by Eqs. (A1) and (A2).

It is clear that while the marginal distribution $Q(y_C)$ can be obtained directly by inserting Eqs. (A6), (A7), (A8), and (A9) in Eq. (A5), the rapidity distributions of the P^* and T^* fireballs may be obtained by considering y_C as a function of $\vec{M}$ and y_j $(j=P^*,T^*)$. For the sake of completeness, we also give the corresponding formulas explicitly:

$$R(y_j)=\int d\vec{M}\,P(M)g(\vec{M},y_j),\quad \vec{M}\in\Omega(\vec{M};y_j)\ ,\quad (A10)$$

where $j=P^*,T^*$, and

$$g(\vec{M},y_j)=\begin{cases} q(y_C(M,y_j)\,|\,\vec{M})\,|\,\partial y_C/\partial y_j\,|\ , & \text{for } y_j\in\Omega(y_j;\vec{M}) \\ 0, & \text{otherwise}\ , \end{cases}\quad (A11)$$

$|\,\partial y_C/\partial y_j\,|$ is the Jacobian due to variable transformation. $\Omega(y_j;M)$ denotes the set of all y_j ($j=P^*,T^*$) satisfying the conditions given in (A1), (A2), and (A3) for given $\vec{M}$. $\Omega(\vec{M};y_j)$ is defined similarly.

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[12]We start from an invariant expression [see, e.g., B. Touschek, Nuovo Cimento 58B, 295 (1968)], and the papers cited therein)

$$E\,d^3n/dp^3=C\exp(-\beta_\mu p^\mu)\ ,$$

$$\beta_\mu=u_\mu/T,\quad p^\mu=Eu^\mu\ ,$$

where u_μ is the four-velocity, T the temperature of the system, and E is the energy of the observed particle in the rest frame of that system. C is a normalization constant. By using the identities

$$E\,d^3n/dp^3=d^3n/dy\,d\vec{p}\ ,$$

$$p_\perp{}^2=E^2(\cosh y)^{-2}-m^2\ ,$$

where $\vec{p}_\perp$, y, and m are the transverse momentum, the rapidity and the mass of the produced particles (all of them are taken to be pions), respectively, we obtain

$$\frac{dn}{dy}=\pi c\int_0^{(p_{\perp\max})^2} d(p_\perp{}^2)\exp\left[-\frac{1}{T}(p_\perp{}^2+m^2)^{1/2}\cosh y\right]\ .$$

The value for $p_{\perp\max}$ is determined by the mass (M) of the fireball, and the constant C is determined by the condition

$$\int dy\frac{dn}{dy}=n\ .$$

The result of dn/dy as a function of y and the fireball mass M can be written as

$$\frac{dn}{dy}=\frac{n}{(\cosh y)^2}\frac{I(M,y)}{K(M)}\ .$$

Here,

$$I(M,y)=\left[1+\frac{m}{T}\cosh y\right]e^{-(m/T)\cosh y}+\left[1+\frac{M}{T}\right]e^{-M/T}$$

and

$$K(M)=\int dy\,I(M,y)/(\cosh y)^2\ ,$$

where the integral is taken over the entire allowed rapidity region. Numerical calculations show that approximation as given in Eq. (12) is sufficiently good, provided that the mass of the fireball is much larger than that of the produced particles (the overwhelming part of which are pions).

[13]In order to compare the calculated result directly with the data, the final result is given in terms of pseudorapidity (η) distributions. The transformation from dn/dy has been carried out in the approximation in which all the produced particles are considered as pions, and the magnitude of the transverse momenta are given by the standard empirical average value $\frac{1}{3}$ GeV/c.

[14]See, e.g., M. Jacob, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Illinois, 1973), p. 373; R. Slansky, Phys. Rep. 11, 99 (1974); P. Carruthers, Ann. N.Y. Acad. Sci. 229, 91 (1974); L. Foa, Phys. Rep. 22, 1 (1975); L. Van Hove and M. Jacob, Phys. Rep. 62C, 1 (1980); and the papers cited therein. In particular, it is interesting to see that the energy independence of the plateau height has already been suspected by Carruthers (see the third paper above) a decade ago.

[15]See Refs. 1 and 6 and the papers cited therein.

[16]S. Pokorski and L. Van Hove, Acta Phys. Pol. B5, 229 (1974); Nucl. Phys. B86, 243 (1975); L. Van Hove, Acta Phys. Pol. B7, 339 (1976).