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Is pionization universal?

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Jet universality, restricted to the pionization region, is discussed both from the theoretical and experimental points of view. We review the notion of topological jets corresponding to hadronization of strings. We argue that the topological jet universality is broken by soft "ladder" gluons present in soft processes but not in hard ones. A multiperipheral model for quark jets which takes into account the spin $\frac{1}{2}$ of the quark predicts a rapidity plateau of the same height as that of a hadron jet (for one cut Pomeron). We show that the q^2 independence of $\langle n_{\rm ch}\rangle$ in deep-inelastic lepton scattering favors this equality, which is contrary to topological jet universality. We propose a new test of the topological jet universality based on the local compensation of quantum numbers.

I. INTRODUCTION: DIFFERENT KINDS OF JET UNIVERSALITY

The production of hadrons in jets seems now to be a general feature of high-energy inelastic reactions (hadron-hadron, hadron-nucleus, lepton-hadron, e^+e^- collisions). Two main characteristics of a standard jet[1] are (i) limited transverse momenta with respect to the jet axis, typically

$$\langle k_T\rangle \sim 300 \text{ MeV}/c \ , \tag{1.1}$$

and (ii) the existence of a "pionization region," or a "rapidity plateau," for $m_{\rm hadron} \ll p_{\parallel} \ll p_{\rm jet}$, with an inclusive spectrum of the form

$$E\frac{dN}{dp_{\parallel}}=\frac{dN}{dY}=C \tag{1.2}$$

Y is rapidity, and $\langle k_T\rangle$ and C are assumed not to vary with $p_{\rm jet}$.

It is now customary to classify the jets according to the nature of the fragmenting object: (a) hadron-fragmentation jets (typically in low-p_T inelastic hadron-hadron collisions), (b) quark-fragmentation jets (two-jet events in e^+e^- reactions, current fragmentation in deep-inelastic lepton scattering, a major part of the large-p_T jets in hadron-hadron collisions, etc.), and (c) gluon-fragmentation jets (typically in three-jet events of e^+e^- reactions). Within each class, the pionization, and in particular the parameters $\langle k_T\rangle$ and C, are assumed to be independent of the quantum numbers (flavor, helicity,...) of the fragmenting object. This is in fact a property of the multiperipheral model,[2] the Regge-Mueller approach,[3] the cascade models,[4] and the string or color-tube models.[5–8] The question is whether or not the pionization is also class independent. If the answer is yes, we shall say that we have "jet universality," even if the fragmentation regions ($p_{\parallel} \gtrsim 0.2 p_{\rm jet}$) turn out to be different. A good introduction to this problem has been given by Kinoshita.[9] Several kinds of jet universality have been proposed:

(1) Naive jet universality (NJU). This kind of universality merely identifies the pionization regions of quark and hadron jets. Rejecting diffractive and large-p_T events in hadron-hadron collisions, and three-or-more-jet events in e^+e^- annihilation, one gets

$$C_{e^+e^-}=C_{hh} \tag{1.3}$$

and, for the asymptotic multiplicity at equivalent energy,

$$\langle n\rangle_{e^+e^-} \sim \langle n\rangle_{hh} \ . \tag{1.4}$$

(2) Topological jet universality (TJU). Dual quark-line diagrams for low-p_T hadronic reactions (Fig. 1) lead to the notion of "topological jets," or "sheets," "chains," "emitting quark lines," "strings," or simply "jets."[5,10–19] One can consider a topological jet as the decay product of a superheavy, elongated dual string[5] or "color-triplet flux tube." This is pictured in Fig. 2.

Dual quark-line diagrams, displaying topological jets, can also be drawn for current-induced reactions[20] (Fig. 3) and high-p_T hadronic reactions[21,22] (Fig. 4). TJU states that, in their respective center-of-mass frames, all the topological jets pionize in the same way (in particular, in rapidity plateaus of a universal height C and a universal intrinsic $\langle k_T\rangle$). Let us denote by $C_{hh}^{(1)}$ [$C_{hh}^{(2)}$] the plateau height due to all diagrams of type 1(a), i.e., the cut Reggeon [1(b), i.e., the cut Pomeron]. Ignoring multi-Pomeron cuts, which become important at CERN SPS collider energies ($\sqrt{s} \sim 500$ GeV) and are treated in Refs. 15(b),(c),(e),(f), we have first

$$C_{hh} \simeq C_{hh}^{(2)} = 2C_{hh}^{(1)} \ . \tag{1.5}$$

This first relation can in fact be derived in dual topological unitarity (DTU) in the multi-Regge limit.[23] TJU further gives[24]

$$C_{hh}^{(1)} = C_{e^+e^-} \equiv C_q \ , \tag{1.6}$$

whence, instead of (1.3),

$$C_{hh} \simeq 2C_{e^+e^-} \ . \tag{1.7}$$

It is frustrating that, when comparing the average

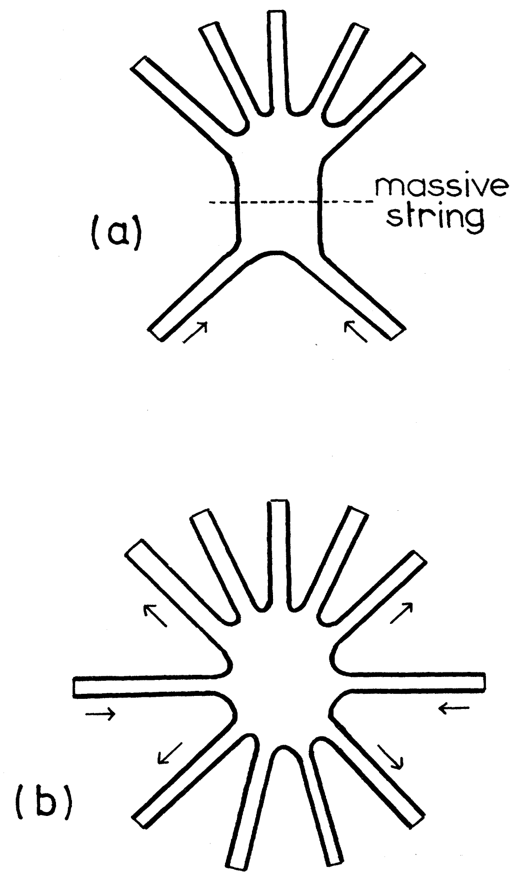

FIG. 1. Topological jets in meson-meson scattering. (a) Regge-resonance component of the Harari-Freund duality. (b) Pomeron-background component.

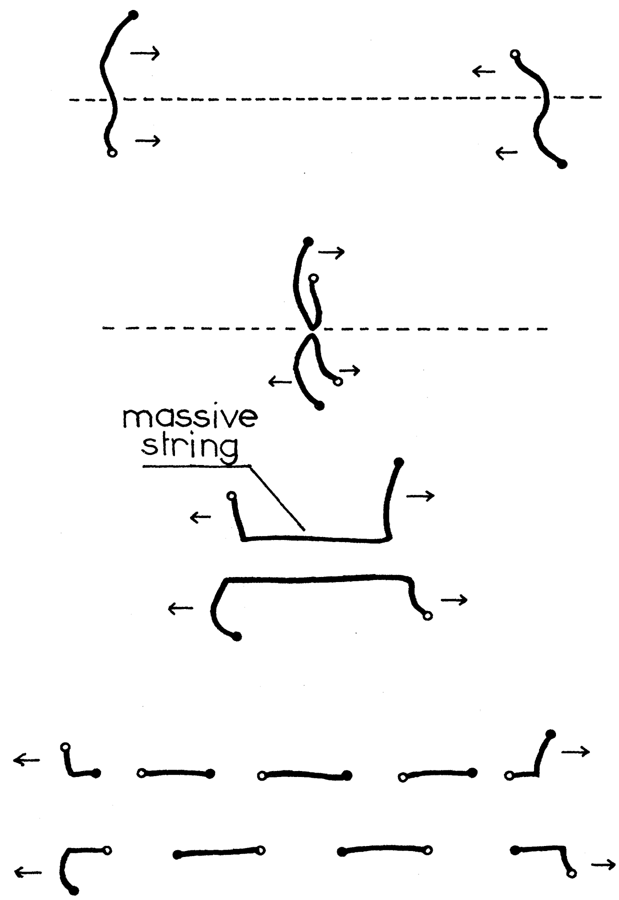

FIG. 2. String scenario corresponding to Fig. 1 (b).

charged multiplicities in e^+e^- and pp reactions, some analysis agrees with NJU,[25] while other ones agree with TJU.[15(a)–(e)] We shall come back to this question in Sec. III.

Quantum chromodynamics has motivated new pictures of the jets:

(1) The color bremsstrahlung model. Low-p_T inelastic hadron-hadron reactions are initiated by the exchange of one gluon between the target and the projectile,[26] which become octet-colored. We have then[27]

$$C_{hh}=C_{\text{gluon}}=2\left[1-\frac{1}{N_c^2}\right]^{-1}C_{\text{quark}}=\tfrac{9}{4}C_{\text{quark}} \ . \tag{1.8}$$

(2) The QCD branching process. This process[28], for high-q^2 current-induced reactions, leads to jets of partons apparently very different from the standard hadron jets. It predicts

$$\langle k_T\rangle_{e^+e^-}\sim as^{1/2}/\ln s \ , \tag{1.9}$$

$$\langle n\rangle_{e^+e^-}\sim ce^{b\sqrt{\ln s}} \ . \tag{1.10}$$

(3) The dual topological multijet picture. It is possible to reconcile the QCD branching process (and perhaps the color bremsstrahlung model) with standard jets: as a first step, the QCD branching process is at work in a region of space of size Q_0^{-1} somewhat smaller than the hadronic scale, until the partons have virtual mass squared $k^2\sim Q_0^2$. Then these partons fly apart, fragmenting in standard jets; this is the second step or hadronization. Equation (1.10) should therefore apply to the number of standard jets. This multijet picture is in fact implicitly assumed in the standard phenomenological analysis of e^+e^- reactions in two-jet events, three-jet events, etc. We must warn however, that this picture seems to be in contradiction with "preconfinement"[29] (color-singlet clusters of low mass should not *a priori* give standard jets).

The multijet picture can also be given a dual topology (see Fig. 5), having thus a string interpretation. This has been realized, semiclassically at least, by the Lund model for quark and gluon jets.[6] According to this model, a gluon is at a corner of a broken-line string (see Fig. 6). The fact that two strings are attached to the gluon makes the associated rapidity plateau (at large gluon energy) twice higher than for a quark jet:

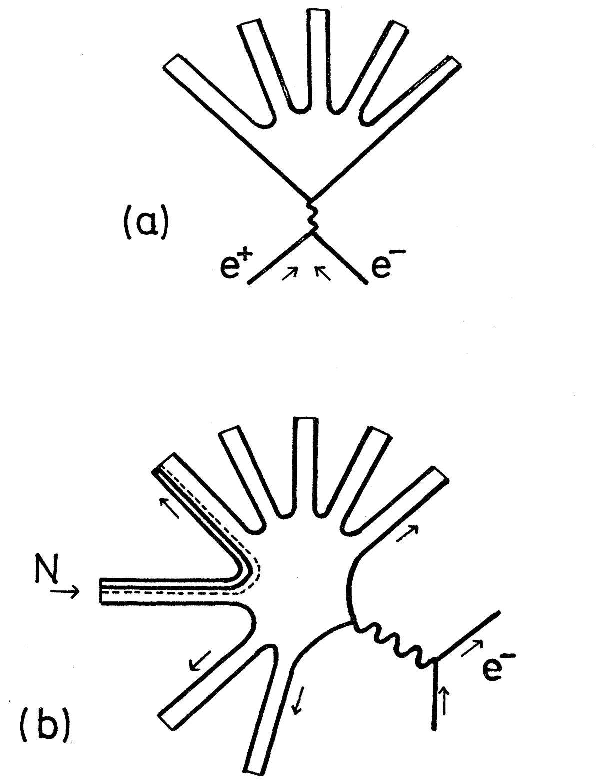

FIG. 3. Dual topological diagrams for (a) e^+e^- annihilation into hadrons and (b) deep-inelastic electron scattering (sea component).

$$C_g = 2C_q \ . \tag{1.11}$$

In fact, it is more convenient to associate jets with strings than with partons. An equivalent statement has been made in Refs. 30 and 31. The factor 2 above may be con-

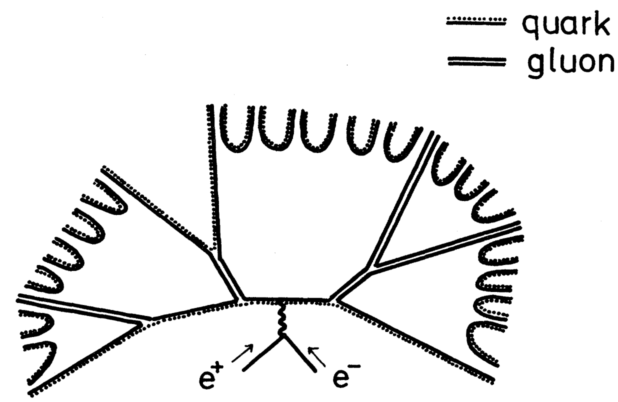

FIG. 5. Planar diagram for a "seven-jet event" in e^+e^- annihilation, in accordance with the Lund model. Soft gluons are not drawn.

sidered as the lowest-order term in a $1/N_c$ expansion of (1.8), and this may be related to having selected only planar QCD graphs (as Fig. 5).

Having this dual topological multijet picture in mind, we finally formulate TJU in the following form:

$$C_{hh} \simeq C_{hh}^{(2)} = 2C_{hh}^{(1)} \ , \tag{1.12a}$$

$$C_{hh}^{(1)} = C_q \ , \tag{1.12b}$$

$$C_g = 2C_q \ , \tag{1.12c}$$

ignoring possible $1/N_c^{\,2}$ corrections [(1.12a) again neglects multi-Pomeron cuts].

Among the above relations, (1.12b) is the most audacious, since it relates soft and hard collisions. In the next section, we shall criticize it from the theoretical point of view, and present a multiperipheral model which breaks TJU. We shall discuss the experimental situation in Sec.

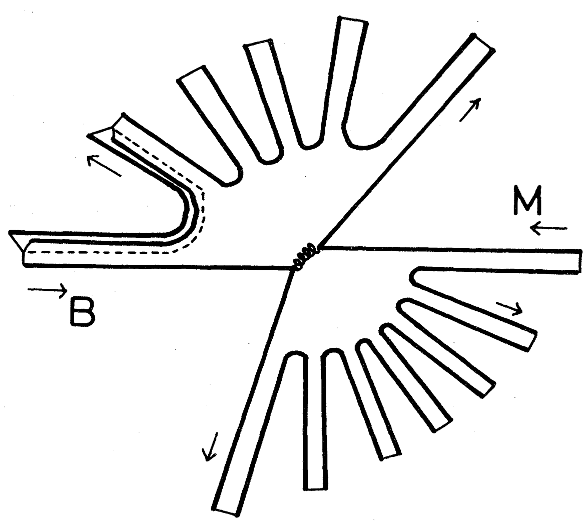

FIG. 4. Dual topological diagram for high-p_T meson-baryon scattering (valence-valence hard scattering).

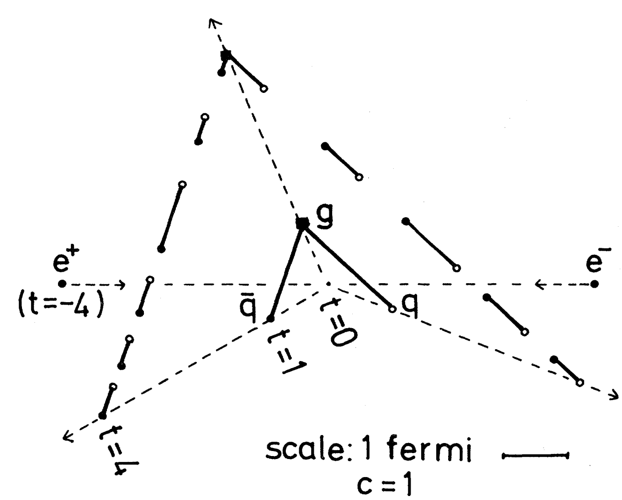

FIG. 6. Scenario for a three-jet event in e^+e^- annihilation according to the Lund model.

III. In Sec. IV, we propose a new test of TJU, based on local compensation of quantum numbers. Our considerations are summarized in Sec. V.

II. A THEORETICAL POINT OF VIEW

Let us compare the most representative topological hadron-fragmentation and quark-fragmentation jets represented in Figs. 1(a) and 3(a), respectively. We assume no large-k_T final particles. In these diagrams, an arbitrary large number of virtual soft gluons and quark loops is understood. Corresponding complete diagrams are shown in Figs. 7(a) and 7(b), which are planar, i.e., of lowest order in the $1/N_c$ expansion.[32] From these pictures, we see a qualitative difference between the two reactions. In meson-meson scattering, we can attach gluons to the bottom line (g_1 to g_5). These "vertical" gluons even play an important role in Reggeizing the "horizontal" meson exchanged in the multiperipheral picture. No such gluons are present in e^+e^- annihilation; in fact the black triangle of Fig. 7(b) represents a short-distance process; any gluon emitted from it would be "hard" (due to the uncertainty relation) and would therefore give a three-or-more-jet event.

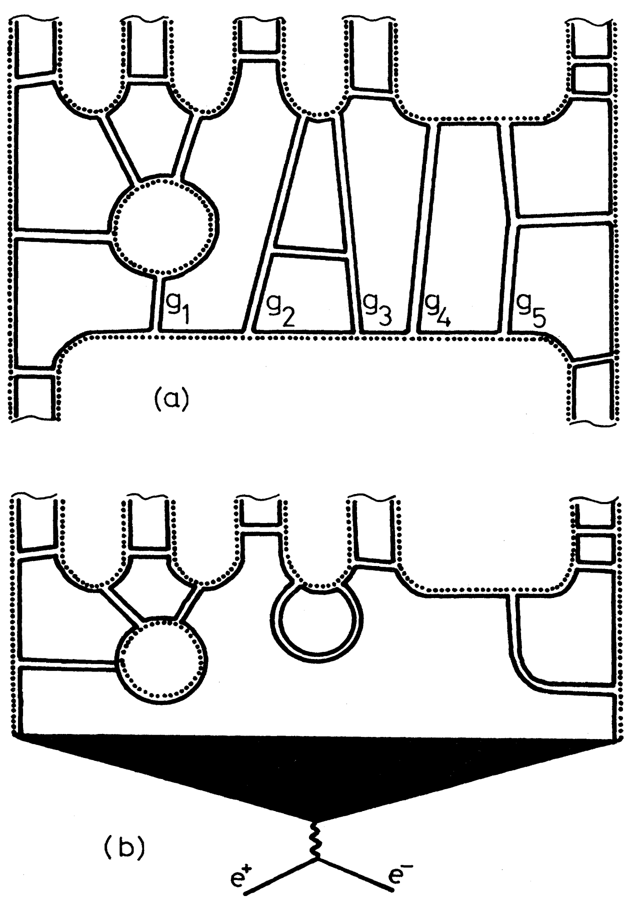

FIG. 7. Planar QCD diagrams corresponding to (a) Fig. 1(a); (b) Fig. 3(a), in the 't Hooft—Veneziano representation.

From this qualitative difference, we expect then the jets represented in Figs. 1(a) and 3(a) to be different, i.e., we are ready to give up TJU. In the following, we shall assume the existence of topological jets but give quantitative arguments in favor of a violation of TJU at least between quark and hadron jets.

It is widely accepted that soft hadron jets are generated by a multiperipheral mechanism; Figs. 1(a) and 1(b) can be deformed into multiperipheral configurations like Figs. 8(a) and 8(b). We have grouped some of the final particles in resonant "clusters"[33] and added soft "ladder" gluons to represent the binding of the quarks in the external or the exchanged particles.

As a first remark, they are ladder gluons connecting the upper and the lower topological jets in Fig. 8(b). Thus, contrary to an implicit assumption of TJU, the two jets are correlated (not only by global conservation of energy, momentum, and quantum numbers). In particular, the lo-

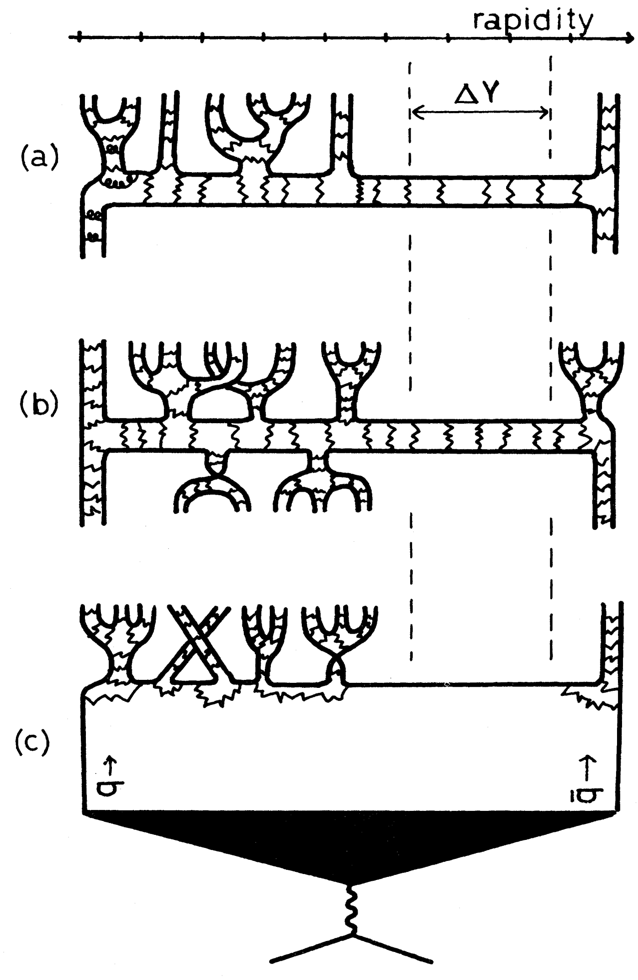

FIG. 8. Quark-duality diagrams in multiperipheral configurations: (a) hadron-hadron scattering, Regge-resonance component; (b) hadron-hadron scattering, Pomeron-background component; (c) e^+e^- annihilation (two-jet event). Ladder gluons are drawn. Resonant clusters and rapidity gaps are exhibited.

cal compensation (in rapidity space) of transverse momentum[34(a)] does not hold separately for the two jets. This is supported by the result of Ref. 18. The local correlation between the two jets is essentially a quantum-mechanical effect, whereas independent evolution of the two strings is a classical hypothesis. Accidentally, both quantum and classical models give the same prediction (1.5) for the multiplicities, and TJU *restricted to soft hadronic reactions* is probably a good approximation.

If one wants to give any physical meaning to (unrestricted) TJU, it seems necessary (but not sufficient) to explain the quark jets by a multiperipheral mechanism. For Fig. 3(a) [or 7(b)], one is naturally led[35] to the configuration Fig. 8(c) which describes a two-step process:

(a) $e^+e^- \to q\bar{q}$ (QED),

(b) $q\bar{q} \to$ hadrons (multiperipheral).

In this process, q and $\bar{q}$ are assumed to be not far off the mass shell, for we restrict ourselves to typical two-jet ($q\bar{q}$) events; if the quark were highly virtual, it would radiate one or more hard gluons before hadronizing, and this would result in a three-or-more jet event (as in Fig. 5). For the same reason, we discard gluon emission from the black triangle in Figs. 7–9. Nevertheless, the multiperipheral mechanism generalizes to multijet events as well; in Fig. 5, for instance, we should have five multiperipheral chains in one-to-one correspondence with the five string segments in the Lund picture [a connection between string decay and multiperipheral quark chain was noted in Refs. 5(a), Sec. 6.1]. The amplitude corresponding to processes (a) and (b) is, ignoring spin, color, and flavor indices,

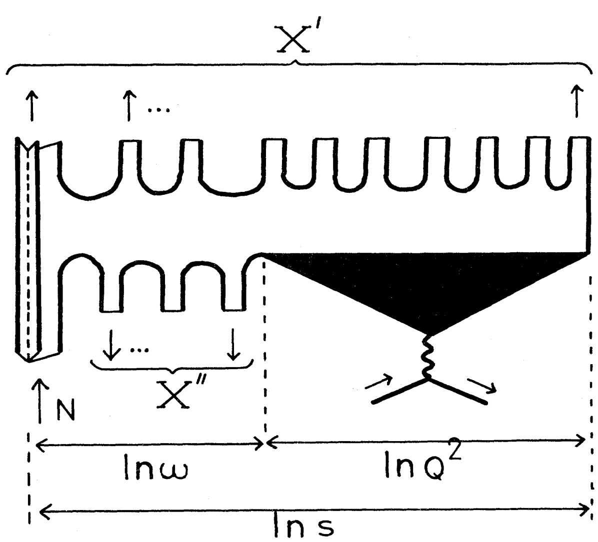

FIG. 9. Quark duality diagram for deep-inelastic electron scattering in a multiperipheral configuration. Cluster decays are not displayed.

$$T^{e^+e^-\to f}(p_{e^+},p_{e^-},p_1\cdots p_{n-1}) = \int d^4q D(-\bar{q}) T^{e^+e^-\to q\bar{q}}(p_{e^+},p_{e^-},q) D(q) \times T^{q\bar{q}\to f}(q,\bar{q},p_1\cdots p_{n-1}) . \tag{2.1}$$

$D(q)$ is the quark propagator. $T^{q\bar{q}\to f}$ is the multiperipheral quark$\to$hadron amplitude, the initial quarks being off the mass shell but not too far; this can be ensured by a softness of the first and the nth quark-hadron vertices. The same is true for the exchanged quarks in the multiperipheral chain, and this should be the origin of the jet structure of the final state (limited k_T, rapidity plateau,. . .). This model [Eq. (2.1)] is in fact too simple and leads to a logarithmic violation of asymptotic freedom and/or confinement (see Appendix). Nevertheless it might contain part of the truth and offers a (quantum mechanical) explanation of the qualitative similarities between hadron and quark jets. However, the different natures of the exchanged objects (Reggeon versus quark) are expected to induce quantitative differences:

(1) The probability of a rapidity gap. Let us consider an interval $[Y_1,Y_2]$ of the kinematical rapidity plateau of a jet, with $\Delta Y = Y_2 - Y_1 \gg 1$, and look for the probability $\mathcal{P}(Y_1,Y_2)$ to have no particle in $[Y_1,Y_2]$. Multiperipheral models lead to translational invariance in rapidity space inside the plateau, so we have

$$\mathcal{P}(Y_1,Y_2)=\mathcal{P}(\Delta Y) . \tag{2.2}$$

For hadron jets [Figs. 8(a) and 8(b)] the multi-Regge model gives[36]

$$\mathcal{P} \sim e^{-\gamma_h \Delta Y} \tag{2.3}$$

with

$$\gamma_h = 1 - 2\alpha_{\text{in}}(0) + \alpha_{\text{out}}(0) . \tag{2.4}$$

The "input trajectory" α_{in} is that exchanged between adjacent vertices and the "output trajectory" α_{out} governs the total cross section. Planar unitarity ($\alpha_{\text{in}}=\alpha_{\text{out}}=\alpha_M \simeq 0.5$) gives

$$\gamma_h^{(1)} \simeq 0.5 \quad [\text{Fig. 8(a)}] \tag{2.5a}$$

and cylinder unitarity ($\alpha_{\text{in}}=\alpha_M$, $\alpha_{\text{out}}=\alpha_{\text{Pomeron}}=1$) gives

$$\gamma_h^{(2)} \simeq 1 \quad [\text{Fig. 8(b)}] . \tag{2.5b}$$

For a multiperipheral quark jet [Fig. 8(c)], one has, by a derivation analogous to that of (2.3) and (2.4) (see Appendix),

$$\mathcal{P} \sim e^{-\gamma_q \Delta Y} \tag{2.6}$$

with

$$\gamma_q = 2(1 - \text{spin of the quark}) = 1 . \tag{2.7}$$

(2.7) is obtained from (2.4) by the substitution $\gamma_h \to \gamma_q$, $\alpha_{\text{in}}(0) \to$ spin of the quark, and setting

$$\alpha_{\text{out}}(0) = 1 . \tag{2.8}$$

This last relation expresses the fact that the quarks hadronize with probability one. (It was already given in a parton dual resonance model[37] and in the massive-quark model.[19(a)] In our model we do not Reggeize the quarks and we do not relate α_{out} to the usual Pomeron, since it comes from *planar* unitarity in $q\bar{q}$ scattering).

In hadron-hadron collision, (2.3) and (2.4) are still valid for Y_1 or Y_2 or both at their kinematical limits. In the case $Y_2=Y_{max}$ for instance, we have an isolated large-z final particle such that

$$\Delta Y \simeq \ln\frac{1}{1-z} \quad \left[z \equiv \frac{(E+r_{\|})_{\text{particle}}}{(E+r_{\|})_{\text{jet}}} \right] , \tag{2.8}$$

and (2.3) and (2.4) results from the triple-Regge formula. For a quark jet, $\mathscr{P}(Y_1, Y_{max})$ is related to the threshold behavior of the fragmentation function:

$$\mathscr{P}(Y_1,Y_{max}) \sim \int_{1-\epsilon}^{1} D(z)dz \ , \quad \epsilon=e^{-\Delta Y} . \tag{2.9}$$

Then

$$D(z) \sim (1-z)^{\gamma_{qF}-1} \Longleftrightarrow \mathscr{P}(Y_1,Y_{max}) \sim e^{-\gamma_{qF}\Delta Y} . \tag{2.10}$$

This is connected with (2.6) if

$$\gamma_q = \gamma_{qF} . \tag{2.11}$$

The dimensional-counting rules give[38]

$$\gamma_{qF}=2 , \tag{2.12}$$

which differs from our γ_q (2.7). On the other hand, we have proposed,[39] as an alternative to the dimensional-counting rules, a model where γ_{qF} is given by (2.7). In order to draw the less model-dependent conclusion, we consider equally the three plausible possibilities

$$\begin{aligned} &(\mathrm{A})\ \gamma_q=\gamma_{qF}=2 \ \ \text{(dimensional-counting rules)}\\ &(\mathrm{B})\ \gamma_q=\gamma_{qF}=1 \ \ \text{(model of Ref. 39)}\\ &(\mathrm{C})\ 1=\gamma_q\neq\gamma_{qF}=2 \ \ \text{(no connection between the central plateau and the end of the spectrum)} . \end{aligned} \tag{2.13}$$

In any of these cases, we have $\gamma_q > \gamma_h^{(1)}$ [given by 2.5(a)], and TJU is not valid concerning the rapidity-gap distributions. [Independently of the assumptions (A), (B), or (C), we also stress that, in the vicinity of $z=1$, TJU is in contradiction with the standard triple-Regge behavior for nondiffractive reactions such as $\pi^- p \to \pi^0 X$, $pp \to nX$. The latter predicts a flat spectrum (the Reggeon-Reggeon-Pomeron term):

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dz} \underset{z\to 1}{\sim} (1-z)^{\alpha_{\text{Pomeron}}(0)-2\alpha_M(0)} \sim \text{constant} ,$$

while the rules of Refs. 15(a) (second work), 15(b), 15(c), 16(b) for TJU give

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dz} \underset{z\to 1}{\sim} \int_z^1 \frac{dx}{x}(1-x)^{-\alpha_M(0)} D\left[\frac{z}{x}\right]$$

$$\sim (1-z)^{\gamma_{qF}-1/2} .$$

The compatibility between the two formulas would require $\gamma_{qF} \simeq \frac{1}{2}$, i.e.,

$$\frac{d\sigma}{dz}(e^+e^- \to \text{hadron}+X) \propto (1-z)^{-1/2} ,$$

which is ruled out experimentally. This fact is known, but not often mentioned in the literature. The contribution of our work is to generalize the argument to the central region.]

(2) The heights of the plateaus. In the multi-Regge limit (weak coupling; small density) of multiperipheral models, the clusters form an uncorrelated gas[40] and we have

$$\mathscr{P}\ (\text{rapidity gap}) \sim e^{-\gamma\Delta Y} \Longleftrightarrow \rho_{cl} \equiv \frac{dN_{\text{cluster}}}{dY} = \gamma . \tag{2.14}$$

At large cluster density (i.e., small invariant energy of two adjacent clusters), threshold or t_{min} effects act as repulsive forces. For hadron-hadron collisions, (2.14) gives $\rho_{cl}=\gamma_h^{(2)}=1$, which is consistent with standard cluster analysis,[41] so we expect the corrections to be not too large. Anyway, we can assume that ρ_{cl} is a monotonic function of γ such that

$$\gamma < \gamma' \Longrightarrow \rho_{cl}(\gamma) < \rho_{cl}(\gamma') \le \frac{\gamma'}{\gamma}\rho_{cl}(\gamma) . \tag{2.15}$$

From (2.15) on, the equality option of $\le$ will refer to the weak coupling limit. Next we assume a universality of cluster decay, in particular of the mean number $\bar{k}$ of particles per cluster. Then

$$\frac{dN_{\text{part}}}{dY} \equiv C = \bar{k}\rho_{cl} \quad (\bar{k} \text{ universal } \sim 3) . \tag{2.16}$$

Gathering (2.5), (2.13), (2.15), and (2.16), we get

$$0.5 \le C_{hh}^{(1)}/C_{hh}^{(2)} < 1 , \tag{2.17}$$

$$1 < C_q/C_{hh}^{(2)} \le 2 \quad (\text{case A}) \tag{2.18a}$$

or

$$C_q = C_{hh}^{(2)} \quad (\text{case B or C}) . \tag{2.18b}$$

The weak-coupling + dimensional-counting rule result $C_q=2C_{hh}^{(2)}$ has already been given by Kinoshita (Ref. 10, Sec. 4.2). For its part, our multiperipheral model gives the same result $C_q=C_{hh}$ as NJU. This is related to the fact that the quark and the Reggeon have the same intercept $\frac{1}{2}$. But this is accidental and we may expect that NJU is violated in more detailed properties. For instance, a set of adjacent clusters can have a total charge $0, \pm 1$ in a quark jet and 0, ± 1, ± 2 in a two-sheet hadron jet. Thus local compensation of quantum numbers[34(b)] (LCQN) should be stronger in e^+e^- than in pp collisions. To our knowledge, this has not yet been investigated. (We shall use extensively LCQN in Sec. IV.)

In any of the cases A, B, and C, we have $C_{hh}^{(1)} < C_q$, which is a violation of TJU. In the color-tube model itself (which should *a priori* lead to TJU), this inequality has been argued for as a consequence of different diameters of the color tubes.[42]

III. EXPERIMENTAL SITUATION

As we have said in the Introduction, the comparison of $\langle n_{\rm ch}\rangle$ in pp and e^+e^- reactions seems unable to decide between NJU and TJU. It must be said that the e^+e^-/pp comparison is not straightforward; for instance:

(1) In the "naive" comparison, one removes one[25(a)] or two[25(b),(c),(d)] "leading protons" from the final state, according as one considers one or both hemispheres.

(2) In the two-topological-jet picture,[15] one has to take into account the mass distribution of the two massive strings, and to estimate the corrections due to multi-Pomeron cuts.[15(b),(c),(e),(f)]

The fact that the second approach does not predict the pp multiplicities to be markedly larger than the e^+e^- ones (up to CERN ISR energy) can be explained by two effects:

(i) the rapidity plateaus of the two topological jets overlap little, at ISR energies.

(ii) the height of the e^+e^- plateau rises with energy or equivalently, the e^+e^- multiplicity increases faster than $\ln s$ (this fact shows the difficulty in defining C_q).

Another significant quantity is the dispersion

$$D=[\langle n_{\rm ch}{}^2\rangle-\langle n_{\rm ch}\rangle]^{1/2} .$$

Taking $D^{e^+e^-}$ ($\sim 0.36\langle n_{\rm ch}\rangle^{e^+e^-}$) or D^{e^-p} as input, TJU explains roughly the observed $D^{pp}\sim 0.55\langle n_{\rm ch}\rangle^{pp}$ [Refs. 15(b),(d),(e)]. But the naive comparison seems to work also[25(b)] [see however Ref. 25(f)] where the opposite opinion is sustained].

Deep-inelastic lepton scattering is a reaction where we can compare NJU and TJU with no theoretical bias and the minimum experimental ones. At large values of the Bjorken variable $\omega=-2p\cdot q/q^2$, one can divide the whole rapidity phase space in two adjacent "plateaus" (see Fig. 9):[43] The "hadronic plateau" of length $\ln\omega$, populated by a hadron fragmentation jet, and the "current plateau" of length $\ln|q^2|$ populated by the fragmentation jet of the struck quark. The total length of the two plateaus is

$$\ln\omega+\ln|q^2|\simeq\ln s . \tag{3.1}$$

By varying q^2 (i.e., ω) at fixed s, one only changes the relative lengths of the hadronic and current plateaus. NJU then implies that *the average multiplicity is a function of s only* (no q^2 dependence).[43(b)] On the contrary, TJU predicts an *increase* of $\langle n_{\rm ch}\rangle$ when $\omega\to\infty$;[44] this increase is entirely in the topological jet X'' of Fig. 9. (At large ω, the struck quark is most probably a sea quark, i.e., X'' is not empty.) Present experiment, however, strongly suggests q^2 independence of $\langle n_{\rm ch}\rangle$,[45] as predicted by NJU.

In Ref. 11(c), this independence was interpreted as a temporary effect due to the necessity of sharing the energy between the two chains in the sea events. Now we have data at much higher energies ($s\sim 10^2$ GeV^2); Allen *et al.*,[45] for instance, report measurements of the multiplicities in neutrino collisions at ω as large as 30. Let us estimate the increase of the charged multiplicity expected from TJU between, let us say, $\omega=3$ (valence alone) and $\omega=30$. In the latter case the sea contribution is as large as the valence one (see, for instance, the parametrizations of Barger and Phillips[46]). Let us assume that, in sea events, X'' contains at least one typical cluster $\pi^+\pi^-\pi^0$:

$$\langle n_{\rm ch}(X'')\rangle_{\rm sea}\gtrsim 2 . \tag{3.2}$$

In TJU, this contribution to $\langle n_{\rm ch}\rangle$ is partly compensated by a reduction $-\delta n'$ due to a lower energy available for the upper jet X' in Fig. 9. Two cases are to be considered according as we have a sea quark or a sea antiquark:

(a) Sea-quark events ($\nu d\to\mu u$). The final baryon is in the upper jet X', as in Fig. 9. X'' is likely much slower, in the center-of-mass frame, than this final baryon (leading-particle effect), so that the reduction of $m_{X'}{}^2$ is much less than 50%. Taking the fit of Allen *et al.*,

$$\langle n_{\rm ch}\rangle=1.33\ln s+0.37 \tag{3.3}$$

and applying it to X', we get

$$\delta n'=\langle n_{\rm ch}(X')\rangle_{\rm valence}-\langle n_{\rm ch}(X')\rangle_{\rm sea} \ll 1.33\ln 2=0.9 . \tag{3.4}$$

(b) Sea-antiquark events ($\nu\bar u\to\mu^- d$). The leading baryon is now in X''. On one hand, X' has a smaller mass than in sea-quark events, but on the other hand it is a purely mesonic jet, so it has a larger multiplicity than a baryonic jet of same mass. These two effects cancel more or less (both are due to the leading-baryon effect), and (3.4) should still hold.

Subtracting (3.4) and (3.2), and taking equal weights for the sea and the valence at $\omega=30$, we get

$$\langle n_{\rm ch}\rangle_{\omega\sim 30}-\langle n_{\rm ch}\rangle_{\omega\sim 3}\gtrsim 0.5-1 . \tag{3.5}$$

This is the prediction of TJU in the experiment of Allen *et al.*[45] Instead, the experimental curve $\langle n_{\rm ch}\rangle$ versus Q^2 is flat, within error bars of $\pm 0{,}3$. This was the prediction of NJU or our multiperipheral model [cf. Eq. (2.18b)]. Due to the theoretical and experimental uncertainties, we can only say that deep-inelastic lepton scattering *favors* NJU or $C_{hh}^{(2)}=C_q$ over TJU ($C_{hh}^{(2)}=2C_q$). To get a more decisive answer, it would be helpful to make experiments at larger ω, for instance, $\omega\sim 100$, with the same precision on $\langle n_{\rm ch}\rangle$. This would add typically one more cluster in X'', without changing $\delta n'$.

IV. PROPOSAL FOR A MORE ELABORATE TEST OF TJU

We have seen (Sec. III) that the major uncertainty in the TJU test is about the number ν of topological jets which contribute to dN/dY (i.e., which overlap) in the observed rapidity interval. This number ν can be estimated by looking at the local compensation of quantum numbers (LCQN)[34(b)] or at the distribution in charge transfer.[47] The charge transfer at fixed rapidity Y is

$$Q(Y)=\sum_{Y_i<Y} Q_i-Q_{\text{left-moving incoming particle}} , \tag{4.1}$$

where Q_i is the charge of the ith outgoing hadron. LCQN states that $\langle Q(Y)\rangle$ is constant in the central region and that $Q(Y)$ has short-range fluctuations: $Q(Y+\Delta)$ and $Q(Y)$ are uncorrelated for $\Delta \gtrsim 2$.

A. Hadron-hadron reactions

In low-p_T hadron-hadron collisions,

$$Q(Y)=\sum_{j=1}^{\nu} q_j(Y) , \tag{4.2}$$

where q_j is the contribution of the jth topological jet; it is the charge of the emitting quark or antiquark, plus possibly some emitted hadrons, which cross the rapidity Y from the left to the right (see Fig. 8). Equation (4.2) includes the case of a cut Reggeon [e.g., Fig. 8(a)] as a particular configuration of the cut Pomeron, i.e., that where one of the jets is empty.

At fixed Y, the hypothesis that topological jets decay independently gives

$$D_Q{}^2(Y) \equiv \langle Q(Y)^2\rangle - \langle Q(Y)\rangle^2 = \bar{\nu}(Y)a , \tag{4.3}$$

where $\bar{\nu}$ is the average number of chains ($\sim$twice the average number of cut Pomerons) overlapping at rapidity Y,

$$\bar{\nu}(Y)=(dN/dY)/C_{hh}^{(1)} , \tag{4.4}$$

and a the fluctuation for one chain, which is *a priori* a constant, like $C_{hh}^{(1)}$:

$$a=\langle q(Y)^2\rangle - \langle q(Y)\rangle^2 \,|_{\text{one topological jet}} . \tag{4.5}$$

From (4.3) and (4.4), TJU first predicts that

$$B_{hh}=D_Q{}^2(Y)/(dN/dY)=a/C_{hh}^{(1)} \tag{4.6}$$

is independent of energy.[48] (This was also predicted by the multiperipheral *neutral-cluster* model.[49]) This agrees well with experiments up to 207 GeV/c,[50] with

$$B_{hh}^{(\text{charged particle})} \simeq 0.72-0.85 . \tag{4.7}$$

It may be difficult to measure the signs of the charges of very fast particles to determine Q. But we can restrict ourselves to a rapidity interval $I=[Y-\delta, Y+\delta]$, and measure the partial charge

$$Q'=\sum_{Y_i \in I} Q_i = Q(Y+\delta)-Q(Y-\delta) . \tag{4.8}$$

For 2δ greater than the correlation length (~ 2), LCQN gives

$$D_{Q'}{}^2 \equiv \langle Q'^2\rangle - \langle Q'\rangle^2 \underset{\delta>1}{\simeq} D_Q{}^2(Y-\delta)+D_Q{}^2(Y+\delta) \simeq 2D_Q{}^2(Y) . \tag{4.9}$$

B. Lepton-induced reactions

In a quark jet [Figs. 8(c) or 9] we have, instead of (4.2)

$$Q(Y)=q_1(Y)-q_0 , \tag{4.10}$$

where q_1 is defined as in (4.2) ($\nu=1$) and q_0 is the charge of the fragmenting quark or antiquark. In charged-current neutrino reactions on a valence quark, q_0 is fixed and we have

$$D_Q{}^2=a\,(\nu N, \bar{\nu} N \text{ reactions}) . \tag{4.11}$$

In other cases, q_0 takes the values $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{2}{3}$, and $+\frac{1}{3}$ with probabilities $p\alpha$, $p(1-\alpha)$, $\bar{p}\bar{\alpha}$, and $\bar{p}(1-\bar{\alpha})$, respectively (p for quark, $\bar{p}$ for antiquark; $p+\bar{p}=1$). Let p_u the probability to "create a $u\bar{u}$ quark pair in the color tube" or to "find the emitting quark line, at a given Y, in the state u"[51]

$$\langle q_1(Y)\rangle_{\text{quark jet}} = -\langle q_1(Y)\rangle_{\text{antiquark jet}} = p_u - \tfrac{1}{3} , \tag{4.12}$$

$$p_u \simeq 0.41-0.48 . \tag{4.13}$$

In the general case, one finds

$$D_Q{}^2(Y)=a+p\alpha(1-\alpha)+\bar{p}\bar{\alpha}(1-\bar{\alpha})+p\bar{p}(2p_u-\alpha-\bar{\alpha})^2 \tag{4.14}$$

[a is defined by (4.5)]. In deep-inelastic electron or muon scattering on a valence quark ($\bar{p}=0$), we have

$$D_Q{}^2=a+\frac{4u(x)d(x)}{[4u(x)+d(x)]^2} \simeq \begin{cases} a+\frac{8}{81} & (e^-p, \mu p) \\ a+\frac{2}{9} & (e^-n, \mu n) \end{cases} . \tag{4.15}$$

In e^+e^- reactions (two-jet events), (4.14) gives

$$D_Q{}^2=\left\{\begin{array}{l} a+\frac{2}{9}+(p_u-\frac{2}{3})^2 \text{ below} J/\psi \\ a+\frac{4}{25}+(p_u-\frac{4}{5})^2 \text{ between } J/\psi \text{ and } \Upsilon \text{, and above } t\bar{t} \\ a+\frac{24}{121}+(p_u-\frac{8}{11})^2 \text{ between } \Upsilon \text{ and } t\bar{t} \end{array}\right\} \simeq a+0.3 . \tag{4.16}$$

As in hadron-hadron reactions [Eq. (4.6)], we define

$$B_{\text{cur}}=a_{\text{cur}}/C_{\text{cur}}=a_{\text{cur}}/(dN/dY)_{\text{cur}} , \tag{4.17}$$

where a_{cur} is measured from (4.11), (4.15), or (4.16).

C. Comparison between hadron-hadron and current-induced reactions

A $\bar{\nu}$-independent test of TJU lies in the ratio

$$r=\frac{B_{\text{cur}}}{B_{hh}} \quad (=1 \text{ for TJU}) . \tag{4.18}$$

If TJU is violated, it may mean not only $C^{(1)}_{hh}\neq C_q$ but also $a_{hh}\neq a_{\text{cur}}$, so it does not imply necessarily $r\neq 1$. In fact, we can decompose a into two terms

$$a=a^{(0)}+a' \tag{4.19}$$

$a^{(0)}$ is the strong-ordering limit (see Ref. 48),

$$a^{(0)}=p_u(1-p_u)\simeq\tfrac{1}{4} , \tag{4.20}$$

and is (nearly) universal. a' is due to overlapping clusters [e.g., in the upper jet of Fig. 8(b)], twisting clusters [lower jet of 8(b)] and possible other mechanisms. a' is likely to be proportional to the cluster density per chain (like $D_Q{}^2$ in the neutral cluster model[49]), so that if $a^{(0)}$ were zero, r would be unity, whether or not TJU is true.

Our multiperipheral model, in the weak-coupling (low-density) limit, gives $a'=0$, $a_{hh}\simeq a_{\text{cur}}$, and from (2.17) (with $\leq\rightarrow=$), (2.18b), (4.6), (4.17), and (4.18)

$$r=\tfrac{1}{2} . \tag{4.21}$$

Let us try to calculate r from the present published data. For B_{hh}, we have the result (4.7). In neutrino reactions, Grässler *et al.*[45] give

$$\begin{aligned} a(\bar{\nu}p)&=0.8-1.0 ,\\ a(\nu p)&=1.0-1.2 . \end{aligned} \tag{4.22}$$

Under the assumption $C=d\langle N_{\text{ch}}\rangle/d\ln s$, the same experiment implies

$$C_{\nu p}=C_{\bar{\nu}p}\simeq 1.43 \tag{4.23a}$$

but a direct measurement[52] gives

$$C_{\bar{\nu}p}=1.8 \quad (4<W<12 \text{ GeV}) . \tag{4.23b}$$

Thus

$$\begin{aligned} \frac{0.8}{1.8}&=0.45\lesssim B_{\bar{\nu}p}\lesssim\frac{1.0}{1.43}=0.7 ,\\ \frac{1.0}{1.8}&=0.56\lesssim B_{\nu p}\lesssim\frac{1.2}{1.43}=0.84 . \end{aligned} \tag{4.24}$$

In e^+e^- reactions, Berger *et al.*[53] plot the distribution in $|Q|_{\text{jet}}$ at $W=30$ GeV, from which we deduce

$$D_Q{}^2\simeq 2.1\rightarrow a(e^+e^-)\simeq 1.8 . \tag{4.25}$$

The height of the plateau is about 2.5, thus

$$B_{e^+e^-}\simeq 0.72 . \tag{4.26}$$

The large discrepancy between (4.22) and (4.25) may be due to gluon jets which increase $D_Q{}^2$ (and C) in e^+e^- reactions.

Thus, $r<1$ is favored, but additional data, including ones from pp and $p\bar{p}$ colliders, are necessary to conclude.

V. SUMMARY AND CONCLUSIONS

We have considered two possible schemes of jet universality, the "naive" one (NJU) and the "topological" one (TJU). Restricting ourselves to the pionization region, we found theoretical objections for both. On one hand, the local compensation of quantum numbers should be stronger ($D_Q{}^2$ smaller) in a quark jet than in a hadron jet, in violation of NJU. On the other hand, TJU is based on the independent decays of semiclassical color tubes or strings, and does not take into account important quantum effects, such as the exchange of soft virtual "ladder" gluons between two quark lines in a hadron jet. If one assumes that all jets are generated by multiperipheral mechanisms, one is led to predict smaller rapidity gaps and a larger density in quark jets than in soft topological jets; this violates TJU, and NJU could be accidentally true if only rapidity densities are considered.

Concerning the experimental situation, the compared pp and e^+e^- multiplicities are interpreted here as a success of NJU and there as a success of TJU. Thus we looked at deep-inelastic lepton scattering, where TJU predicts an increase of $\langle n_{\text{ch}}\rangle$ when $Q^2\rightarrow 0$ at fixed s. Present experiments show no significant variation in Q^2, in accordance with NJU or our multiperipheral model. We have estimated—with very crude assumptions—what the increase should be under the TJU hypothesis in one of these experiments (Allen *et al.*[45]), and found it could have been detectable. However, the theoretical and experimental uncertainties are still a little too large, and it would be helpful to have equally precise measurements of $\langle n_{\text{ch}}\rangle$ at larger ω to settle this important question.

Finally, we have given the general lines of a new test of TJU, which measures the effective number of topological jets by means of the fluctuations of charge transfer or of local compensation of charge.

Additional remarks. QED in one+one dimension (QED$_2$) has been applied to quark jets.[54] It gives a coherent state of mesons with a rapidity density equal to one, therefore satisfying (2.6)—(2.7). QCD$_2$, to lowest order in $1/N_c$, gives jet universality.[55] However it is pointed out[55(b)] that jet universality is most likely broken in $3+1$ dimensions. On the other hand the quarks are "Reggeized" in QCD$_2$,[55(b)] and our γ_q defined by (2.6) is related to β_q of Ref. 55 by $\gamma_q=1+2\beta_q$. This property, which is due to the linear potential, might survive in $3+1$ dimensions. For massless quarks, one has $\beta_q=0$ and (2.7) should still be valid.

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APPENDIX: RAPIDITY-GAP DISTRIBUTION IN A QUARK JET. DERIVATION OF FORMULAS (2.6) AND (2.7)

We consider the reaction $e^+e^-\rightarrow$hadrons and want to calculate, in our multiperipheral model (Fig. 8), the probability $\mathscr{P}$ to have no particle in a rapidity range $[Y_1, Y_2]$. This defines a subset $h\subset H$ of possible final hadronic states, and we have

$$\mathscr{P}=\sigma_h/\sigma_H\ , \tag{A1}$$

where σ_h and σ_H are partial and total hadronic cross sections, modulo the flux factor:

$$\sigma_h \equiv \sum_{n\in h} |T^{e^+e^-\to n}|^2 \text{ (similarly for } \sigma_H) \ . \tag{A2}$$

From Eq. (2.1), we have, still ignoring quark indices,

$$\sigma_h=\int d^4q\, D(q)D(-\bar q)\int d^4q' D^*(q')D^*(-\bar q')[T^{e^+e^-\to q\bar q}T^{*e^+e^-\to q'\bar q'}]\sum_{n\in h} T^{q\bar q\to n}T^{*q'\bar q'\to n} \tag{A3}$$

with q, $\bar q$, q' and $\bar q'$ not far from the mass shell. $\bar q$ and $\bar q'$ are fixed by energy-momentum conservation. The main contribution comes from jetlike n states, which imply small angle between $\vec q$ and $\vec q\,'$. Let us define

$$\sum_{n\in h} T^{q\bar q\to n}T^{*q'\bar q'\to n}\equiv I_h(q^2,q'^2,\bar q^2,\bar q'^2,s,t)\ , \tag{A4}$$

$$s\equiv(q+\bar q)^2\ ,\ t\equiv(q-q')^2\ . \tag{A5}$$

In particular, for $h=H$, we have

$$I_H=\mathrm{Im}T^{q\bar q\to q'\bar q'}\ . \tag{A6}$$

For $q_\mu=q'_\mu$, I_h/I_H is the gap probability in the collision of off-mass-shell quarks, and we may expect a result analogous to (2.3) and (2.4) with

$$\begin{aligned}&\gamma_h \Longrightarrow \gamma_q\ ,\\ &\alpha_{\rm in}^{(0)} \Longrightarrow S_q=\tfrac{1}{2}\ \text{(spin of the quark)}\end{aligned} \tag{A7}$$

[$\alpha_{\rm out}^{(0)}$ has no obvious counterpart for the moment]. Thus we are led to generalize (2.3) and (2.4) to the intermediate states of quark-antiquark unitarity at $t\neq 0$. For this purpose, we copy the calculation of Reggeon loops [see, for instance, Ref. 33 (first work)].

The unitarity diagram with a rapidity gap is given in Fig. 10. We have, adapting formula (5) and (32) of Ref. 33 (first work),

$$\begin{aligned}I_h=\int d^4k\,\gamma(q^2,q'^2,t)(s_1s_2)^{\alpha(t)}\\ \times[V(t,k^2,k'^2)]^2\left[\frac{s}{s_1s_2}\right]^{2S_q}\\ \times\gamma(\bar q'^2,\bar q^2,t)\theta(s_1^{\max}-s_1)\theta(s_2^{\max}-s_2)\end{aligned} \tag{A8}$$

with

$$\begin{aligned}&s_1=(q-k)^2\simeq -q^-k^+\ ,\\ &s_2(k+\bar q)^2\simeq \bar q^+k^-\ ,\\ &k^2\simeq -\vec k_T^{\,2}\ ,\\ &k'^2=(q'+k-q)^2\simeq-(\vec k_T+\vec q\,'_T)^2\ ,\\ &t\simeq -\vec q\,'^2_T\ ,\\ &\ln s_1^{\max}\simeq Y_1+\ln\sqrt{s}\ ,\\ &\ln s_2^{\max}\simeq \ln\sqrt{s}-Y_2\ .\end{aligned} \tag{A9}$$

We have chosen the center-of-mass frame with $\vec q_T=0$, $q^3\simeq-\frac{1}{2}\sqrt{s}$, and used the lightlike variables $k^\pm=k^0\pm k^3$. $\alpha(t)$ is the "Pomeron" of $q\bar q$ scattering[37,19(a)] to be given later. Integrating (A8) on

$$d^4k=\tfrac{1}{2}d^2\vec k_T dk^+dk^-\simeq\frac{1}{2s}d^2\vec k_T ds_1 ds_2\ , \tag{A10}$$

we get

$$I_h\sim\beta(q^2,q'^2,\bar q^2,\bar q'^2,t)s^{\alpha(t)}e^{-\Delta Y[\alpha(t)-2S_q+1]}\ . \tag{A11}$$

(A11) applies equally to $\Delta Y=0$ ($h=H$).

In (A3), we can approximate the square brackets by $|T^{e^+e^-\to q\bar q}_{\text{on shell, Born}}|^2$, i.e., we evaluate T for the pair of on-shell quark and antiquark having the same total four-momentum and the same axis as the virtual one, and we ignore QCD corrections (Sudakov form factor, collinear gluon emission). We have also

$$d^4q\,d^4q'\simeq(\tfrac{1}{8}dq^2d\bar q^2d\Omega_q)(\tfrac{1}{2s}dq'^2\bar q'^2d^2\vec q\,'_T)\ , \tag{A12}$$

so that we rewrite (A3) as

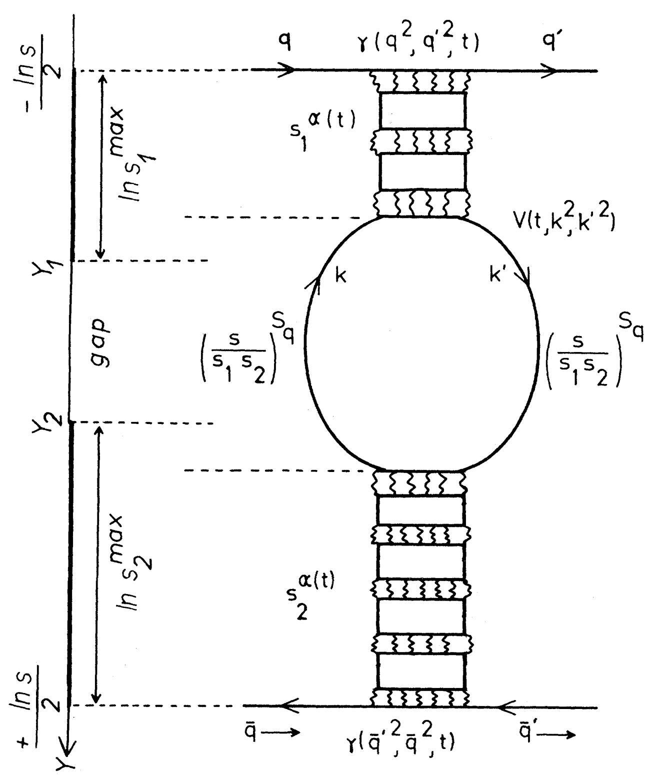

FIG. 10. Unitarity diagram with a rapidity gap in the intermediate state, in quark-antiquark hadronization.

$$\sigma_h = \frac{1}{16s}\int d\Omega_q \,|\, T^{e^+e^-\to q\bar{q}}_{\text{on shell, Born}} \,|^2 \int dq^2 d\bar{q}^2 D(q) D(-\bar{q}) \int dq'^2 d\bar{q}'^2 d^2\vec{q}\,'_T D^*(q') D^*(-\bar{q}') \beta(q^2, q'^2, \bar{q}^2, \bar{q}'^2, t) \times s^{\alpha(t)} e^{-\Delta Y[\alpha(t) - 2s_q + 1]} , \tag{A13}$$

which can apply also at $\Delta Y = 0$ ($h = H$). Asymptotic freedom + confinement imply that

$$\sigma_H \equiv \sum_{n\in H} |\, T^{e^+e^-\to n} \,|^2$$

$$\simeq \frac{1}{32\pi^2}\int d\Omega_q \,|\, T^{e^+e^-\to q\bar{q}}_{\text{on shell, Born}} \,|^2 . \tag{A14}$$

The compatibility of (A14) and (A13) for $h = H$ requires [see also Refs. 37 and 19(a)]

$$\alpha(t) = 1 . \tag{A15}$$

Such a fixed pole is not what we expect from the ladder graph of Fig. 10. If we take instead

$$\alpha(t) = 1 + \alpha' t + O(t^2) \tag{A16}$$

we have

$$\mathscr{P} \equiv \sigma_h / \sigma_H \sim e^{-\Delta Y(2 - 2s_q)} \quad \text{(modulo powers of } Y) , \tag{A17}$$

which is the announced result (2.6)—(2.8), but

$$\sigma_H \equiv \sigma(e^+e^- \to 2 \text{ jets})$$

$$\sim \frac{\sigma_{\text{Born}}(e^+e^- \to q\bar{q} \text{ on shell})}{\alpha' \ln s} \tag{A18}$$

instead of (A14) (see also Ref. 37). Thus the multiperipheral model is unrealistic by a logarithmic factor. Presumably one should consider more complicated graphs, where any number of soft gluon lines are added to Fig. 10.

A gluon exchanged parallel to the quark chain [as in Fig. 8(c), between the fourth and fifth clusters] does not modify the result (A17): the spin of the quark is replaced by

$$\alpha \text{ (quark}- \text{ gluon cut)} = S_q + S_g - 1 = S_q . \tag{A19}$$

There is a possibility that the summation of graphs with an arbitrary number of additional gluons "Reggeizes" the quark, as in QCD_2.[55(b)] In that case $\alpha_q(0) \le S_q$. Then (2.7) would be a lower bound, and TJU would be even more broken.

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