Neutrino-pair bremsstrahlung with generation change of the charged lepton

A. Pich
Department of Theoretical Physics, University of Valencia, Burjasot, Valencia, Spain
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The ratio $R_{\nu\bar{\nu}} \equiv \sigma(l\,^{A}Z \to l\,^{A}Z \sum_{\nu} \nu\bar{\nu}) / \sigma(l\,^{A}Z \to l'\,^{A}Z \nu_l \bar{\nu}_{l'})$, where l and l' are distinct charged leptons, is evaluated in the framework of the standard theory. The dependences on the number of generations, the helicity of the incoming lepton, and the lepton masses are analyzed. It is shown that in spite of the high energies considered lepton-mass effects are important, modifying the $R_{\nu\bar{\nu}}$ estimate for vanishing lepton masses by a factor of about three.

I. INTRODUCTION

Neutrino-pair creation in bremsstrahlung processes of the type $l^{-}\,^{A}Z \to l^{-}\,^{A}Z \nu\bar{\nu}$ has been considered recently as a possible way to extract information about the number of lepton generations.[1-4] At typical incoming energies of several hundred GeV, the calculated cross section turns out to be[2] of the order of $10^{-40} \times N$ cm^2, where N is the number of neutrino types, perhaps too small to allow the experimental observation of the process if the number of lepton doublets is small. Nevertheless, the special features of the differential leptonic distributions[4] (the scattered charged-lepton peaks at low energies and emerges at appreciable angles) seem to indicate that the possibility of doing an actual experiment is not ruled out in the near future.

As depicted in Fig. 1, there are two different kinds of contributions to the nuclear coherent process, characterized by the charge of the gauge boson exchanged. The neutral-current contribution (diagrams 1 and 2) is the same for all the neutrino species, while the charged-current one (diagrams 3, 4, and 5) is only present when ν refers to the doublet-partner neutrino of the incident lepton.

It seems *a priori* that the different behavior of one of the generations (to which the incoming lepton belongs) may obscure the dependence of the cross section in the number of neutrino types. However, there is no problem if one realizes that for right-handed helicity of the incoming negative lepton, the charged-current contribution is suppressed by lepton-mass factors, due to its $V-A$ character.[2] Furthermore, for the opposite helicity one can relate the amplitudes associated with diagrams 3 and 4 with those associated with diagrams 1 and 2 by means of a Fierz transformation. Diagram 5 is suppressed by one additional boson propagator, so one can safely neglect it and write the following relation between charged- and neutral-current amplitudes:[4]

$$T_{\rm CC}(\lambda) \simeq \frac{2\delta_{\lambda,-1/2}\delta_{l,l'}}{1-2\sin^2\theta_W} T_{\rm NC}(\lambda) , \qquad (1)$$

where l and l' refer to the incident lepton and the charged partner of the outgoing neutrino, respectively.

Adding the contribution of all neutrino types, the cross section will be then proportional to some function of N and λ,

$$\sigma \simeq f(N,\lambda)\sigma_{\rm NC}(\lambda,E_1) , \qquad (2)$$

$$f(N,\lambda) \equiv \left[1+\frac{2\delta_{\lambda,-1/2}}{1-2\sin^2\theta_W}\right]^2 + (N-1) .$$

Here $\sigma_{\rm NC}(\lambda,E_1)$ refers to the neutral-current contribution only.

It was argued by Tung[1] that if one considers the similar process $l^{-}\,^{A}Z \to l'^{-}\,^{A}Z \nu_l \bar{\nu}_{l'}$, where l and l' are distinct charged leptons, the ratio

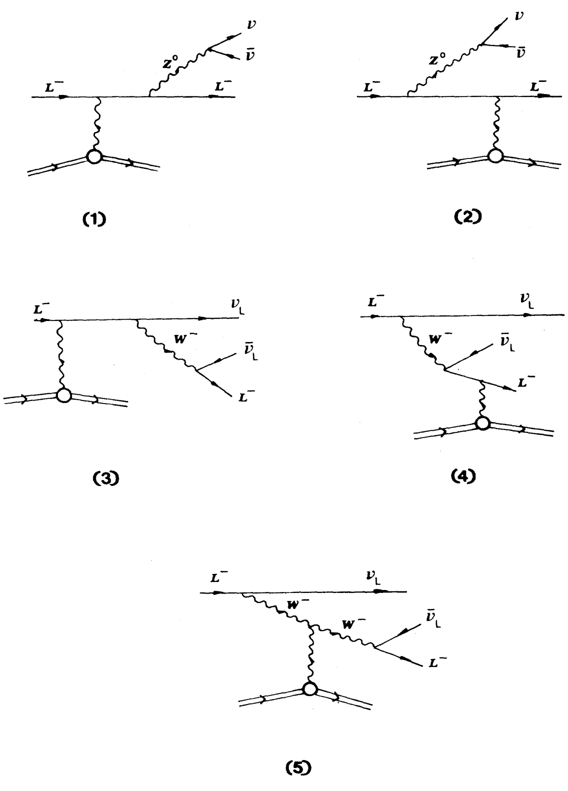

FIG. 1. Feynman diagrams contributing to the bremsstrahlung of neutrino pairs induced by a high-energy charged lepton in the nuclear Coulomb field.

$$R_{\nu\bar{\nu}} \equiv \frac{\sigma\left[l\,^{A}Z \to l\,^{A}Z \sum_{\nu} \nu\bar{\nu}\right]}{\sigma(l\,^{A}Z \to l'\,^{A}Z \nu_l \bar{\nu}_{l'})} \tag{3}$$

is equal to some function of N only, because the dynamical content of the processes factorizes, as shown by the Fierz relation between charged and neutral amplitudes.

In this paper, we study the validity of the main hypothesis implicit in Tung's reasoning, i.e., that one can completely neglect lepton masses in the high-energy limit. We show that, although one can safely neglect masses with respect to incoming energies, the comparison between mass and momentum of the virtual photon is relevant, leading to nontrivial mass dependence. So, one must correct the $R_{\nu\bar{\nu}}$ estimate with a multiplicative function of m_l and $m_{l'}$. This implies, of course, that one needs to do the entire calculation of the cross section in order to determine the correct value for $R_{\nu\bar{\nu}}$.

II. $R_{\nu\bar{\nu}}$ CALCULATION

The bremsstrahlung process $l\,^{A}Z \to l'\,^{A}Z \nu_l \bar{\nu}_{l'}$ with $l \neq l'$, involves the charged-current sector only and, obviously, produces a definite pair of neutrinos, the doublet partners of l^- and l'^-. Neglecting lepton masses, as a first approximation, we can write

$$\sigma_{l\neq l'} \simeq \left[\frac{2}{1-2\sin^2\theta_W}\right]^2 \delta_{\lambda,-1/2}\sigma_{\rm NC}(E_1,\lambda)\ , \tag{4}$$

where $\sigma_{\rm NC}(E_1,\lambda)$ is the same cross section which appears in Eq. (2).

For right-handed helicity of the incoming negative lepton, the process is suppressed by lepton-mass factors, so the interesting case is $\lambda=-\frac{1}{2}$. From Eqs. (2), (3), and (4) we easily obtain the following expression for the rate:

$$R_{\nu\bar{\nu}}(\lambda=-\tfrac{1}{2})\,|_{m=0} \simeq 2(1-\sin^2\theta_W)+N(\tfrac{1}{2}-\sin^2\theta_W)^2\ . \tag{5}$$

To calculate $R_{\nu\bar{\nu}}$ for unpolarized incoming leptons, we need the explicit dependence of $|T_{\rm NC}|^2$ on λ. As shown in Ref. 4, it amounts to a global factor $(4\sin^2\theta_W-1+2\lambda)^2$ only, so we have

$$R_{\nu\bar{\nu}}(\text{unpolarized})\,|_{m=0} \simeq 2(1-\sin^2\theta_W) + N[(\tfrac{1}{2}-\sin^2\theta_W)^2+\sin^4\theta_W]. \tag{6}$$

It was shown in Ref. 2 that, although one can safely ignore lepton masses in most of the dynamical calculations of the amplitudes, they play a crucial role when compared with virtual photon momenta, as they give the scale which controls the leptonic propagator poles. In fact, a different behavior was found for electrons and muons, the cross section being a factor about three times larger in the electron case for incoming energies of 300 GeV.

These results were obtained in the "elastic" case, where $l=l'$. However, it is not difficult to perform the calculation maintaining two different leptonic masses $m_l \neq m_{l'}$. The generalization of the phase-space formulas is straightforward. We have found that the results for the cross section remain valid if one substitutes m for $m_{l'}$ everywhere, $m_{l'}$ being the mass of the outgoing charged lepton.

Explicitly, one obtains

$$\begin{aligned}
&\sigma(lZ \to l'\nu_l\bar{\nu}_{l'}) \simeq \left[\frac{2}{1-2\sin^2\theta_W}\right]^2 \delta_{\lambda,-1/2}\sigma_{\rm NC}^{l,l'}\ ,\\
&\sigma_{\rm NC}^{l,l'} \simeq (4\sin^2\theta_W-1+2\lambda)^2 \frac{Z^2\alpha^2G^2}{216\pi^3}E_1 \int_{\underline{x}_1}^{\bar{x}_1} \frac{dx_1}{\sqrt{x_1}}\,|F(x_1)|^2[\mathscr{L}(x_1,m_{l'})+\ln(2E_1\sqrt{x_1}/m_{l'}^2)]\ ,\\
&\mathscr{L}(x_1,m) \equiv -\tfrac{35}{24}-10\frac{m^2}{x_1}-(1+2m^2/x_1-20m^4/x_1^2)\frac{1}{f}\ln\left[\frac{f+1}{f-1}\right]\ ,\\
&f \equiv (1+4m^2/x_1)^{1/2}\ ,
\end{aligned} \tag{7}$$

where x_1 is the square of the virtual-photon mass ($x_1 \equiv -q^2$), $F(x_1)$ the nuclear form factor, and E_1 the incoming-lepton energy in the laboratory frame.

It is evident now how to correct our estimates (5) and (6) for $R_{\nu\bar{\nu}}$. As $m_l \neq m_{l'}$, the dynamical factors do not factorize and we have

$$R_{\nu\bar{\nu}} \simeq R_{\nu\bar{\nu}}\,|_{m=0}\,\theta(m_l,m_{l'})\ , \tag{8}$$

where $\theta(m_l,m_{l'})$ is the rate between the x_1 integrations for the two masses

$$\theta(m_l,m_{l'}) \equiv \frac{\int_{\underline{x}_1}^{\bar{x}_1} \frac{dx_1}{\sqrt{x_1}}\,|F(x_1)|^2[\mathscr{L}(x_1,m_l)+\ln(2E_1\sqrt{x_1}/m_l^2)]}{\int_{\underline{x}_1}^{\bar{x}_1} \frac{dx_1}{\sqrt{x_1}}\,|F(x_1)|^2[\mathscr{L}(x_1,m_{l'})+\ln(2E_1\sqrt{x_1}/m_{l'}^2)]}\ . \tag{9}$$

TABLE I. Values of $a(m_\mu)$, $a(m_e)$, and b for $^{56}_{26}$Fe and $^{208}_{82}$Pb. All the coefficients are given in GeV.

	$a(m_\mu)$	$a(m_e)$	b
$^{56}_{26}$Fe	-0.834	-0.899	0.158
$^{208}_{82}$Pb	-0.587	-0.550	0.103

Our computation is valid not only for the ratio of the two total cross sections, but also for the ratio of the virtual-photon mass distributions if we replace in Eq. (8) the $\theta(m_l,m_{l'})$ function by

$$\Theta(x_1,m_l,m_l)\equiv\frac{\mathscr{L}(x_1,m_l)+\ln(2E_1\sqrt{x_1}/m_l^2)}{\mathscr{L}(x_1,m_{l'})+\ln(2E_1\sqrt{x_1}/m_{l'}^2)}\;. \qquad (10)$$

III. NUMERICAL RESULTS AND DISCUSSION

To perform the x_1 integration, it is convenient to write the $\theta(m_l,m_{l'})$ function in the form

$$\theta(m_l,m_{l'})=\frac{a(m_l)+b\ln(2E_1/m_l)}{a(m_{l'})+b\ln(2E_1/m_{l'})}\;, \qquad (11)$$

where the $a(m)$ and b coefficients are defined by

$$a(m)\equiv\int_{\underline{x}_1}^{\bar{x}_1}\frac{dx_1}{\sqrt{x_1}}\,|F(x_1)|^2[\mathscr{L}(x_1,m)+\ln(\sqrt{x_1}/m)]\;,$$

$$b\equiv\int_{\underline{x}_1}^{\bar{x}_1}\frac{dx_1}{\sqrt{x_1}}\,|F(x_1)|^2\;. \qquad (12)$$

Explicit calculations have been made for $^{56}_{26}$Fe and $^{208}_{83}$Pb, using realistic nuclear form factors.[5,6] The results are given in Table I for muons and electrons. We see that the mass dependence of the $a(m)$ function is very soft, so the main dependence of $\theta(m_l,m_{l'})$ on the leptonic masses comes from the scale in the logarithmic terms with the energy.

TABLE II. Values of $\theta(m_e,m_\mu)$ at different incoming energies for $^{56}_{26}$Fe and $^{208}_{82}$Pb.

	$\theta(m_e,m_\mu)$	
E_1 (GeV)	$^{56}_{26}$Fe	$^{208}_{82}$Pb
100	3.17	4.08
300	2.46	2.93
500	2.27	2.65
700	2.17	2.50
900	2.10	2.41

In Table II, values of $\theta(m_e,m_\mu)=1/\theta(m_\mu,m_e)$ are given for typical incoming energies. $\theta(m_e,m_\mu)$ is always greater than one, indicating that the mass dependence tends to give a larger cross section for outgoing electrons, independent of which is the incoming lepton. At 300 GeV, the $\theta(m_e,m_\mu)$ enhancement amounts to a factor 2.5 for ^{56}Fe decreasing slowly when the energy grows. For ^{208}Pb, the behavior is similar with slightly larger factors.

It is apparent from these results that the $R_{\nu\bar{\nu}}$ estimate for vanishing lepton masses is substantially modified in spite of the high energies we are considering. This is so because there is another scale in the process, i.e., the virtual-photon mass, which is[7] of the same order as the muon mass ($\langle x_1\rangle^{1/2}\sim m_\mu$). The $\theta(m_l,m_{l'})$ function effect on the ratio is then crucial in order to determine the number of neutrino types. For $N=3$ and $\sin^2\theta_W\simeq\frac{1}{4}$, the ratio $R_{\nu\bar{\nu}}$ (unpolarized) becomes 0.76 for incoming muons at 300 GeV, to be compared to 1.88 as given by Eq. (6).

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