Comparison of energy dependence of transverse momentum of dimuons produced in pN and $\pi^- N$ interactions with quantum-chromodynamic predictions

B. Cox and P. K. Malhotra*
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
(Received 1 April 1983; revised manuscript received 25 August 1983)

We have analyzed the dependence of the average transverse momentum squared $\langle p_T^2 \rangle$ of dimuons produced in pN and $\pi^- N$ interactions on s, the square of the center-of-mass energy. The presently available data indicate a linear increase of $\langle p_T^2 \rangle$ with s in both reactions, with the rate of increase being approximately twice as large in the $\pi^- N$ reaction for $M/\sqrt{s}=0.28$ as in the pN reaction for $M/\sqrt{s}=0.22$. The value of $\langle p_T^2 \rangle$ extrapolated to $s=0$, which is interpreted as the intrinsic $\langle k_T^2 \rangle$ of the π^- and p constituents, is the same within errors for both reactions. First-order QCD consistently predicts a lower $\langle p_T^2 \rangle$ at all s than is observed in the data.

A well known prediction of perturbative quantum chromodynamics (QCD) is that the average transverse momentum squared $\langle p_T^2 \rangle$ of dimuons produced in hadronic collisions is given to order α_s by the relationship[1-3]

$$\langle p_T^2 \rangle = \langle k_T^2 \rangle + \alpha_s(Q^2) f(\tau, x_F, \ln Q^2) s \ , \tag{1}$$

where α_s is the running coupling constant, M is the mass of the dimuon, s is the center-of-mass energy squared, $\tau = M^2/s$, and $\langle k_T^2 \rangle$ is the sum of the intrinsic transverse momentum squared of the constituents that take part in the interaction. Equation (1) implies that $\langle p_T^2 \rangle$ should grow linearly with s at a given τ, Q^2, and x_F. A linear relationship has been reported.[4-7] The aim of this paper is to extract quantitative information from presently existing data on the s variation of $\langle p_T^2 \rangle$ in $\pi^- N$ and pN reactions and to compare the observed data with the predictions of first-order QCD.

The perturbative expression for the differential cross section to first order in α_s for dimuon production can be written as[8]

$$\frac{d\sigma_P}{dM\,dy\,dp_T^2} = \frac{d\sigma_A}{dM\,dy\,dp_T^2} + \frac{d\sigma_C}{dM\,dy\,dp_T^2}$$

$$= \frac{16\alpha^2}{27}\frac{s}{M}\int_{x_1^{\min}}^1 dx_1 \frac{x_1 x_2 \alpha_s}{x_1 s + u - M^2}\frac{(\hat{t}-M^2)^2+(\hat{u}-M^2)^2}{s^2\hat{t}\hat{u}}\sum_i e_i^2 [q_i^{h_1}(x_1,Q^2)\bar{q}_i^{h_2}(x_2,Q^2)+(1\leftrightarrow 2)]$$

$$+\frac{2\alpha^2}{9}\frac{s}{M}\int_{x_1^{\min}}^1 dx_1 \frac{x_1 x_2 \alpha_s}{x_1 s + u - M^2}\frac{\hat{s}^2+\hat{u}^2+2M^2\hat{t}}{-\hat{s}^3\hat{u}}\sum_i e_i^2 [q_i^{h_1}(x_1,Q^2)G_i^{h_2}(x_2,Q^2)+(1\leftrightarrow 2)] \ . \tag{2}$$

Here σ_A and σ_C represent the contributions of the order-α_s quark-antiquark-annihilation and Compton-scattering diagrams shown in Figs. 1(b) and 1(c), respectively. The q_i's and G_i's in Eq. (2) are the quark and gluon distribution functions, and the various kinematic variables are given by

$$x_1^{\min} = -u/(s+t-M^2) \ ,$$
$$x_2 = [-x_1 t - (1-x_1)m^2]/(x_1 s + u - m^2) \ ,$$
$$t = M^2 - \sqrt{s}\,M_T e^{-y} \ , \quad u = M^2 - \sqrt{2}M_T e^{y} \ ,$$
$$\hat{s} = x_1 x_2 s \ , \quad \hat{t} = x_1 t + (1-x_1)M^2 \ ,$$
$$\hat{u} = x_2 u + (1-x_2)M^2 \ ,$$
$$M_T^2 = p_T^2 + M^2 \ ,$$

and

$$\alpha_s = 12\pi/[25\ln(Q^2/\Lambda^2)]$$

with $Q^2 = M^2$ in expression (2).

Equation (2) is useful only for large p_T since it has a singularity at $p_T = 0$. Furthermore, it ignores the intrinsic transverse momentum k_T of the constituents. Altarelli, Parisi, and Petronzio[1] have given the following prescription which uses this intrinsic k_T to regularize the $p_T = 0$ divergence:

$$\frac{d\sigma}{dM\,dy\,dp_T^2} = \int d^2 q_T \frac{d\sigma_P}{dM\,dy\,dp_T^2}[f(\vec{p}_T - \vec{q}_T) - f(p_T)]$$

$$+\pi f(p_T)\frac{d\sigma_{\rm DY}}{dM\,dy} \ . \tag{3}$$

Here $\vec{q}_T = \vec{p}_T - \vec{k}_T$, $\sigma_{\rm DY}$ is the Drell-Yan cross section [see Fig. 1(a)], and f is the soft k_T distribution. We have

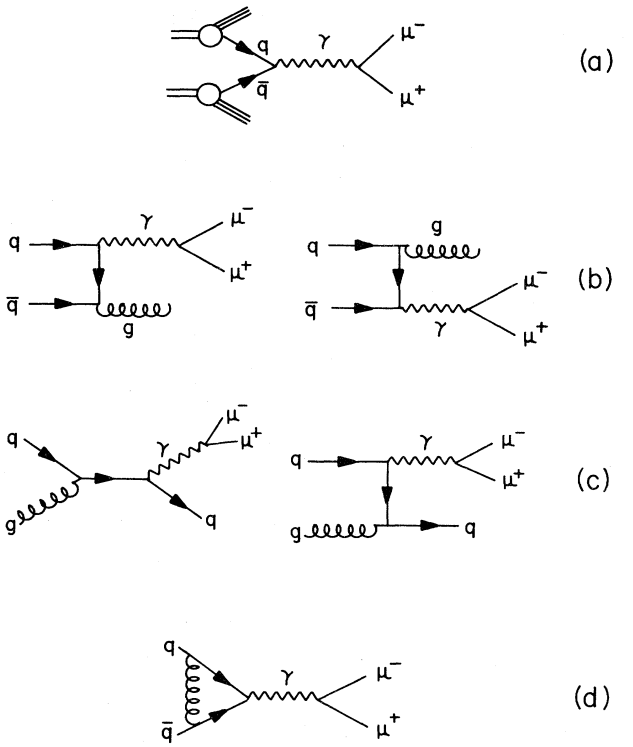

FIG. 1. (a) Drell-Yan process. (b) $O(\alpha_s)$ QCD quark-antiquark-annihilation process. (c) $O(\alpha_s)$ QCD Compton-scattering process. (d) Vertex correction to the Drell-Yan process with virtual gluon exchange.

assumed the k_T distribution to have a Gaussian form with the normalization

$$\int f(p_T)d^2p_T=\frac{1}{\pi\langle k_T{}^2\rangle}\int d^2p_T\exp[-p_T{}^2/\langle k_T{}^2\rangle]$$
$$=1\ . \tag{4}$$

The structure functions used in this calculation of $\langle p_T{}^2\rangle$ are given by

$$\begin{aligned}
xu_p&=u_0x^{0.52-0.16\bar{s}}(1-x)^{2.79+0.77\bar{s}}\ ,\\
xd_p&=d_0x^{0.52-0.16\bar{s}}(1-x)^{3.79+0.77\bar{s}}\ ,\\
xS_p&=(0.26+0.18\bar{s})(1-x)^{7.8+0.78\bar{s}}\ ,\\
xG_p&=3.06(1-x)^{5.0}\ ,\\
xV_\pi&=V_0x^{0.4}(1-X)^{0.9}\ ,\\
xS_\pi&=0.24(1-x)^{6.9}\ ,\\
xG_\pi&=2.0(1-x)^{3.0}
\end{aligned} \tag{5}$$

with $\bar{s}=\ln[(Q^2/0.25)/\ln(20/0.25)]$, where Q^2 is in GeV^2. The valence-quark distributions are normalized to obtain the proper number of valence quarks for π's and protons. We have assumed that the strange quark-antiquark pairs are suppressed by a factor of 2 compared to $u\bar{u}$ or $d\bar{d}$ pairs. The quark distribution functions for the proton correspond to the CERN-Dortmund-Heidelberg-Saclay (CDHS) results[9] while the quark distribution functions for the pion correspond to the NA3 results.[6,10] The Q^2 dependence is quite small over the range of interest. We have used a Q^2-independent gluon distribution for the proton corresponding to counting rules; the conclusions arrived at here are not affected if the power index is varied in the range 5.0 ± 1.0.

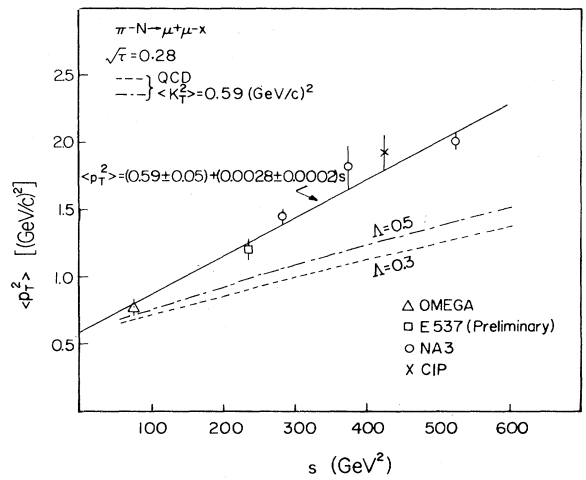

FIG. 2. $\langle p_T{}^2\rangle$ versus s for dimuons produced in π^--nucleon interactions. The solid curve is a linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ.

When we use the structure functions (5) in Eq. (3) and integrate over p_T, we obtain the Q^2-dependent Drell-Yan cross section which is known to be lower than the experimental cross sections for dimuon production by π's, p's, or antiprotons by factors in the range 1.6 to 2.6 [compilations of K factors$\equiv(d\sigma/dM\,dy)_{\rm data}/(d\sigma/dM\,dy)_{\rm DY}$ for various reactions are given, for example, in Refs. 7 and 11—14]. At the level of the accuracy of the data these K

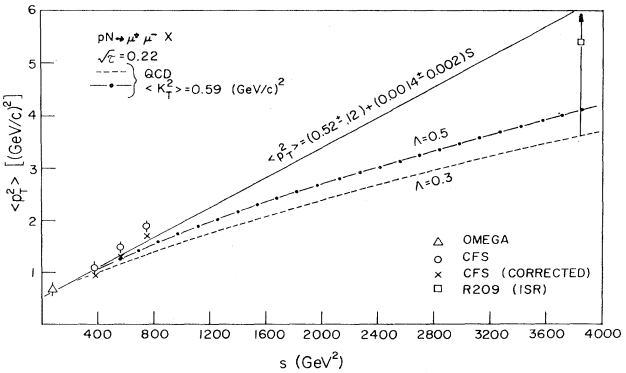

FIG. 3. $\langle p_T{}^2\rangle$ vs s for dimuons produced in p-nucleon interactions. The solid curve is the linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ.

TABLE I. Fits of $\langle p_T^2 \rangle = A + B(\tau)s$ to existing dimuon data.

Fit	Reaction	$\sqrt{\tau}$	Intercept (A) [(GeV/c)^2]	Slope (B)	χ^2/DF
$\langle p_T^2 \rangle$	$\pi^- N$	0.28	0.59±0.05	$(2.8\pm0.2)\times10^{-3}$	1.2
$\langle p_T^2 \rangle$	pN	0.22	0.52±0.11	$(1.4\pm0.2)\times10^{-3}$	0.3

factors have been determined to be independent of τ and y. Theoretical justifications of constant K factors have been provided by several authors[15] who have carried out a full $O(\alpha_s)$ calculation and have confirmed the observed independence of K of τ and y for $\tau < 0.5$. In these calculations the largest part of correction to the cross section for $\tau \leq 0.3$ (i.e., for the range of the current experiments) is found to arise from the vertex-correction diagram [Fig. 1(d)]. This diagram, however, contributes no p_T other than the k_T of the constituents to the dimuons. In addition, more complete calculations to order α_s^2 which have been done by Ellis, Martinelli, and Petronzio,[16] indicate that K is only a slowly varying function of p_T. Therefore, we have used empirical K factors assumed to be independent of τ, x_F, and p_T in fitting the dimuon p_T distributions at various energies.

Using Eq. (3), the structure functions given by (5), and the empirical K factors, we have been able to fit[13] published dimuon transverse-momentum distributions for proton interactions at 400 and 2050 GeV/c, for π^- interactions from 125 to 280 GeV/c, and for $\bar{p}$ interactions at 125 GeV/c. While the agreement of the calculations with the proton data is quite good at a given s, a $\langle k_T^2 \rangle$ varying from 0.9 (GeV/c)^2 at $\sqrt{s} = 23.4$ GeV to 1.5 (GeV/c)^2 at $\sqrt{s} = 62$ GeV is required. Similarly $\langle k_T^2 \rangle$ must vary from 0.9 (GeV/c)^2 at $\sqrt{s} = 15.3$ GeV to 1.2 (GeV/c)^2 at $\sqrt{s} = 22.8$ GeV for the π^- reaction. Thus, we find that QCD to order α_s with a constant $\langle k_T^2 \rangle$ is unable to describe the existing p_T data at different energies.[13]

To make explicit the failure of first-order QCD with constant $\langle k_T^2 \rangle$ to explain the transverse-momentum distribution of dimuons at different energies, we have compared the observed second momenta of the p_T distributions of the data to the $\langle p_T^2 \rangle$ predicted by (3). Figure 2 shows the existing measurements[5,6,17,18] of $\langle p_T^2 \rangle$ for $\sqrt{\tau} = 0.28$ and $x_F > 0$ at different values of s for the $\pi^- N$ reaction. Figure 3 shows $\langle p_T^2 \rangle$ for the pN data[5,19,20] for $\sqrt{\tau} = 0.22$. The pN data points from the Columbia–Fermilab–Stony Brook (CFS) experiment[19] are for $\langle y \rangle$ ranging from 0.03 for 400 GeV/c to 0.40 for 200 GeV/c. To take into account the expected dependence of $\langle p_T^2 \rangle$ on y we have used the QCD formalism described above to correct the CFS data to correspond to $y \geq 0$. Linear fits of the form $\langle p_t^2 \rangle = A + Bs$ are shown for both sets of data. The details of the fits are given in Table I. As shown in the figures and in Table I, the $s = 0$ intercept is the same within errors for the pN and $\pi^- N$ data but the slope of the rise of $\langle p_T^2 \rangle$ with s is approximately twice as large for the π^- data as for the pN data.

Since within errors the intercepts $\langle P_T^2 \rangle_{s=0}$ are the same and since we interpret $\langle P_T^2 \rangle_{s=0}$ to be equal to $\langle k_T^2 \rangle$ from Eq. (1), we have set $\langle k_T^2 \rangle = 0.59$ (GeV/c)^2 in the QCD calculations. The results of the calculation of $\langle P_T^2 \rangle$ with this choice of $\langle k_T^2 \rangle$ are shown in Figs. 2 and 3 for two different values of Λ (0.3 and 0.5 GeV/c). The predictions of QCD fall well below the data in the case of both the pN and the $\pi^- N$ reactions. However, the calculations do show a steeper slope for $\pi^- N$ than for pN in qualitative agreement with the data.

In carrying out the computations we have restricted the calculations to $p_T \leq 10$ GeV/c. In order to check the effect of this truncation we have repeated the calculation for $s = 3850$ GeV^2 (since the effect is expected to be maximum at large s) but extending p_T to $p_T \leq 15$ GeV/c. (Note that kinematics restrict the maximum value of p_T to 29.5 GeV/c for $\sqrt{\tau} = 0.22$.) For $\Lambda = 0.5$ GeV/c, we obtain $\langle p_T^2 \rangle = 4.20$ (GeV/c)^2 compared to $\langle p_T^2 \rangle = 4.07$ (GeV/c)^2 for $p_T \leq 10$ GeV/c. Thus, the effect of truncation of the p_T distribution is fairly small for $s \leq 3850$ GeV^2. In order to explore the source of the curvature seen in Fig. 3, we have repeated calculations by using a constant value of 0.3 for α_s. This yields a very nearly straight-line behavior for $\langle p_T^2 \rangle$ vs s. Thus, the source of the curvature is almost entirely due to the $\ln(Q^2)$ dependence of α_s.

In conclusion we have examined the behavior of $\langle p_T^2 \rangle$ as a function of s for the high-mass dimuons produced in pN and $\pi^- N$ interactions. We find that the data are consistent within errors with the same $\langle p_T^2 \rangle$ intercept at $s = 0$ for p and π^-, but that the rise of $\langle p_T^2 \rangle$ with s is approximately twice as fast for the $\pi^- N$ reaction as for the pN reaction. It may, however, be noted that the values of τ used for pN ($\sqrt{\tau} = 0.22$) and $\pi^- N$ ($\sqrt{\tau} = 0.28$) are different which, in principle, would lead one to expect different slopes for the two cases. We have interpreted the $s = 0$ intercept as the sum of the $\langle k_T^2 \rangle$ of the constituents of the proton and pion participating in the interaction and have used the observed intercept $\langle k_T^2 \rangle = 0.59$ GeV$^2/c^2$ in the first-order QCD prescription of Altarelli *et al.* to calculate the expected behavior of $\langle p_T^2 \rangle$ with s. We find that this $O(\alpha_s)$ formalism of QCD to (with the assumption of a p_T-independent K factor and $Q^2 = M^2$) is unable to account for the observed energy dependence of $\langle p_T^2 \rangle$ with the disagreement worsening at larger s. We therefore conclude that if the p_T distribution of the dimuons has to be understood on the basis of QCD, it is necessary to consider diagrams representing higher orders in α_s.

*On leave from the Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India.

[1]G. Altarelli, G. Parisi, and R. Petronzio, Phys. Lett. 76B, 351 (1978); 76B, 356 (1978).

[2]E. L. Berger, in *New Results in High Energy Physics—1978,* edited by R. S. Panvini and S. E. Csorna (AIP, New York, 1978).

[3]F. Halzen and D. M. Scott, in *Lepton Pair Production,* proceedings of the Moriond Workshop, Les Arcs, France, edited by J. Trân Thanh Vân (Editions Frontréres, Dreux, France, 1981), p. 297.

[4]J. K. Yoh *et al.*, Phys. Rev. Lett. 41, 684 (1978).

[5]M. Corden *et al.*, Birmingham-CERN-Polaiseau Collaboration, paper submitted to European Physical Society Conference on High Energy Physics, 1981 (unpublished).

[6]J. Badier *et al.*, NA3 Collaboration, Report No. LPNHE/X 82/02, 1982 (unpublished).

[7]I. R. Kenyon, Rep. Prog. Phys. 45, 1261 (1982).

[8]F. Halzen and D. M. Scott, Phys. Rev. D 18, 3378 (1978); K. Kajantie and R. Raitio, Nucl. Phys. B139, 72 (1978).

[9]J. G. H. deGroot *et al.*, CDHS Collaboration, Phys. Lett. 82B, 456 (1979).

[10]J. Badier *et al.*, Phys. Lett. 89B, 145 (1979); Report No. CERN/EP 80-148, 1980 (unpublished).

[11]L. Lyons, Prog. Part. Nucl. Phys. 7, 169 (1981).

[12]B. Cox, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982,* edited by P. Petiau and M. Pornuef [J. Phys. (Paris) Colloq. 43, (1982)]; B. Cox, in Proceedings of the Drell-Yan Workshop, Fermilab, 1982 (unpublished), p. 63.

[13]P. K. Malhotra, in Proceedings of the Drell-Yan Workshop, Fermilab, 1982 (unpublished), p. 217.

[14]E. L. Berger, in Proceedings of the Drell-Yan Workshop, Fermilab, 1982 (unpublished), p. 1.

[15]G. Altarelli, R. K. Ellis, and G. Martinelli, Nucl. Phys. B157, 461 (1979); J. Kubar-Andre and F. E. Paige, Phys. Rev. D 19, 221 (1979); J. Kubar *et al.*, Nucl. Phys. B175, 251 (1980).

[16]R. K. Ellis, G. Martinelli, and R. Petronzio, Phys. Lett. 104B, 45 (1981).

[17]E. Anassontzis *et al.*, E-537 (Athens-Fermilab-McGill-Michigan-Shandong) Collaboration, Report No. FERMILAB-CONF-82/50-EXP, presented at the 21st International Conference on High Energy Physics, Paris, 1982 (unpublished).

[18]K. J. Anderson *et al.*, Phys. Rev. Lett. 42, 944 (1979).

[19]A. S. Ito *et al.*, CFS Collaboration, Phys. Rev. D 23, 604 (1981).

[20]D. Antreasyan *et al.*, R209 Collaboration, Phys. Rev. Lett. 47, 12 (1981).