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Implications from the b-decay measurements

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The recent measurements of the bottom-quark b lifetime and the ratio $\Gamma_{b\to u}/\Gamma_{b\to c}$ are used to analyze the quark-mixing phenomena. We report the implications of these measurements on the Kobayashi-Maskawa angles and phase, K_{e3} and hyperon decays, CP-violation parameter ϵ, t-quark mass, $K^0 \leftrightarrow \bar{K}^0$ transition dynamics, B^0-$\bar{B}^0$ mixing and CP violation, and nonleptonic decays and leptonic production of heavy quarks.

The b lifetime now has been measured[1] by the MAC and Mark II Collaborations to be, respectively,

$$\tau_b = 1.8 \pm 0.6 \pm 0.4 \text{ psec}$$
$$= 1.20^{+0.45}_{-0.36} \pm 0.30 \text{ psec} \quad . \tag{1}$$

This somewhat unexpectedly long lifetime of the b, together with the limit[2]

$$\Gamma_{b\to u}/\Gamma_{b\to c} \leq 0.05 \quad , \tag{2}$$

obtained by the CLEO and CUSB collaborations, puts very stringent bounds on the quark-mixing matrix. In this paper we systematically analyze the implications from these b-decay measurements in the framework of the Kobayashi-Maskawa (KM) model.[3-5] Much of the discussions given here are based upon formulations given previously.[6,7] First we give the allowed region of the KM angles. Secondly, based upon this allowed region of s_1, s_2, s_3, and s_δ, we use the CP-violation parameter ϵ from $K_L \to \pi\pi$ to give further constraints. In the box-graph calculation, in order to fit ϵ, as emphasized by Ginsparg, Glashow, and Wise,[8] the t-quark mass m_t must exceed a certain minimum value for a long-lived b quark. For a given allowed value of $m_t > m_{t,\min}$, the values for s_2 and s_3 can be calculated in terms of s_δ. We also find that δ can only be in the first and the second quadrants.[9] Thirdly, we study the consequences of trying to fit $\Delta m = m_{K_L} - m_{K_S}$. We find that Δm cannot be fitted based upon the simple box graph,[6] given the b lifetime, Eq. (1). Here it may be argued that the box-graph calculation is not as reliable as for ϵ, since in calculating Δm the light-quark $u\bar{u}$ intermediate state contributes in addition to the heavy intermediate states, while in calculating ϵ only the heavy intermediate states contribute. This disagreement with Δm indicates that the box-diagram calculation does not give an adequate account for the low-energy intermediate-state contributions. We find that such low-energy contributions can be substantial. These estimates of the contribution from low-energy intermediate states provide useful information for future theoretical dynamical calculations. Finally, we discuss the implications of these new results of quark mixing on other weak-interaction processes, e.g., B^0-$\bar{B}^0$ mixing, CP violation, nonleptonic decay rates, neutrino production of heavy quarks, and even the K_{e3} and hyperon decays.

With the use of the simple W-emission diagram (i.e., like the diagram for $\mu \to \nu_\mu e \bar{\nu}_e$), the lifetime of the b quark is given by [see Eqs. (4.13) and (4.14) of Ref. 6]

$$\Gamma_b = (G_F m_b^5/192\pi^3)(2.95|V_{cb}|^2 + 6.33|V_{ub}|^2) \quad , \tag{3}$$

where the numerical coefficients[10] are the phase-space factors based on the constituent quark masses: $m_u = m_d = 0.3$, $m_s = 0.5$, $m_c = 1.5$, and $m_b = 4.9$ in GeV units. Using the experimental information in Eq. (2) and the phase-space factor of Eq. (3), we obtain

$$|V_{ub}/V_{cb}| \leq 0.14 \quad . \tag{4}$$

Since the $b \to u$ contribution is so small, we can neglect it in Eq. (3) and thus calculate $|V_{cb}|$ from the b-lifetime measurements, Eq. (1),

$$|V_{cb}|^2 = (0.003\,46 \text{ psec})/\tau_b$$
$$= 2.47\times10^{-3},\ 3.46\times10^{-3},\ 5.77\times10^{-3} \quad , \tag{5}$$

for $\tau_b = 1.4$, 1.0, 0.6 psec, respectively. Given $|V_{cb}|^2$, we obtain the bound on $|V_{ub}|$, i.e.,

$$|V_{ub}| \leq 6.96\times10^{-3},\ 8.24\times10^{-3},\ 1.06\times10^{-2} \quad , \tag{6}$$

for $\tau_b = 1.4$, 1.0, 0.6 psec, respectively. From

$$|V_{cb}|^2 = |c_1 c_2 s_3 + s_2 c_3 e^{i\delta}|^2$$

and

$$|V_{ub}|^2 = s_1^2 s_3^2 \quad ,$$

we can find the allowed region in s_2 and s_3 as given in Fig. 1. We can see that s_2 and s_3 now are restricted to a very small triangle region bounded by

$$s_2 \lesseqgtr |V_{cb}| \pm s_3, \quad s_3 < |V_{ub}|_{\max}/s_1 \quad , \tag{7a}$$

with[4] $s_1 = 0.23$, depending on the value of δ (note that $\delta = 0°$ provides the lower bound for s_2 and $\delta = 180°$ the upper bound). With the use of the values of Eqs. (5) and (6), these bounds become

$$s_2 \lesseqgtr (0.0497,\ 0.0588,\ 0.0760) \pm s_3 \quad , \tag{7b}$$

$$s_3 < 0.030,\ 0.036,\ 0.046 \quad , \tag{7c}$$

for $\tau_b = 1.4$, 1.0, 0.6 psec, respectively. As now both s_2 and s_3 are small, V_{cd} and V_{cs} are close to the values given by $-V_{us}$ and V_{ud}, respectively,

$$V_{cs} \approx -0.227, \quad V_{cd} \approx 0.97 \quad . \tag{7d}$$

Note that these bounds are much more stringent than those previously obtained. Especially interesting are the bounds (6) and (7). Since they are much better determined than from hyperon and K_{e3} decays,[4] they now can be used as an input for the K_{e3} and hyperon decay fits in order to study

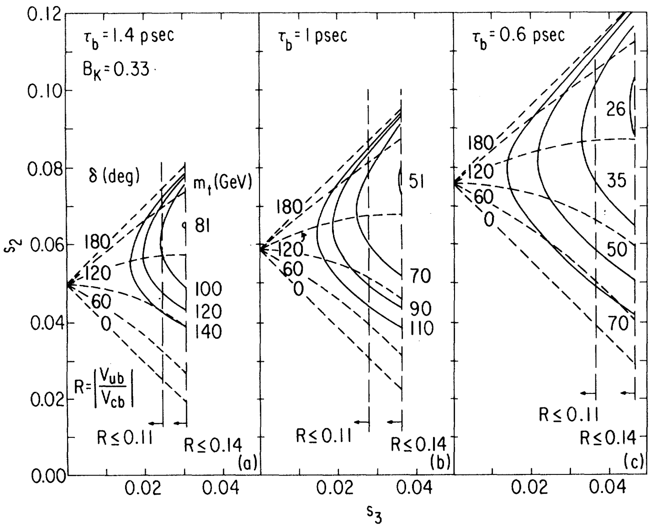

FIG. 1. The allowed regions of s_2 and s_3 are given. The region bounded by the long-dashed lines and the $\delta=0°$ and 180° lines are from $|V_{ub}/V_{cb}|$ and $|V_{cb}|$ restrictions only. The solid lines are from fitting ϵ based upon the box-graph calculation for $B_K=0.33$ and various t-quark masses.

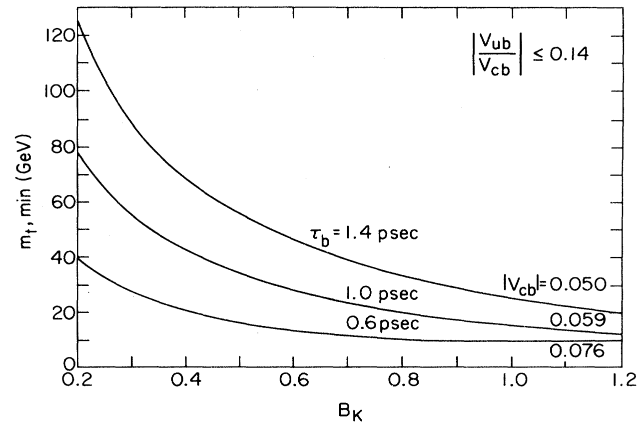

FIG. 2. The minimum value of m_t ($m_{t,\min}$) for fitting ϵ, as a function of B_K for various τ_b.

other dynamical properties in the decay, e.g., SU(3)-symmetry breaking.

The CP-violation parameter ϵ is given by the box-graph calculation,[6,7]

$$\mathrm{Re}\epsilon = -\tfrac{1}{2}\mathrm{Im}M_{12}/\Delta m \ , \tag{8}$$

$$\mathrm{Im}M_{12} = -\frac{G_F{}^2 m_W{}^2 B_K f_K{}^2 m_K}{12\pi^2}\sum_{i,j}^{c,t}\eta_{ij}\mathrm{Im}(\lambda_i\lambda_j)A_{ij} \ , \tag{9}$$

where $\lambda_i = V_{id}^* V_{is}$, and A_{ij} is equal to $-\bar{E}(x_i,x_j)$ in Eqs. (2.12) and (2.13) of Ref. 11. Notice that the u quark does not contribute, since $\mathrm{Im}(\lambda_u\lambda_u)=0$. η_{ij} are the QCD leading-logarithmic correction factors, $\eta_{cc}=0.7$, $\eta_{tt}=0.6$, and $\eta_{ct}=0.4$, as taken from Ref. 12. The factor $-B_K f_K{}^2$ is the infamous uncertainty in the calculation of the matrix element

$$\langle K^0|[\bar{d}\gamma_\mu(1-\gamma_5)s]^2|\bar{K}^0\rangle = -\tfrac{4}{3}B_K f_K{}^2 m_K \ ,$$

where $B_K=1$, if the vacuum-insertion calculation is used. Here we treat both m_t and B_K as free parameters. Note that in Eq. (8) the experimental value of Δm is used.[13] For too low values of m_t, the solution from the ϵ constraint lies outside the allowed boundary, as given in (7c). As m_t increases, the solution moves inside the allowed domain given by (7c) and approaches the other two boundaries given by (7b). The direct observation of V_{ub} from b decay and the measurement of m_t from the t-quark discovery in the future could determine all the mixing angles based upon this model. In Fig. 2 we give the minimum t-quark mass $m_{t,\min}$ required by fitting ϵ, as B_K varies from 0.2 to 1.2. We see that for τ_b in the psec range the smallest $m_{t,\min}$ for $B_K \sim 1$ (the vacuum-insertion result) is about 20 GeV. For $B_K=0.33$, as given in some calculations,[14] the value of $m_{t,\min}$ rises rapidly from 30 to 90 GeV as τ_b increases from 0.6 to 1.4 psec. Conversely, given the experimental bound on m_t (currently $m_t > 21$ GeV from PETRA experiments), from Fig. 2, a maximum value of B_K can be obtained for a precise value of τ_b. For given B_K and $m_t \geq m_{t,\min}$, s_2 and s_3 are completely determined in terms of δ. In Fig. 1(a)–1(c) we give such points. Here we restrict our discussions to $B_K>0$; then only $s_\delta>0$ regions are allowed. For $B_K<0$, from this ϵ restriction, the solutions are simply given by $s_\delta \to -s_\delta$.

Given the narrow region of s_2 and s_3, we give one typical example from Fig. 1(b), for the case of $B_K=0.33$, $\tau_b=1.0$ psec, $m_t=m_{t,\min}=50.6$ GeV, $\delta=132°$, $s_2=0.077$, and $s_3=0.036$,

$$V_{ij} = \begin{pmatrix} 0.9737 & 0.228 & 0.008\,22 \\ -0.227 & 0.972\ -i0.0021 & -0.0162+i0.0568 \\ -0.0174 & 0.0504+i0.0267 & 0.670\ -i0.740 \end{pmatrix} . \tag{10}$$

Next we give the box-graph calculation of $\mathrm{Re}M_{12}$, which is simply given by the same expression Eq. (9) for $\mathrm{Im}M_{12}$ with $\mathrm{Im}(\lambda_i\lambda_j)$ replaced by $\mathrm{Re}(\lambda_i\lambda_j)$. Now the u quark does contribute since $\mathrm{Re}(\lambda_u\lambda_u)\neq 0$. However, the box graph has been demonstrated to be inadequate in estimating the u-quark contribution.[15] In Fig. 3 we give

$$[(\Delta m)_{\mathrm{box}} = -2\,\mathrm{Re}M_{12}]/[(\Delta m)_{\mathrm{expt}} = 3.52\times10^{-15}\ \mathrm{GeV}]$$

as a function of B_K for given $m_t=m_{t,\min}$ for that particular value of B_K. We see that for small values of B_K, we need large compensation from low-energy intermediate contributions. Here the sign of B_K being positive is important; $B_K>0$ gives $(\Delta m)_{\mathrm{box}}>0$ as required by experiment. If B_K is taken to be negative, then $(\Delta m)_{\mathrm{box}}<0$, opposite in sign comparing to the experiment so even more positive contributions from other sources are needed to compensate the wrong result given by $\mathrm{Re}M_{12}$. It is interesting to note that $\Delta m(K_L-K_S)_{\mathrm{box}}$ is essentially independent of τ_b in the range of 1.4–0.6 psec, i.e., $m_{t,\min}\approx 30$–90 GeV. This is due to the fact that s_2 and s_3 are so small that the real part of $V_{td}^* V_{ts}$ is much smaller than $V_{cd}^* V_{cs}$, so that the charm contribution actually dominates. Interestingly, this reminds us why the early estimate of the charm-quark mass from $(\Delta m)_{\mathrm{box}}$ by Gaillard and Lee[16] using $B_K=1$ was quite reasonable.

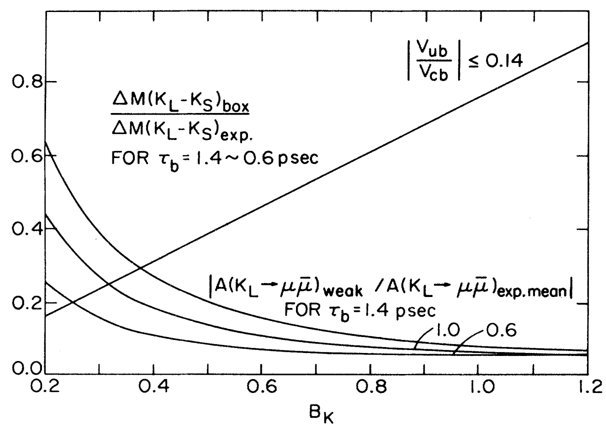

FIG. 3. Comparison with experiment of theoretical calculations for $\Delta m = m_{K_L} - m_{K_S}$ from the box-graph calculation, and the dispersive part of the $K_L \to \mu\bar{\mu}$ amplitude $A(K_L \to \mu\bar{\mu})$ from the pure weak-interaction quark-diagram (see Ref. 6) calculations, at $m_t = m_{t,\min}$.

As emphasized in Ref. 6, similar uncertainties exist in calculating the dispersive part of $K_L \to \mu\bar{\mu}$. Given all the parameters, we can also calculate the dispersive part of the amplitude of $K_L \to \mu\bar{\mu}$ from the purely weak-interaction quark diagrams, $A(K_L \to \mu\bar{\mu})_{\text{weak}}$ (see Sec. III of Ref. 6 for details). In Fig. 3 we also give the comparison with experiments of the theoretical calculation,

$$A(K_L \to \mu\bar{\mu})_{\text{weak}}/A(K_L \to \mu\bar{\mu})_{\text{expt mean}} \ ,$$

at $m_t = m_{t,\min}$. We see that the discrepancy can be very large, especially in the large-B_K region. Note that the decreasing behavior of

$$A(\bar{K}_L \to \mu\bar{\mu})_{\text{weak}}/A(\bar{K}_L \to \mu\bar{\mu})_{\text{expt mean}}$$

as B_K increases is mainly from the decreasing behavior of $m_{t,\min}$ as B_K increases. While $\Delta m(K_L \to K_S)_{\text{box}}$ has a factor B_K, it thus increases linearly with B_K. Although $A(K_L \to \mu\bar{\mu})_{\text{weak}}$ grows with m_t, it would not be a large fraction of A_{expt} unless m_t is much greater than 100 GeV.

Next we calculate the B^0-$\bar{B}^0$ mixing probability.[7] The B^0-$\bar{B}^0$ mixing parameter $r(B)$ is the time-integrated probability $(B^0 \to \bar{B}^0)$ of B^0 becoming $\bar{B}^0$ relative to that $(B^0 \to B^0)$ of B^0 staying as B^0, $r(B) = (B^0 \to \bar{B}^0)/(B^0 \to B^0)$ and for $\bar{r}(B) = (\bar{B}^0 \to B^0)/(\bar{B}^0 \to \bar{B}^0)$. The CP violation can give rise to a difference in such mixing between particle and antiparticle, i.e.,

$$a(B) = [\bar{r}(B) - r(B)]/[\bar{r}(B) + r(B)] \neq 0 \ .$$

In Fig. 4 we give $r(B)$ and $a(B)$. We see that for a very conservative guess of $B_b f_B{}^2 = (0.1 \text{ GeV})^2$ we have appreciable mixing in $\bar{B}_s$, B_s as noted before.[6,7] The mixing $\bar{B}_d$, B_d is much smaller. Note that the mixing parameter is very sensitive to the parameter $B_b f_B{}^2$; if the value $B_b f_B{}^2 = (0.33 \text{ GeV})^2$, as given in some calculations,[6] is used, then the mixing of B_d, $\bar{B}_d$ can be appreciable.

Using now much better determined V_{ij}, we can also use

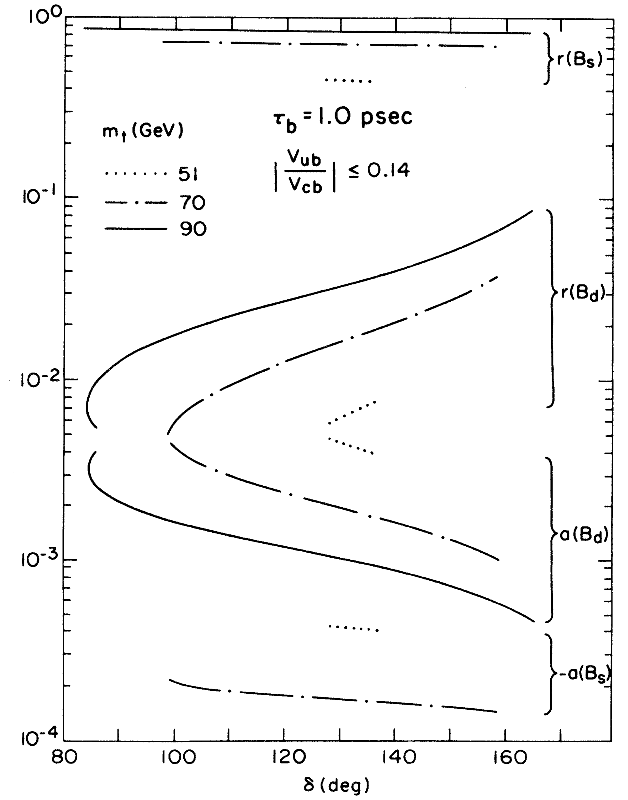

FIG. 4. The B^0-$\bar{B}^0$ mixing parameter $r(B)$, see Eqs. (11), and the CP-violation parameters $a(B)$ [see Eq. (12)] are given for B_s and B_d states.

them as input in the calculation of heavy-quark decays and productions. For example,[7]

$$\Gamma(D^+ \to \pi^0\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+) = \tfrac{1}{2}|V_{cd}/V_{cs}|^2 = 0.027 \ .$$

Another implication is that now $V_{us}/V_{ud} \approx -V_{cd}/V_{cs}$, the apparent deviation of $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-)$ from 1 has to be explained from other sources,[7,17] such as substantial SU(3)-symmetry breaking.

Previously, the $\mu^+\mu^-$ inclusive production from ν and $\bar{\nu}$ scattering were used to estimate the products of the quark-mixing matrix and the distribution functions of nucleons.[6,18] With the knowledge of V_{cs} and V_{cd}, see Eq. (7d), we can learn more about the sea-quark distributions. From the CERN-Dortmund-Heidelberg-Saclay experiment,[19] the measurement of $|V_{cs}/V_{cd}|^2 = 2S/(U+D) = 1.19 \pm 0.09$ in $\nu N \to \mu^-\mu^+X$ implies $2S/(U+D) = 0.065 \pm 0.005$; also a similar analysis from $\bar{\nu}N \to \mu^+\mu^-X$ gives $2S/(\bar{U}+\bar{D}) = 0.52 \pm 0.07$, which indicates substantial SU(3) breaking in the sea-quark distribution.

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