# Quark-antiquark contribution to the hadroproduction of $\psi$ in the color-singlet model

L. Clavelli\*

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

P. H. Cox and B. Harms

Department of Physics, University of Alabama, University, Alabama 35486 (Received 22 July 1983)

We consider the contribution to  $\psi$  hadroproduction due to the elementary subprocess  $q\bar{q} \rightarrow \psi GG$ . The color-singlet nature and observed spin-parity of the  $\psi$  are incorporated and the normalization of the cross section is predicted by the  $\psi$  wave function at the origin as measured in the leptonic  $\psi$  decay. We expect this process to dominate the difference in  $\psi$  production from particle versus antiparticle beams. The energy dependence of  $\psi$  production as well as distributions in  $x_F$  and  $P_T$  are predicted.

#### I. INTRODUCTION

Owing to the high momentum transfers required to create a bound state of heavy quarks from light-particle beams, the hadroproduction of  $\psi$  and  $\Upsilon$  has been extensively investigated in the last few years as a test of perturbative QCD.<sup>1</sup>

The early QCD predictions<sup>2,3</sup> for these processes were based on the lowest-order graphs for the production of charmed quarks in quark-antiquark or gluon-gluon annihilation:

 $G+G \rightarrow c+\overline{c}$ , (1.1a)

$$q + \overline{q} \rightarrow c + \overline{c}$$
 (1.1b)

It is assumed, in this model, that some fraction F of those  $c\overline{c}$  pairs having joint invariant mass greater than  $2m_c$  but less than  $2m_D$ , twice the *D*-meson mass, would appear as a  $\psi$  meson. This idea was given the name of "semilocal duality."

Since the fraction F cannot be calculated, the magnitude of the  $\psi$ -production cross section in this model is somewhat uncertain (F, in fact, is found to be beam-energy dependent). In addition, the  $c\bar{c}$  state in (1.1a) and (1.1b) is produced with zero transverse momentum so that the large observed average  $P_T$  is not described by this model. Its greatest success has been in predicting the dependence of the cross section on  $x_F = 2P_{||}/\sqrt{s}$ , although Barger, Keung, and Phillips<sup>3</sup> have noted that the range of validity of the model is restricted at high energy to small  $x_F$ .

It is also not clear if the model has a well-defined theory of higher-order corrections since one does not know what fraction of any radiated gluons are to be included in the  $\psi$ . Furthermore, the lowest-order QCD graphs for processes (1.1a) and (1.1b) yield  $c\bar{c}$  final states with the wrong color and/or spin-parity to produce a  $\psi$ . It is therefore necessary to assume that the required shedding of color and spin-parity occurs nonperturbatively without affecting the  $x_F$  distribution.

In 1979, it was suggested<sup>4</sup> that this shedding of color, or "color bleaching," might be perturbatively calculable by

emitting a final-state gluon in process (1.1a). One then has the process

$$G + G \rightarrow \psi + G$$
 . (1.2)

The amplitude for this process can be obtained by crossing the three-gluon-decay amplitude of the  $\psi$ . The colorsinglet nature and correct spin-parity of the  $\psi$  are then automatically incorporated and the normalization of the amplitude is likewise determined from  $\psi$  decay. The reaction (1.2) was discussed by Baier and Ruckl<sup>5</sup> and shown to yield a fair description of the  $P_T$  dependence of  $\psi$  production once the intrinsic  $k_T$  of the gluons in the hadrons is taken into account. A similar calculation for  $\psi$  photoproduction, replacing one of the initial-state gluons by a photon, has also been done.<sup>6</sup> In Ref. 5, about 35% of the  $\psi$ production is attributed to process (1.2) (the remaining 65% coming from production of P-wave charmonium states which then decay into  $\psi$ ). This 35% requires an  $\alpha_s = 0.37$  which may be consistent with the best present jet measurement<sup>7</sup>:

$$\alpha_s(30 \text{ GeV}) = 0.13 \pm 0.023$$
, (1.3)

but is too high to be consistent with the measurement in charmonium decays<sup>8</sup>:

$$\alpha_s(4.5 \text{ GeV}) = 0.158 \pm 0.011$$
 (1.4)

As in charmonium decays,<sup>8</sup> it seems reasonable to assume that charmonium production probes the strong coupling constant at the scale of the heavy-quark mass, i.e., half the charmonium mass. Equation (1.3), extrapolated via the renormalization group, would predict  $\alpha_s(m_c)=0.30^{+0.2}_{-0.1}$  while (1.4) would require  $\alpha_s(m_c)=0.22\pm0.02$ . Since the cross section for (1.2) is proportional to  $\alpha_s^3$ , using (1.4) would reduce the amount of  $\psi$  production attributable to this process to 5% to 9% with a similar effect in  $\psi$  photoproduction. In addition, it is now known<sup>9</sup> that only about 30% of produced  $\psi$ 's are decay products of *P*-wave charmonium states. If the rest of  $\psi$  production is due to other processes or to QCD radiative corrections, it is difficult to understand the success of (1.2) in predicting the

shape of the various distributions.

The other parton-level processes which can contribute to  $\psi$  production in the charmonium normalized colorsinglet model are

$$G + q \to \psi + G + q \tag{1.5}$$

and

$$q + \bar{q} \to \psi + G + G . \tag{1.6}$$

Each of (1.2), (1.5), and (1.6) forms the leading term in a well-defined, infrared-finite series of QCD corrections to the basic process. One might optimistically hope, based on the experience of Ref. 8, that these corrections amount to no more than 25% to 35% of each Born term. The singularity in process (1.5) when the initial- and final-state quarks are collinear is eliminated by a renormalization of the gluon distribution functions in the beam hadrons. We reserve the study of this contribution to  $\psi$  production to a later paper.

We would like to investigate the extent to which these three contributions, when added together, describe the differential and total cross sections for inelastic  $\psi$  production with a value of  $\alpha_s$  consistent with that found in other measurements such as Eqs. (1.3) and (1.4). In this article we calculate the matrix element for (1.6) and present the distributions in  $x_F$  and transverse momentum of the  $\psi$  as well as the energy dependence of the total cross section. This matrix element is simply related to the decay amplitude of  $\psi$  into two gluons and a lepton pair<sup>10</sup> and to one of the QCD corrections to hadronic  $\psi$  decay considered in Ref. 8. It is also related in the limit of negligible lightquark mass to the amplitude<sup>11</sup> for  $e^+e^- \rightarrow \psi GG$ .

We find that the resulting distributions are in satisfactory agreement with experiment and the cross section is comparable in magnitude to the lower-order process (1.2). We therefore expect that consideration of the three processes (1.2), (1.5), and (1.6) may each yield important contributions to hadronic  $\psi$  production and similar distributions in transverse momentum and  $x_F$ . Thus, the sum of a few QCD processes may predict all the features of the data with a value of  $\alpha_s$  compatible with other measurements.

#### II. CALCULATION OF PRODUCTION CROSS SECTIONS

In the calculations below, we assume that the  $\psi$  meson is a  $J^P = 1^-$  bound state of two essentially free quarks, each of which carries, therefore, half the momentum and half the mass of the  $\psi$ . The vertex factor for the  $\psi \rightarrow c\bar{c}$ coupling in the charmonium model is

$$V = \frac{1}{2} \frac{\psi(0)}{\sqrt{N_c M_{\psi}}} \mathscr{E}(\mathcal{P}_{\psi} + M_{\psi}) \lambda^0 , \qquad (2.1)$$

where  $\psi(0)$  is the value of the  $\psi$  wave function at the origin in momentum space. It can be determined from the leptonic width of the  $\psi$  (4.8 keV) to be

$$\psi(0)^2 = (3.92 \times 10^{-3} \text{ GeV}) M_{\psi}^2$$
 (2.2)

 $\lambda^0$  is the unit matrix in the space of  $N_c$  colored quarks.

The amplitude for the annihilation of a quark-antiquark pair into  $\psi$  and two gluons (see Fig. 1) is given by

$$M = -i(4\pi\alpha_s)^2 (\text{Tr}T_a T_b T_c) \frac{\psi(0)}{\sqrt{N_c M_{\psi}}} \frac{Q_{\mu}^a H_{\mu}^{oc}}{D} + \text{perms} ,$$
(2.3)

where  $T_i$ , i=a,b,c, are color matrices,  $\alpha_s$  is the strong coupling constant, and the factors  $Q_{\mu}$ ,  $H_{\mu}$ , and D are given by

$$Q^{a}_{\mu} = \overline{u}(P_{1})\gamma_{\mu}T_{a}v(P_{2}) , \qquad (2.4)$$

$$H^{bc}_{\mu} = \operatorname{Tr} \mathscr{C}(\psi)(P+m)\mathscr{C}_{4}{}^{b}(P_{4}+P+m) \times \gamma_{\mu}(-P_{3}-P+m)\mathscr{C}_{3}{}^{c} , \qquad (2.5)$$

$$D = 4k^{2}P_{3} \cdot P P_{4} \cdot P . \qquad (2.5)$$

There are six independent permutations of Fig. 1 corresponding to the six different orderings of the gluon momenta. To form the appropriate differential cross sections we average  $M^2$  over initial spins and colors and sum over final spins and colors. The resulting matrix element squared is given in the Appendix.

The parton-level cross section is given by

$$d\hat{\sigma} = d\Omega \frac{1}{2k^2} \frac{1}{4N_c^2} \sum_{\text{spins}} |M|^2 , \qquad (2.6)$$

where  $d\Omega$  is the differential of invariant phase space.

In order to compare the predictions of the process shown in Fig. 1 with experimental data, we must fold in the parton distribution functions in the beam and target:

$$d\sigma = dx_1 dx_2 \overline{q}(x_1) q(x_2) d\hat{\sigma} . \qquad (2.7)$$

The following CERN-Dortmund-Heidelberg-Scalay (CDHS) and NA3 distribution functions are used<sup>12</sup>:

$$\bar{u}_{\pi}(x_1) = \bar{u}_{\pi 0} x_1^{-0.6} (1 - x_1)^{0.9}$$
, (2.8)

$$u_p(x_2) = u_{p0} x_2^{a} (1 - x_2)^{b} , \qquad (2.9)$$

$$d_p(x_2) = d_{p0}u_p(x_2)(1-x_2)/u_{p0} , \qquad (2.10)$$

$$a = 2.79 + 0.77\overline{s}$$
, (2.11)

$$b = -0.48 - 0.16\overline{s} , \qquad (2.12)$$

$$\bar{s} = \ln[\ln(M_{\psi}^2/\Lambda^2)/\ln(20 \text{ GeV}^2/\Lambda^2)]. \qquad (2.13)$$

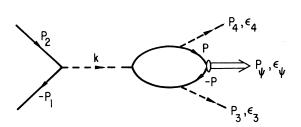


FIG. 1. Typical Feynman graph for the production of  $\psi$  from a  $q\bar{q}$  initial state. Emission of two gluons is required by color and angular momentum conservation. Five additional graphs related to this one by permuting the order of the gluons on the quark line are not shown.

We have taken  $\Lambda = 0.120$  GeV to agree with the results of Refs. 7 and 8. The distributions are normalized to the number of valence quarks in the pion and proton. Since the experimental data comes from the scattering off targets containing roughly equal numbers of protons and neutrons, we use "nucleon" structure functions defined by averaging over protons and neutrons. We assume that the up-quark distribution in the proton is the same as the down-quark distribution in the neutron and vice versa.

In Fig. 2 we show the predictions of the model at 125 GeV/c for  $d\sigma/dx_F$ , where  $x_F$  is the ratio of the  $\psi$  longitudinal momentum to the maximum longitudinal momentum in the overall c.m. frame. The data points were taken from Ref. 13. The model predicts the correct shapes of the  $x_F$  distributions for both  $\pi^-$  and  $\overline{p}$  beams. In order to compare the shapes of the distributions, we use unrealistic values of  $\alpha_s$  (0.31 for  $\overline{p}N$  and 0.36 for  $\pi^-N$ ) such that the quark-antiquark-annihilation contribution to  $\psi$  production saturates the experimental cross sections. Values of  $\alpha_{e}$  between 0.30 and 0.37 have been commonly used in previous  $\psi$ -production calculations.<sup>5,6</sup> However, the results of Refs. 7 and 8 suggest values of  $\alpha_s$  from 0.20 to 0.24. The theoretical curves in Fig. 2 should be multiplied by  $(\alpha_s/0.31)^4$  in the case of  $\overline{p}$  and  $(\alpha_s/0.36)^4$  in the case of  $\pi^-$ . For any reasonable values of  $\alpha_s$ , the  $q\bar{q}$  contribution we have calculated dominates over the gluon-gluon contribution of Ref. 5. The  $x_F$  distributions of the present model seem to be as good as those of the semilocal-duality models of Refs. 2 and 3.

A major weakness of the fusion models of Refs. 2 and 3 was their inability to describe the  $P_T$  dependence of  $\psi$  production. In Fig. 3 we show the corresponding distribu-

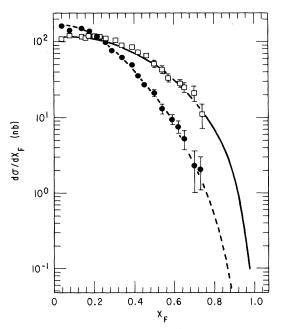


FIG. 2. Feynman-x distribution of the produced  $\psi$  in 125-GeV/c  $\pi^-N$  (solid curve) and  $\overline{p}N$  (dashed curve) collisions compared to Fermilab data (Ref. 13).

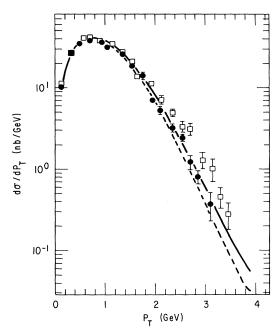


FIG. 3. Transverse-momentum distribution of the produced  $\psi$  in 125-GeV/c  $\pi^-N$  (solid curve) and  $\overline{p}N$  (dashed curve) collisions compared to Fermilab data.

tions for the present  $q\bar{q}$ -annihilation model compared with the 125-GeV/c data of Ref. 13. As in Ref. 5 we incorporate intrinsic parton transverse momentum by smearing the bare distribution of Eq. (2.7) by a Gaussian form:

$$\frac{d\sigma}{dP_T} = \int d^2 q_T h(\vec{\mathbf{P}}_T - \vec{\mathbf{q}}_T) \frac{d\sigma}{dq_T} , \qquad (2.14)$$

where

$$h(\vec{\mathbf{P}}_{T} - \vec{\mathbf{q}}_{T}) = \frac{1}{4\pi\sigma^{2}} e^{-(\vec{\mathbf{P}}_{T} - \vec{\mathbf{q}}_{T})^{2}/4\sigma^{2}}.$$
 (2.15)

We use the same value of  $\sigma^2$  (namely 0.2304 GeV<sup>2</sup>/c<sup>2</sup>) as Ref. 5. Again the theoretical curves should be multiplied by the factors of  $(\alpha_s/0.31)^4$  in the case of  $\bar{p}N$  and  $(\alpha_s/0.36)^4$  for  $\pi^-N$ . For smaller values of  $\alpha_s$ , the  $q\bar{q}$  process still contributes a significant fraction of the observed cross section. The shape of the  $P_T$  distributions agrees well with experimental results and could clearly be made better by increasing  $\sigma^2$  slightly.

The contribution of this process to the total forward  $(x_F > 0)$  cross section for  $\psi$  production by  $\pi^-$ , shown in Fig. 4 for  $\alpha_s = 0.362$ , exhibits the characteristic sharp rise from zero at threshold to an approximately flat region for s greater than 400 GeV<sup>2</sup>. The shape of the curve is controlled by the large-x behavior of the quark distribution functions at low energy ( $s < 100 \text{ GeV}^2$ ) and by the small-x behavior at high energy ( $s > 100 \text{ GeV}^2$ ). Therefore, including the sea-quark contributions, which we have neglected, would lead to a further slow growth of the cross section. The dashed curve in Fig. 4 is the function 500 exp( $-10M_{\psi}/\sqrt{s}$ )(nb) which has been shown to describe the experimental energy dependence.<sup>14</sup> For a

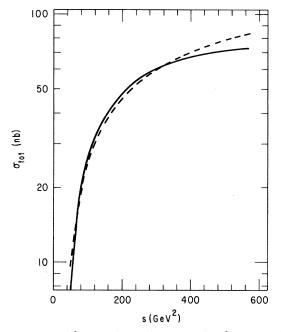


FIG. 4. Total forward  $(x_F > 0)$  cross section for production of  $\psi$  in  $\pi^- N$  collisions as a function of energy. The theoretical result (solid curve) should be multiplied by  $[\alpha_s(m_c)/0.362]^4$ . The dashed curve is the empirical fit to data. See text.

realistic value of  $\alpha_s$ , the fraction of inclusive  $\psi$  production attributable to process (1.6) is  $[\alpha_s(m_c)/0.362]^4$ . At asymptotically high energies, reaction (1.2) and perhaps (1.5) should dominate because of the dominance of the gluon structure function at small x. The difference between the  $\pi^- p$  and  $\pi^+ p$  cross sections must continue to be dominated by the present  $q\bar{q}$  process since the gluon-gluon and quark-gluon processes cancel out of this difference. This model predicts an equal and opposite difference between the  $\pi^- n$  and  $\pi^+ n$  cross sections so that for multinucleon targets, the difference tends to cancel.

Reaction (1.6) is the lowest-order QCD contribution to the difference

$$\Delta \sigma \equiv \sigma(\bar{p}N \rightarrow \psi X) - \sigma(pN \rightarrow \psi X) \; .$$

In Fig. 5 we show the range of theoretical predictions for  $\alpha_s(m_c)$  between 0.20 and 0.24 as a function of the centerof-mass energy squared. The theoretical curves scale as  $\alpha_s(m_c)^4$ . The data point at 424 GeV<sup>2</sup> is derived from Ref. 15. The lower-energy points are from the conference report of Badier.<sup>16</sup>

## **III. CONCLUSION**

The results of this investigation encourage us to believe that the charmonium decay amplitudes crossed to the production region can quantitatively describe the features of hadronic  $\psi$  production. The difference between particle and antiparticle production of  $\psi$ , varying as  $\alpha_s(m_c)^4$ , could become a sensitive test of QCD and perhaps an accurate method of measuring the strong coupling constant.

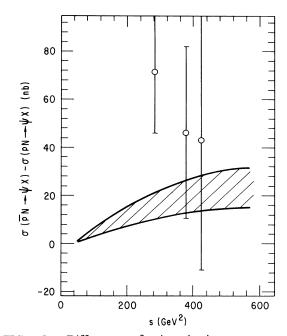


FIG. 5. Difference of  $\psi$ -production cross sections  $\Delta\sigma = \sigma(\bar{p}) - \sigma(p)$  as a function of energy. The region between the two curves is the range of predictions for  $\alpha_s(m_c) = 0.22 \pm 0.02$ .

Our results, added to those of Ref. 5, are probably already inconsistent with the values of  $\alpha_s$  (30 GeV) greater than 0.15 as suggested by some jet experiments,<sup>17</sup> but are in good agreement with the values suggested in Refs. 7 and 8.

Further study of hadronic  $\psi$  production could lead to useful information about quark and gluon distribution functions in hadrons, about intrinsic  $k_T$  of partons, and perhaps even about details of charmonium binding.

## ACKNOWLEDGMENTS

The authors would like to acknowledge discussions on  $\psi$  production with S. Jones of the University of Alabama, with D. Duke of Florida State University, and with J. Donohue of the University of Bordeaux. One of us (B.H.) would like to thank Fermilab for hospitality during a visit, when much of this work was completed.

#### APPENDIX

In the zero-binding-charmonium approximation used in this article, each charmed quark in the  $\psi$  is on the mass shell with half the  $\psi$  four-momentum,

$$P_{\mu} = P_{\psi\mu}/2 , \qquad (A1)$$

and thus half the  $\psi$  mass,

$$m = M_{\psi}/2 . \tag{A2}$$

The sum over  $\psi$  polarizations is therefore accomplished through the tensor

$$\sum \epsilon_{\lambda}(\psi)\epsilon_{\lambda}^{*}(\psi) = -g_{\lambda\lambda'} + \frac{P_{\lambda}P_{\lambda'}}{m^{2}} .$$
 (A3)

The square of the matrix element of (2.3), summed over final-state spins and colors, can be written

$$\sum |M|^{2} = F_{c} (4\pi\alpha_{s})^{4} \frac{\psi(0)^{2}}{N_{c}M_{\psi}} \frac{4|M_{c}|^{2}}{D'^{2}}, \qquad (A4)$$

where  $F_c$  is the color factor and

$$D' = P_1 \cdot P_2 P \cdot P_3 P \cdot P_4 (P \cdot P_3 + P \cdot P_4 + P_3 \cdot P_4) .$$
 (A5)

With the particles identified as in Fig. 1 the reduced matrix element squared is

$$|M_{c}|^{2} = -4(P_{1} \cdot P_{2} + 2m_{1}^{2})m^{2}[-8m^{4}(P_{3} \cdot P_{4})^{2} - 4(m^{2}P_{3} \cdot P_{4} - P \cdot P_{3}P \cdot P_{4})B_{2} - (P_{3} \cdot P_{4})^{4} + 8P \cdot P_{3}P \cdot P_{4}(P_{3} \cdot P_{4})^{2} - 2P \cdot P_{3}P \cdot P_{4}m^{-2}P_{3} \cdot P_{4}B_{1}^{2}] + 4P_{1} \cdot P_{3}P_{2} \cdot P_{3}m^{2}[-4m^{2}(P_{3} \cdot P_{4})^{2} + B_{1}B_{3}] + 4P_{1} \cdot P_{4}P_{2} \cdot P_{4}m^{2}[-4m^{2}(P_{3} \cdot P_{4})^{2} + B_{1}B_{4}] - 8m^{2}[P_{1} \cdot P_{3}P_{2} \cdot P_{4} + P_{1} \cdot P_{4}P_{2} \cdot P_{3} - (P_{1} \cdot P_{2} + m_{1}^{2})P_{3} \cdot P_{4}][m^{2}P_{3} \cdot P_{4}B_{1} - P \cdot P_{3}P \cdot P_{4}m^{-2}B_{1}^{2}].$$
(A6)

Here *m* is the charmed-quark mass and  $m_1$  (which we hereafter neglect) is the light-quark mass. The four *B*'s are given by

$$B_1 = -P_3 \cdot P_4 - 2P \cdot P_3 - 2P \cdot P_4 , \qquad (A7a)$$

$$B_2 = -4P \cdot P_3 P \cdot P_4 - P_3 \cdot P_4 B_1 , \qquad (A7b)$$

$$B_3 = (P_3 \cdot P_4)^2 - 4P \cdot P_4 P_3 \cdot P_4 - 4(P \cdot P_4)^2, \qquad (A7c)$$

$$B_4 = (P_3 \cdot P_4)^2 - 4P \cdot P_3 P_3 \cdot P_4 - 4(P \cdot P_3)^2$$
. (A7d)

The denominator  $D'^2$  in (A4) vanishes at threshold like  $(k^2 - M_{\psi}^2)^6$ . However, the same expression can be factored out of the matrix element squared times the phase-space element so that the cross section has no infrared singularities. Nevertheless some care is required in the numerical integration because of the high power of vanishing factors in the numerator and denominator. The antiquark and quark momenta  $P_1$  and  $P_2$ , respectively, are related to the hadron-beam and target-nucleon momenta by

$$P_{1\mu} = x_1 P_{b\mu}$$
, (A8)

$$P_{2\mu} = x_2 P_{t\mu} . \tag{A9}$$

The phase-space differential is

$$d\Omega = \frac{d^3 P_{\psi}}{2E_{\psi}} \frac{d^3 P_3}{2E_3} \frac{d^3 P_4}{2E_4} \frac{\delta^4 (k - P_4 - P_3 - P_{\psi})}{(2\pi)^5} .$$
(A10)

We write this as

$$d\Omega = d\hat{t} \, d\hat{u} \, d\cos\theta_{\psi} d\phi_3 / (2\pi^4 k^2) , \qquad (A11)$$

where

$$\hat{t} = (P_3 + P_{th})^2 , \qquad (A12)$$

$$\hat{u} = (P_4 + P_{\psi})^2 , \qquad (A13)$$

and  $\cos\theta$  is the angle between the  $\psi$  momentum and the antiquark momentum  $P_1$  in the quark-antiquark rest frame  $(\vec{k}=0)$ :

$$\cos\theta_{J} = \frac{P_{1} \cdot k P_{\psi} \cdot k - P_{1} \cdot P_{\psi} k^{2}}{(P_{1} \cdot k) [(P_{\psi} \cdot k)^{2} - M_{\psi}^{2} k^{2}]^{1/2}} .$$
(A14)

The angle  $\phi_3$  is defined in the frame in which

$$\vec{\mathbf{P}}_1 + \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_{\psi} = 0 \tag{A15}$$

with  $\vec{P}_{\psi}$  along the z axis and  $\vec{P}_1$  in the x,z plane. In this frame

$$\hat{P}_1 \cdot \hat{P}_{\psi} = \cos\theta_1 , \qquad (A16)$$

$$\hat{P}_3 \cdot \hat{P}_{\psi} = \cos\theta_3 , \qquad (A17)$$

and

$$P_1 \cdot (P_4 - P_3) = 2E_1 E_3 (\cos\theta_1 \cos\theta_3 + \sin\theta_1 \sin\theta_3 \cos\phi_3) .$$
(A18)

 $|M_c|^2$  can be trivially integrated over  $\phi_3$  and we have therefore done this integration analytically. The color factor in (A4) is

$$F_{c} = \frac{1}{2} \sum_{a,b,c} \operatorname{Tr} \left[ \frac{T^{a}T^{b}T^{c} + T^{a}T^{c}T^{b}}{2} \right] \\ \times \operatorname{Tr} \left[ \frac{T^{a}T^{b}T^{c} + T^{a}T^{c}T^{b}}{2} \right] \\ = \frac{(N_{c}^{2} - 4)(N_{c}^{2} - 1)}{32N_{c}} .$$
 (A19)

These results, together with Eqs. (2.6) to (2.15), suffice to determine the differential cross sections.

- \*Present address: Department of Physics, Indiana University, Bloomington, IN 47405.
- <sup>1</sup>For an extensive review see Francis Halzen, in Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982, edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. 43 (1982)].
- <sup>2</sup>H. Fritzsch, Phys. Lett. <u>67B</u>, 217 (1977).
- <sup>3</sup>M. Gluck, J. F. Owens, and E. Reya, Phys. Rev. D <u>17</u>, 2324 (1978); M. Gluck and E. Reya, Phys. Lett. <u>79B</u>, 453 (1978); <u>83B</u>, 98 (1979); C. E. Carlson and R. Suaya, Phys. Rev. D <u>18</u>, 760 (1978); Phys. Lett. <u>81B</u>, 329 (1979); V. Barger, W. Y. Keung, and R. J. N. Phillips, Z. Phys. C <u>6</u>, 169 (1980).
- <sup>4</sup>J. Leveille, in Proceedings of the Topical Workshop on the Production of New Particles in Super High Energy Collisions, Madison, Wisconsin, 1979, edited by V. Barger and F. Halzen (University of Wisconsin Press, Madison 1979); and Madison Report No. COO-881-120, 1979 (unpublished); see also V. Barger, J. Leveille, and W. Y. Keung, report, 1979 (unpublished).
- <sup>5</sup>R. Baier and R. Ruckl, Phys. Lett. <u>102B</u>, 364 (1981).
- <sup>6</sup>W. Y. Keung, in *Proceedings of the Z<sup>0</sup> Physics Workshop, Ithaca, New York, 1981*, edited by M. E. Peskin and S.-H. H. Tye (Laboratory of Nuclear Studies, Cornell University, Ithaca, 1981); E. L. Berger and D. Jones, Phys. Rev. D <u>23</u>, 1521

(1981).

- <sup>7</sup>B. Adeva *et al.*, Mark J Collaboration, Phys. Rev. Lett. <u>50</u>, 2051 (1983).
- <sup>8</sup>P. Mackenzie and P. Lepage, Phys. Rev. Lett. <u>47</u>, 1244 (1981).
- <sup>9</sup>Y. Lemoigne *et al.*, Goliath Spectrometer Collaboration, Phys. Lett. <u>113B</u>, 509 (1982).
- <sup>10</sup>J. P. Leveille and D. M. Scott, Phys. Lett. <u>95B</u>, 96 (1980).
- <sup>11</sup>J. H. Kühn and H. Schneider, Z. Phys. C <u>11</u>, 263 (1981).
- <sup>12</sup>J. G. H. deGroot *et al.*, CDHS Collaboration, Phys. Lett.
  <u>82B</u>, 455 (1979); D. Decamp, NA3 Collaboration, in *High Energy Physics—1980*, proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrum (AIP, New York, 1981), p. 149.
- <sup>13</sup>E. Anassontzis *et al.*, Athens-Fermilab-McGill-Michigan-Shandong Collaboration, presented at the 21st International Conference on High Energy Physics, Paris, 1982, Fermilab report Conf-82/50-EXP (unpublished).
- <sup>14</sup>L. Lyons, Prog. Part. Nucl. Phys. <u>7</u>, 169 (1981).
- <sup>15</sup>K. J. Anderson *et al.*, Chicago-Illinois-Princeton Collaboration, Phys. Rev. Lett. <u>42</u>, 944 (1979).
- <sup>16</sup>J. Badier et al., NA3 Collaboration, in High Energy Physics— 1980 (Ref. 12), p. 201.
- <sup>17</sup>W. Bartel et al., JADE Collaboration, Phys. Lett. <u>119B</u>, 239 (1982).