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Spin structure of the nucleon

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Within the framework of the quark-parton model and perturbative QCD the spin-dependent structure functions of the nucleon are discussed. The formalism suggests dramatic behavior for the distribution describing the polarization of the d quark in a polarized proton (or a u quark in a polarized neutron). Possible experimental tests of this result are noted.

A complete description of strong-interaction phenomena must include an understanding of the spin-dependent structure functions of the nucleon. Quantitative knowledge of these structure functions is essential for the development and testing of models of hadronic composition. At high ($Q^2 > 1$ GeV^2) momentum transfers the interaction of hadrons can be described[1] using the parton model; it is with this kinematic region that the following analysis is primarily concerned.

Of particular interest in such an analysis is the polarization of the d quark in a polarized proton (or equivalently, the polarization of the u quark in a polarized neutron). The basic argument is simple.[2] First, as is shown below, sum rules imply that the helicity of the d quark is *generally* opposite to the helicity of its parent proton. Secondly, in the limit that this quark carries all the momentum of its parent, simple perturbative arguments suggest that is should also carry the same helicity. Thus the helicity of a d quark relative to its parent proton should flip (at least once) as a function of its momentum fraction. If an asymmetry involving only the d quark could be constructed (candidates are proposed below), then it should display a sign change as a function of momentum fraction—a rather dramatic experimental signature.

Experimentally[3,4] most knowledge of hadronic spin-dependent structure functions at high momentum transfer has come from the measurement of the asymmetry

$$A = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \tag{1}$$

in deep-inelastic scattering. Here σ is the total absorption cross section of the virtual photon by the nucleon. The subscripts $\frac{1}{2}$ and $\frac{3}{2}$ are the components of the angular momentum of the virtual photon plus nucleon parallel to the virtual-photon momentum.

In the parton model, A is given by[1]

$$A = \frac{2xg_1(x)}{F_2(x)} , \tag{2}$$

with

$$2g_1(x) = \sum_i e_i^2[q_i^+(x) - q_i^-(x)] \equiv \sum_i e_i^2 \Delta q_i(x) , \tag{3a}$$

$$F_2(x) = \sum_i e_i^2[q_i^+(x) + q_i^-(x)] \equiv \sum_i e_i^2 q_i(x) , \tag{3b}$$

where the sum is over quark flavors in the nucleon. The e_i are the electric charges of each quark (in units of $|e|$) and $q_i^+(x)$ $[q_i^-(x)]$ is the distribution in longitudinal-momentum fraction x of quarks of flavor q whose helicity equals [opposes] that of the parent hadron. Thus, for the proton,

$$2g_1^p(x) = \tfrac{4}{9}\Delta u(x) + \tfrac{1}{9}\Delta d(x) , \tag{4a}$$

$$\frac{F_2^p(x)}{x} = \tfrac{4}{9}u(x) + \tfrac{1}{9}d(x) , \tag{4b}$$

where antiquarks and heavy sea quarks have been neglected.

In Eqs. (4) and in the following discussion the notation $u(x)$ is used interchangeably to mean the distribution of u quarks in the proton or the distribution of d quarks in the neutron. Similarly, $d(x)$ refers to d [u] quarks in the proton [neutron]. The implied strong isospin symmetry is a consequence of neglecting electromagnetic effects relative to the strong interaction. Thus the corresponding structure functions for the neutron may be derived from Eqs. (4) by interchanging $u(x)$ and $d(x)$.

Asymmetries at the quark level may also be defined:

$$A_u(x) \equiv \Delta u(x)/u(x) , \tag{5a}$$

$$A_d(x) \equiv \Delta d(x)/d(x) . \tag{5b}$$

Presumably,[1] polarized structure functions reflect the nonperturbative nature of the underlying theory and thus they cannot be calculated solely in perturbation theory. The form of the nonscaling behavior, which can be calculated in perturbation theory, is not relevant here. However, the polarization distributions obey certain sum rules. One such sum rule was derived by Bjorken[5]; in the language of the parton model it is given by

$$\int_0^1 dx[A_u(x)u(x) - A_d(x)d(x)] = \frac{G_A}{G_V} , \tag{6}$$

where $G_A/G_V \cong 1.25$ is the ratio of axial-vector to vector coupling constants in neutron β decay.

A second sum rule is obtained by calculating the expectation value of the contribution of the quark spins, denoted by S_3, to the total proton spin. With the assumption of an unpolarized "sea" of quark-antiquark pairs this sum rule and Eq. (6) give

$$\int_0^1 dx\, A_u(x)\, u(x) = S_3 + \frac{1}{2}\frac{G_A}{G_V} \ , \tag{7a}$$

$$\int_0^1 dx A_d(x)\, d(x) = S_3 - \frac{1}{2}\frac{G_A}{G_V} < 0 \ . \tag{7b}$$

[Note the bound on Eq. (7b).] If the spin of the proton arises entirely from the spin of the quarks, then $S_3 = \frac{1}{2}$. An argument[6] based on SU(3) symmetry produces a sum rule corresponding to Eq. (7) for Ξ^- decay which, along with data[7] on these decays, suggests $S_3 = 0.3$, which is used in the numerical analyses described below. Such a result is in agreement with other sum rules.[8] Note that a polarization of the antiquarks (and gluons) in the sea along the direction of the helicity of the parent hadron, as suggested by a QCD-based analysis,[9] also serves to lower the bound on S_3 $(<\frac{1}{2})$ and on Eq. (7b). Only a large oppositely oriented contribution from orbital angular momentum could change the sign of the bound in Eq. (7b) (i.e., allow $S_3 > \frac{1}{2} G_A/G_V$).

It is reasonable to assume that the quark distributions are dominated by the (unpolarized) sea as $x \to 0$; thus

$$\lim_{x\to 0} \begin{bmatrix} A_u(x) \\ A_d(x) \end{bmatrix} = 0 \ . \tag{8}$$

Perturbative QCD arguments[10] suggest that a quark at $x=1$ carries the spin of the parent nucleon, and thus

$$\lim_{x\to 1} \begin{bmatrix} A_u(x) \\ A_d(x) \end{bmatrix} = 1 \ . \tag{9}$$

As stated above, the sum rule Eq. (7b) along with the boundary condition Eq. (9) implies that $A_d(x)$ changes sign as a function of x. This sign change at some unspecified value of x (call this point x_0) suggests the following form for $A_d(x)$:

$$A_d(x) = \left[\frac{x - x_0}{1 - x_0}\right] x^p \ . \tag{10}$$

This form for $A_d(x)$ satisfies the boundary conditions Eqs. (8) and (9) if p is positive.

In the following analysis x_0 is treated as an adjustable parameter, while $p(x_0)$ is determined by means of the sum rule Eq. (7b). The parton distributions of Field and Feynman[11] are used for this purpose; the numerical results are given in Table I. The corresponding functions $A_d(x)$ are plotted in Fig. 1. Also plotted in Fig. 1 is the function

$$A_d^{\mathrm{CF}}(x) = -\tfrac{1}{3}x^{0.23} \ , \tag{11}$$

TABLE I. Parameters for $A_d(x)$.

	x_0	$p(x_0)$
$A_d^{\mathrm{I}}(x)$	0.25	0.15
$A_d^{\mathrm{II}}(x)$	0.50	0.46
$A_d^{\mathrm{III}}(x)$	0.75	0.97

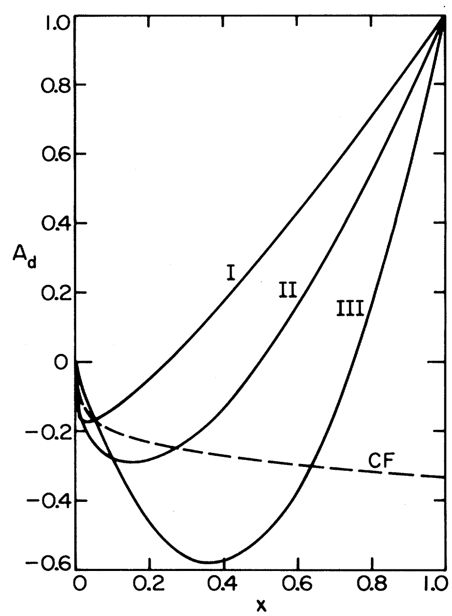

FIG. 1. The d-quark asymmetry function for the four cases A_d^{I}, A_d^{II}, A_d^{III}, and A_d^{CF} plotted as a function of x. Note especially the different behavior as x approaches one.

which is taken from Ref. 12. This asymmetry function does *not* satisfy the perturbative QCD result Eq. (9), as is clear in Fig. 1, although it does satisfy the sum rule Eq. (7). Similar d-quark asymmetry functions have been proposed by other authors[13]; unfortunately they all possess this difficulty.

In order to calculate nucleon asymmetries a u-quark asymmetry function is needed. The function

$$A_u^{\mathrm{CF}}(x) = x^{0.39} \tag{12}$$

proposed in Ref. 12 satisfies the constraints Eqs. (7)–(9) and seems to fit the data. It has therefore been employed in the present analysis.

In Fig. 2(a) the proton asymmetry

$$A_p(x) \cong \frac{\frac{4}{9}A_u(x)\,u(x) + \frac{1}{9}A_d(x)\,d(x)}{\frac{4}{9}u(x) + \frac{1}{9}d(x)} \tag{13}$$

(where heavy quarks and antiquarks have been neglected) is plotted versus x for the various d-quark polarization functions. The parton distributions are from Ref. 11 and the u-quark asymmetry is taken from Ref. 12. For clarity, only the distributions $A_d^{\mathrm{I}}(x)$ and $A_d^{\mathrm{III}}(x)$ defined in Table I are used. As can be seen, the agreement with the data[4] is excellent, particularly for $A_d^{\mathrm{III}}(x)$. At the same time it should also be noted that the choice of d-quark asymmetry function makes little difference in the proton asymmetry. For example, the proton asymmetry calculated using the distribution Eq. (11) is essentially indistinguishable in the plot from the choice $A_d^{\mathrm{III}}(x)$. Hence proton-asymmetry data provide little information about the d-quark asymmetry even at the level of the data in Fig. 1.

Figure 2(b) displays the neutron asymmetry $A_n(x)$, calculated using Eq. (13) with u and d interchanged. The

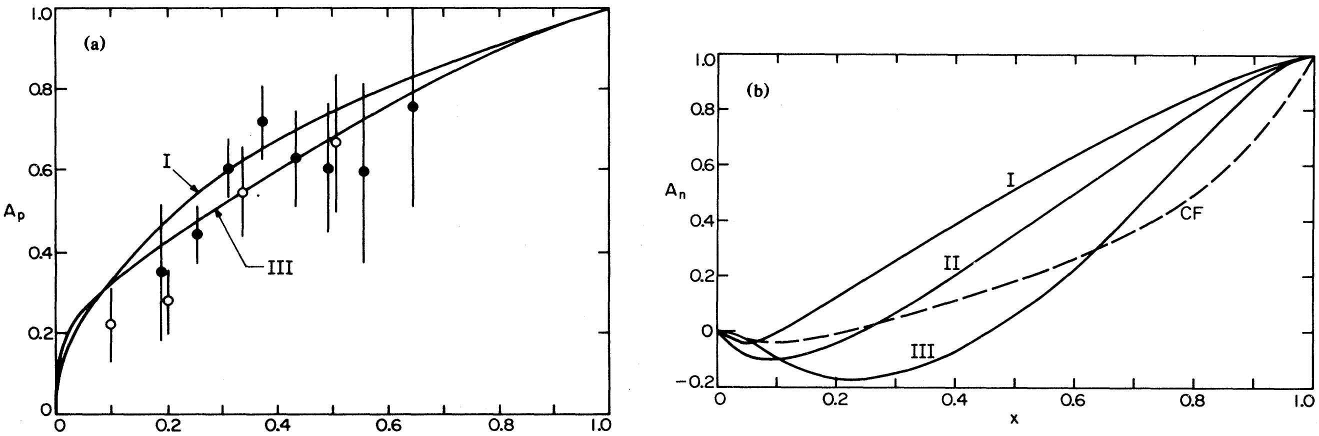

FIG. 2. (a) The proton asymmetry A_p calculated using A_d^{I} and A_d^{III} plotted as a function of x and compared with the data from the summary in the first paper of Ref. 4. For clarity, only two theoretical curves are shown. SLAC E-130 data are represented by closed circles and revised SLAC E-80 data are represented by open circles. (b) The neutron asymmetry $A_n(x)$ plotted as a function of x for each of the d-quark asymmetry functions $A_d(x)$ shown in Fig. 1.

choices of A_d are the same as in Fig. 1. Note that in all four cases the *neutron* asymmetry changes sign as a function of x. Thus the sign change in A_d is not responsible for the sign change in A_n. Also note that using d-quark asymmetry functions $A_d(x)$ which have the boundary condition Eq. (9) as x approaches one gives a much larger neutron asymmetry than predicted by previous models.[12,13]

Of course, in order to observe the dramatic behavior of the d-quark asymmetry seen in Fig. 1 it is necessary to take appropriate combinations of spin-dependent proton and neutron structure functions. Unfortunately, the latter function has not yet been measured. Other possible opportunities to isolate the d-quark asymmetry involve the measurement of asymmetries in polarized lepton-pair production,[2,14,15] polarized hadron-hadron scattering,[16] and polarized large-P_T photoproduction.[17]

It has been shown that current-algebra sum rules and simple arguments from perturbative QCD imply a dramatic result visible in the spin-dependent structure function for the d quark in the proton. Experimental measurements of this distribution should serve as a clean test of these perturbative ideas.[10] Probably the most realistic way to observe this effect experimentally involves a measurement of the polarization asymmetry of the neutron in deep-inelastic scattering, which may be performed in the near future.

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