Self-consistent lowest-order dual-topological-unitarization Regge-trajectory and coupling calculation

L. A. P. Balázs
Physics Department, Purdue University, West Lafayette, Indiana 47907

P. Gauron and B. Nicolescu
Division de Physique Théorique, Institut de Physique Nucléaire, 91406 Orsay and Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, Paris, France
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Recently a generalized topological-unitarization scheme has been developed in which the effect of "sea"-quark loops is taken into account from the beginning. At the lowest-order planar "zero-entropy" level, a self-consistent calculation of the leading Regge trajectory $\alpha(t)$ gives a ground-state mass $m_0=0.13\alpha'^{-1/2}$ and a coupling $g_0{}^2/4\pi$ of the order of the fine-structure constant, suggesting a strong-electroweak unification. This calculation does not entail any free (input) parameters.

During the past several years considerable progress has been made in developing a topological-unitarization (TU) approach for calculating soft-process amplitudes, the domain where most of the data in particle physics is to be found. In striking contrast to many other approaches in the confinement region, quark loops play an essential role from the beginning in TU, a feature which is in harmony with the important role that "sea" quarks are known to play for small values of the fractional quark momentum within hadrons in deep-inelastic lepton-hadron scattering, with the ready formation of hadronic jets, and with the rather large phenomenological values of typical hadronic couplings.

In the TU approach one starts with planar quark duality diagrams of the type shown in Fig. 1, where the solid lines are (purely mathematical) quark lines and the dashed lines carry the four-momenta of the hadrons. Higher-order corrections are then developed via a topological expansion. Earlier ($1/N_{\rm flavor}$) versions of this approach,[1] while unambiguous and successful for $q\bar{q}$ mesons (M_2), could not consistently take into account B_3 (qqq) and M_4 ($qq\bar{q}\,\bar{q}$) states. However, the generalized topological-unitarization scheme of Chew and Poenaru[2] and Stapp[3] provides a consistent way of overcoming this difficulty by effectively writing $B_3=q\delta$ and $M_4=\delta\bar{\delta}$, where $\delta=qq$ is a "diquark" with a certain well-defined topological structure at the lowest-order level. Figure 1 is then generalized by allowing one or more $q\to\bar{\delta}$ line replacements. Higher-order corrections, which would lead to "interactions" of the dashed hadronic four-momentum lines with these $\bar{\delta}$ lines, can then be argued to be small.[2,3]

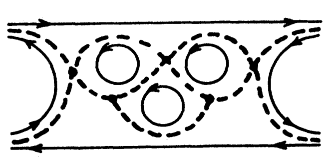

FIG. 1. A typical planar quark-duality diagram.

At the lowest-order "zero-entropy" level the basic (planar) dynamics can be represented symbolically by Fig. 2 (where the left side is a shorthand notation for the sum of the graphs of Fig. 1 and their $q\to\bar{\delta}$ generalizations) and each S-matrix element can be consistently factored into a known spin-momentum factor and a common spin-independent dynamical function T of the external four-momenta. Within this fully relativistic level it can be shown that the usual simple quark-model $q\bar{q}$ and qqq hadronic ground-state families are generated despite the presence of quark loops,[2,3] and that an interesting "topological-supersymmetry" property arises, with $q\bar{q}$-qqq-$qq\bar{q}\,\bar{q}$ degeneracy and with realistic consequences for hadronic cross sections.[4] This topological-supersymmetry property states that at the zero-entropy level the masses of the ground states $q\bar{q}$, qqq, and $qq\bar{q}\,\bar{q}$ have the same value m_0 and that all the ground-state hadronic coupling constants can be computed in terms of a single basic coupling constant g_0. The unitarity relations and dynamical equations satisfied by our T amplitudes turn out to be identical to the equations which would be satisfied by the corresponding S-matrix elements if all the ground-state particles (collectively denoted by z) were spinless. We can then compute in a self-consistent way the zero-entropy constants $m_0{}^2\alpha'$ and g_0, which in turn would permit us to calculate the higher-order contributions; here α' is the slope of the lowest-order Regge trajectory.

For zz scattering at moderate values of t, the dominant contribution to the sum of graphs of Fig. 1 and their $q\to\bar{\delta}$ generalizations is expected to come from the infinite ladder sum of the s-channel unitarity graphs of Fig. 3, which generates a leading "output" Regge trajectory $\alpha(t)$

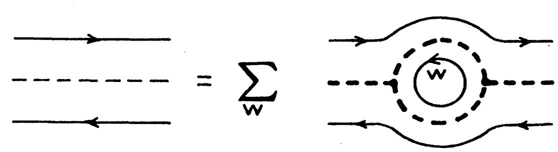

FIG. 2. Zero-entropy dynamics, where w labels all possible q and $\bar{\delta}$ lines.

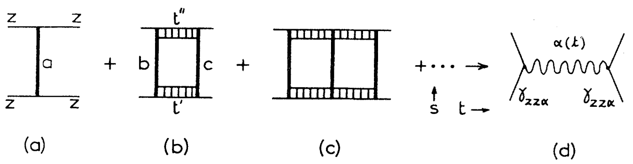

FIG. 3. Infinite ladder sum of s-channel unitarity graphs.

and in which the masses of the vertical-line clusters $a,b,c,\ldots$ must be bounded to avoid double-counting, say, between Figs. 3(a) and 3(b). The ladder exchanges within Figs. 3(b), 3(c), ... themselves have the form of the entire sum of Fig. 3. In practice we will approximate them by the Regge exchange $\alpha(t)$ in what follows.

Figure 1 does not give rise to the usual Regge cuts. When combined with analyticity it therefore implies simple finite-energy sum-rule (FESR) duality, which can be used, for example, to relate Figs. 3(a) and 3(d) and gives

$$\int_{-\infty}^{\bar{s}(t)} ds[\Gamma_0\delta(s-m_0^2)-\gamma_{zz\alpha}^2(t)\nu^{\alpha(t)}\theta(\nu)]=0\ , \qquad (1)$$

where the δ-function term is the lowest nonvanishing contribution of Fig. 3(a) (coming from the ground state z of mass m_0) to the s-channel absorptive part A ($=\mathrm{Im}T$), $\gamma_{zz\alpha}^2(t)\nu^{\alpha(t)}$ is the asymptotic Regge behavior of A, ν is the usual crossing-symmetric variable $(s-u)/2$ or

$$\nu=s+\tfrac{1}{2}(t-4m_0^2)\ , \qquad (2)$$

and the upper limit $\bar{s}(t)$ corresponds to a point between z and the next significant contribution above it, at least if we invoke semilocal duality to prevent double counting. The constant Γ_0 is related to the basic dimensionless zzz coupling g_0 of our TU theory through

$$\Gamma_0=N(N-1)m_0^2g_0^2\ , \qquad (3)$$

where $N=32$ is the multiplicity of each quark loop in Fig. 2 (2 spins $\times 2$ charges $\times 4$ generations $\times 2$ chiralities).[5] Note that Fig. 2 requires $g_0N^{1/2}(N-1)^{1/2}/4\pi$ to be of the order of unity.[5]

In previous papers it was shown that, if we make a certain simple peaking approximation for Fig. 3(b), the ladder dynamics of Fig. 3, when combined with FESR duality, leads to a linear $\alpha(t)$.[6,7] We shall continue to assume this as a first approximation in what follows.

To deal with the graphs of Fig. 3 we make the kinematic-singularity-free Mellin-transform projection:[7]

$$A(j,t)=\int_0^\infty d\nu A(s,t)\nu^{-j-1}\ , \qquad (4)$$

and formally associate a coupling-strength parameter ϕ with each of the clusters $a,b,c,\ldots$. If we then take the [1,1] Padé approximant of the resulting expansion in ϕ, we obtain, for a given t,

$$A(j,t)=W(j,t)/[1-K(j,t)]\ , \qquad (5)$$

where

$$K(j,t)=B(j,t)/W(j,t)\ , \qquad (6)$$

and W and B are the contributions of Figs. 3(a) and 3(b), respectively. Equation (5) is in fact exact if we have a factorizable model. It gives a Regge pole at $j=\alpha(t)$ if

$$K(\alpha(t),t)=1\ , \qquad (7)$$

with a residue

$$\gamma_{zz\alpha}^2(t)=-W(\alpha(t),t)\Big/\left[\frac{\partial}{\partial j}K(j,t)\right]_{j=\alpha(t)}\ . \qquad (8)$$

Figure 3(a) gives, approximately,

$$W(s,t)=\Gamma_0\delta(s-m_0^2)+\gamma_{zz\alpha}^2(t)\nu^{\alpha(t)}\theta(s-\bar{s})\theta(s_0-s)\ . \quad (9)$$

The δ function is again the contribution of the ground state z and the Regge term takes into account all the higher $(s>\bar{s})$ contributions to W in the semilocal duality sense of Eq. (1). Since we are insisting on no double counting between Figs. 3(b) and 3(c) we have inserted a step function $\theta(s_0-s)$ to exclude states above the effective threshold $s=s_0$ of Fig. 3(b). Since s_0 is relatively large and is expected (because of duality) to be a relatively slowly varying function of t, we shall take it to be a constant. In practice, W was further approximated by a single δ function

$$W(s,t)=\Gamma_W(t)\delta(s-s_W(t))\ , \qquad (10)$$

where the s integral over Eq. (10) was required to be equal to the one over Eq. (9) for all s, and the integral over Eq. (9) for $s>s_W$ was required to be equal to the one for $s<s_W$. Exactly the same set of approximations were made for the clusters b and c of Fig. 3(b), with Eq. (1) replaced by the generalized finite-mass sum rule

$$\int_{-\infty}^{\bar{s}(t)} ds[\gamma_{zz\alpha}(t')\gamma_{zz\alpha}(t'')\delta(s-m_0^2) -\gamma_{zz\alpha}(t)g(t',t'',t)\omega^{\alpha(t)-\alpha(t')-\alpha(t'')}\theta(\omega)]=0\ , \qquad (11)$$

and the analog of the right-hand side of Eq. (9) again approximated by a single δ function, this time at $s=s_X(t',t'',t)$; here $\omega=s+\frac{1}{2}(t-2m_0^2-t'-t'')$, and $\bar{s}(t)$ and s_0 are the same as before.

For $t\neq 0$, Fig. 3(b) involves a double integral over the internal momentum transfers t' and t''. We therefore restrict ourselves to the value and first two t derivatives at $t=0$, which can all be reduced to single integrals. We also set

$$s_X(t',t'',t)\simeq s_X(0,0,t)\ , \qquad (12)$$

and introduce a t' cutoff at $\alpha(t')=-\frac{3}{2}$ to avoid the tachyon singularities at $\alpha(t')=-2,-3,\ldots$ which arise because of our incomplete implementation of duality. Neither of these approximations should have too much of an effect on our final results, however, because of the sharp peaking of our integrands for small $|t'|$ and $|t''|$. Finally, an upper cutoff $\theta(s_2-s)$ was introduced into the contribution $B(s,t)$ of Fig. 3(b) to prevent double counting with Fig. 3(c), since each of these should represent, in a duality sense, the entire contribution to A near its own s threshold. Thus, for example,

$$B(s,0)=\frac{1}{16\pi}[s(s-4m_0^2)]^{-1/2}\int_{t_-}^{t_+}dt'\,|T_X(s,t')|^2\theta(\alpha(t')+\tfrac{3}{2})\theta(s-4s_{X_0})\theta(s_2-s)\ , \tag{13}$$

where

$$t_\pm=-\tfrac{1}{2}s+s_{X_0}+m_0^2\pm\tfrac{1}{2}[(s-4s_{X_0})(s-4m_0^2)]^{1/2}\ ,$$

$$s_{X_0}=s_X(0,0,0) \tag{14}$$

and T_X is the "amplitude" for $zz\to XX$ with Regge exchange $\alpha(t')$.

Equations (7) and (8) and their first two derivatives at $t=0$ now lead to six conditions on our parameters

$$F_i=0\ ,\quad i=1,\ldots,6 \tag{15}$$

which can only be solved numerically. We therefore varied our parameters so as to minimize a dimensionless

$$\chi^2=\tfrac{1}{6}\sum_{i=1}^{6}(F_i/\epsilon_i)^2\ , \tag{16}$$

where we have introduced a set of ϵ_i for dimensional reasons and for properly weighting each term in the sum. We must remember that physical restrictions on our parameters result in correlations between them, which induce, in turn, further restrictive constraints on our solutions. In such a highly nonlinear situation, we must therefore begin our search for a solution by dropping one or more of our constraints and then attempting to find a condition where the addition of a further constraint does not change our results in any appreciable way. We must then be close to the final solution, where, of course, the number of parameters is equal to the number of constraints.

We have restricted ourselves to solutions for which any reasonable extrapolation of $\bar{s}(t)$ from $t=0$ leads to $\bar{v}(t)>0$ (which permits our formalism to be well defined, as it stands, for all t). We then found that the solution with the smallest χ^2, which corresponded to $10^{-6}\lesssim|F_i|\lesssim10^{-4}$ for all of the F_i of Eq. (15), led to

$$\alpha' m_0^2\simeq0.017\ ,\quad g_0^2/4\pi\simeq\tfrac{1}{800}\ , \tag{17}$$

$$\alpha'\bar{s}(0)\simeq0.05\ ,\quad \bar{s}'(0)\simeq-1\ ,\qquad \alpha' s_0\simeq0.20\ ,\quad \alpha' s_2\simeq0.80\ . \tag{18}$$

Note that our $\bar{s}(0)$ is not far from the background threshold of $4m_0^2$. On the other hand, if we combine Eq. (1) with the exact relation

$$\alpha'(m_0^2)\Gamma_0=\gamma_{zz\alpha}{}^2(m_0^2)$$

demanded by crossing symmetry, we obtain

$$\bar{s}(m_0^2)=1.03\alpha'^{-1}\gg\bar{s}(0)\ ,$$

which corresponds to a very rapid variation of $\bar{s}(t)$ in the $0<t<m_0^2$ range. It would be interesting to see whether this persists if we were to improve our calculation and perhaps extend it to finite positive t.

Our value of $g_0(\simeq0.4e)$ is close to the value of e, where $e^2/4\pi=\frac{1}{137}$. This is consistent with certain recent ideas about a possible unification of soft hadronic physics and electromagnetic interactions within a topological expansion.[5] Note that our $\alpha' m_0^2\ll1$. This means that there are two different energy scales in our problem, one set by $\alpha'^{-1/2}$ and a smaller one by m_0. The latter suggests the possibility of a PCAC-type (partial conservation of axial-vector current) constraint at the zero-entropy level, with the z playing the role of the pion.

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