Study of muon-number-violating hyperon decays

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Muon-number-violating hyperon decays are discussed, first from a phenomenological point of view and then considering some possible sources of muon-number violation. We show that branching ratios as large as of the order of 10^{-6} for the decays $\Sigma^+ \to p\mu^\mp e^\pm$ and $\Xi^0 \to \Lambda\mu^\mp e^\pm$ cannot be ruled out. We note, however, that such branching ratios are unlikely, since they would require muon-number-violating interactions of very special structure.

I. INTRODUCTION

In the minimal standard $SU(2)_L \times U(1)$ gauge theory of the electroweak interactions[1] involving only a single Higgs doublet, muon number is conserved if the neutrinos are massless. For massive neutrinos the weak eigenstates and the mass eigenstates of the neutrinos do not coincide in general, resulting in muon-number violation (and in other transitions between different lepton families). In the presence of only the three known lepton families the branching ratios of muon-number-violating processes (such as $\mu \to e\gamma$, $\mu^- N \to e^- N$, $K_L \to \mu e$) are orders of magnitude below the present experimental limits.[2,3] This is due to the suppression factors m_ν^2/m_W^2 present in the amplitudes and the experimental limits on the neutrino masses.[4] Larger branching ratios for muon-number-violating processes are possible, however, in many theoretical schemes that go beyond the minimal standard model. The possibilities include existence of flavor-changing neutral gauge bosons (for example, the gauge bosons associated with horizontal gauge interactions,[5] or the gauge bosons present in extended hypercolor theories[6]), existence of flavor-changing neutral Higgs bosons (in the standard model this would require the presence of two or more Higgs doublets[7]), composite models,[8] muon-number violation mediated by light leptoquarks (present in some grand unified theories[9] and in extended hypercolor theories[6]), muon-number violation mediated by supersymmetric partners of the usual $SU(2)_L \times U(1)$ gauge bosons,[10] and existence of new electroweak interactions.[11] Large branching ratios might result also in the minimal standard model if further lepton families, involving heavier neutrinos, exist.

The relative size of the rates of various muon-number-violating reactions depends on the mechanism of muon-number violation, and on the model and its parameters. This underlines the importance of searching for all possible muon-number-violating processes.

In this paper we investigate what information one could obtain from experimental searches for muon-number-violating decay modes of the usual octet hyperons,[12] in particular, as compared to the information provided by muon-number-violating K decays.

In Sec. II we study the muon-number-violating hyperon decays in a phenomenological framework, and consider the constraints imposed on their branching ratios by the available experimental information on muon-number-violating K decays. In Sec. III we discuss the branching ratios in the light of possible sources of muon-number violation, suggested by current theories. In Sec. IV we summarize our conclusions.

II. PHENOMENOLOGICAL CONSIDERATIONS

Considering only decay modes which do not involve neutrinos and/or photons, and which contain no more than three particles in the final state, the following muon-number-violating, lepton-number-conserving decays of $\Sigma^{\pm,0}$, Λ, and $\Xi^{\pm,0}$ are possible:

$$\Sigma^+ \to p\mu^\mp e^\pm \ , \tag{1a}$$

$$\Lambda \to n\mu^\mp e^\pm \ , \tag{1b}$$

$$\Xi^- \to \Sigma^- \mu^\mp e^\pm \ , \tag{1c}$$

$$\Xi^0 \to \Lambda\mu^\mp e^\pm \ , \tag{1d}$$

$$\Xi^0 \to \Sigma^0 \mu^\mp e^\pm \ , \tag{1e}$$

$$\Sigma^0 \to n\mu^\mp e^\pm \ , \tag{1f}$$

$$\Xi^0 \to n\mu^\mp e^\pm \ . \tag{1g}$$

Analogous decay modes are possible for the antihyperons. We note that none of the above decays conserve strangeness. The decays (1f) and (1g) will not be considered in the following, since the former has to compete with the large $\Sigma^0 \to \Lambda\gamma$ rate, and the latter requires a $\Delta S = 2$ effective quark current (and therefore is not expected in the same order as the $\Delta S = \pm 1$ transitions).

As yet, no experimental limits have been set on the branching ratios of (1a)—(1g) or on the branching ratios of the corresponding antihyperon decays. To give some perspective concerning present and future experimental possibilities, we note that experiments carried out to search for rare hyperon decays have so far been sensitive only to branching ratios of the order of 10^{-6} or larger. Sensitivities to branching ratios of the order of 10^{-10}—10^{-12} seem to be the ultimate ones that one can contemplate in the foreseeable future.

Some possible mechanisms for the decays (1a)—(1f) are shown on Fig. 1. Before considering these, we shall discuss the branching ratios of (1a)—(1e) in terms of effective muon-number-violating interactions, represented by general four-fermion couplings. In what follows we shall refer to a four-fermion coupling in which the quark bilinear transforms as a vector, axial-vector, scalar, pseudoscalar and a tensor as a *V*-, *A*-, *S*-, *P*-, and *T*-type coupling, respectively. Their contribution to the branching ratios of $a\to b\mu^{\mp}e^{\pm}$ [a,b=spin-$\frac{1}{2}$ baryons involved in (1a)—(1e)] will be denoted at $B_k(a\to b\mu^{\mp}e^{\pm})$ ($k=V$, A, S, P, or T). Unlike $K\to\mu e$ decays which are sensitive only to *A*- and *P*-type couplings, or $K\to\pi\mu e$ decays to which only *V*-, *S*-, or *T*-type couplings contribute, muon-number-violating hyperon decays receive contributions from couplings of all types.

For our further discussion the following special cases will be of particular interest: (1) an effective interaction involving vector and axial-vector currents; (2) an effective interaction constructed from scalar and pseudoscalar densities; and (3) a tensor-type four-fermion coupling. With the electron mass neglected, there are no interference terms in the $a\to b\mu e$ rates between *V*- and *A*-type or *S*- and *P*-type couplings.

In the following we shall quote the results only for hyperon decays. The results for the corresponding decays of antihyperons are the same since, neglecting corrections of order α (and in the absence of important diagrams with absorptive parts, which we shall assume), the rates of $a\to b\mu^- e^+$ and $a\to b\mu^+ e^-$ are equal (assuming *CPT* invariance) to the rates of $\bar a\to\bar b\mu^+e^-$ and $\bar a\to\bar b\mu^- e^+$, respectively.

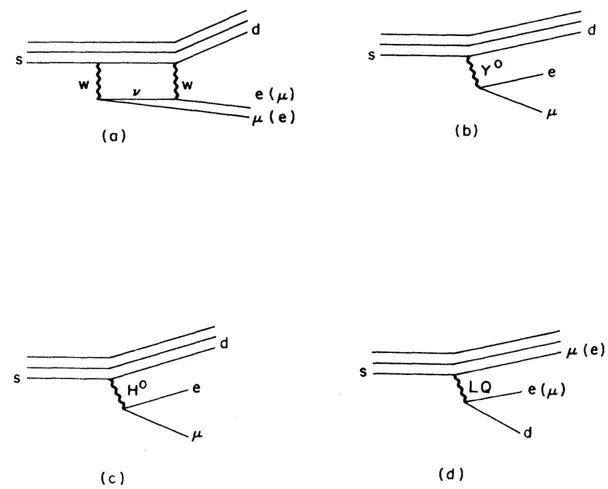

FIG. 1. Possible contributions to the decays (1a)—(1f). (a) Electroweak contribution; (b) contribution of a flavor-changing gauge boson; (c) flavor-changing Higgs-boson contribution; (d) leptoquark contribution.

A. *V*- and *A*-type couplings

The most general local nonderivative effective interaction for $s\to d\mu^{\mp}e^{\pm}$ and $\bar s\to\bar d\mu^{\mp}e^{\pm}$ involving vector and axial-vector currents is of the form[13]

$$H_{V,A}=\frac{G}{\sqrt{2}}[\bar e\gamma^\lambda\mu(g_{VV}\bar s\gamma_\lambda d+\tilde g_{VV}\bar d\gamma_\lambda s)+\bar e\gamma^\lambda\gamma_5\mu(g_{AV}\bar s\gamma_\lambda d+\tilde g_{AV}\bar d\gamma_\lambda s)$$
$$+\bar e\gamma^\lambda\mu(g_{VA}\bar s\gamma_\lambda\gamma_5 d+\tilde g_{VA}\bar d\gamma_\lambda\gamma_5 s)+\bar e\gamma^\lambda\gamma_5\mu(g_{AA}\bar s\gamma_\lambda\gamma_5 d+\tilde g_{AA}\bar d\gamma_\lambda\gamma_5 s)]+\text{H.c.}\ , \quad (2)$$

where g_{jk} and $\tilde g_{jk}$ ($j,k=V,A$) are constants measuring the strength of the couplings relative to $G/\sqrt{2}$ ($G\equiv$ Fermi constant $\simeq 10^{-5}/m_p{}^2$).

Neglecting the induced hadronic form factors (their effect is of the order of a few percent), the q^2 dependence of the form factors that remain (effects amounting at most about 10%), and the mass of the electron, the rates of the decays $a\to b\mu^- e^+$ and $a\to b\mu^+e^-$ are given by[14]

$$\Gamma(a\to b\mu^-e^+)=\frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^3(m_a-m_b)^5[\kappa_V^{ba}|F_1^{ba}|^2(|g_{VV}|^2+|g_{AV}|^2)+\kappa_A^{ba}|G_1^{ba}|^2(|g_{VA}|^2+|g_{AA}|^2)] \quad (3)$$

and

$$\Gamma(a\to b\mu^+e^-)=\frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^3(m_a-m_b)^5[\kappa_V^{ba}|F_1^{ba}|^2(|\tilde g_{VV}|^2+|\tilde g_{AV}|^2)+\kappa_A^{ba}|G_1^{ba}|^2(|\tilde g_{VA}|^2+|\tilde g_{AA}|^2)]\ . \quad (4)$$

In Eqs. (3) and (4), m_a and m_b are the masses of a and b, F_1^{ba} and G_1^{ba} are defined by the matrix elements

$$\langle b(p')|\bar d\gamma_\lambda s|a(p)\rangle=\bar u(p')\left[F_1^{ba}\gamma_\lambda+\frac{F_2^{ba}}{m_a+m_b}\sigma_{\lambda\rho}iq^\rho+\frac{F_3^{ba}}{m_a+m_b}q_\lambda\right]u_a(p) \quad (5)$$

and

$$\langle b(p')|\bar d\gamma_\lambda\gamma_5 s|a(p)\rangle=\bar u(p')\left[G_1^{ba}\gamma_\lambda+\frac{G_2^{ba}}{m_a+m_b}\sigma_{\lambda\rho}iq^\rho+\frac{G_3^{ba}}{m_a+m_b}q_\lambda\right]\gamma_5 u_a(p)\ , \quad (6)$$

respectively; $q=p'-p$. In the limit of SU(3) symmetry, which we shall assume, the form factors F_1^{ba} are related to the Dirac form factors of the neutron and the proton, and G_1^{ba} are linear combinations of the symmetric and antisymmetric reduced matrix elements F and D. The total width of a, the values of m_a-m_b, and the form factors F_1^{ba} and G_1^{ba} are listed in Table I. The quantities κ_V^{ba} and κ_A^{ba} are kinematic factors dependent on $m_\mu/(m_a+m_b)$ and $(m_a-m_b)/(m_a+m_b)$.[15] Their values are given in Table II.

The branching ratios can be written as

$$B_{V,A}(a\to b\mu^-e^+)=\gamma_V^{ba}(|g_{VV}|^2+|g_{AV}|^2)+\gamma_A^{ba}(|g_{VA}|^2+|g_{AA}|^2)\,, \quad (7)$$

$$B_{V,A}(a\to b\mu^+e^-)=\gamma_V^{ba}(|\tilde{g}_{VV}|^2+|\tilde{g}_{AV}|^2)+\gamma_A^{ba}(|\tilde{g}_{VA}|^2+|\tilde{g}_{AA}|^2)\,. \quad (8)$$

The constants γ_V^{ba} and γ_A^{ba}, given in Table III, have been evaluated using $F=0.44$ and $D=0.81$ (Ref. 16).

We observe that $B_{V,A}(\Xi^-\to\Sigma^-\mu e)$ and $B_{V,A}(\Xi^0\to\Sigma^0\mu e)$ are smaller by 2—3 orders of magnitude than the

TABLE I. m_a-m_b, $\Gamma(a\to\text{all})$, and the form factors in Eqs. (3) and (4).

$a\to b$	m_a-m_b (MeV)	$\Gamma(a\to\text{all})$ (eV)	F_1^{ba}	G_1^{ba}
$\Sigma^+\to p$	251.08	8.23×10^{-6}	1	$F-D$
$\Lambda\to n$	176.03	2.50×10^{-6}	$(3/2)^{1/2}$	$(3F+D)/\sqrt{6}$
$\Xi^-\to\Sigma^-$	123.98	4.01×10^{-6}	-1	$-(F+D)$
$\Xi^0\to\Lambda$	199.3	2.27×10^{-6}	$-(3/2)^{1/2}$	$(-3F+D)/\sqrt{6}$
$\Xi^0\to\Sigma^0$	122.4	2.27×10^{-6}	$1/\sqrt{2}$	$(F+D)/\sqrt{2}$

others. This is due to the relatively small values of m_a-m_b in $\Xi^{-,0}\to\Sigma^{-,0}\mu e$. From the remaining decays, which all have comparable rates, $\Sigma^+\to p\mu e$ and $\Xi^0\to\Lambda\mu e$ (Ref. 17) are the better candidates for an experimental study, since $\Lambda\to n\mu e$ requires the more demanding neutron detection.

The only direct experimental information on the coupling constants in Eq. (2) comes from searches for the decays $K_L\to\mu e$ and $K^\pm\to\pi^\pm\mu e$. The contribution of (2) to their branching ratios is given by[18]

$$B(K_L\to\mu^+e^-)=B(K_L\to\mu^-e^+)$$
$$=\frac{1}{\Gamma(K_L\to\text{all})}\frac{1}{8\pi}G^2m_\mu{}^2m_Kf_K{}^2\left[1-\frac{m_\mu{}^2}{m_K{}^2}\right]^2(|g_{VA}^{(+)}+i\epsilon g_{VA}^{(-)}|^2+|g_{AA}^{(+)}+i\epsilon g_{VA}^{(-)}|^2)$$
$$\simeq 63.6(|g_{VA}^{(+)}+i\epsilon g_{VA}^{(-)}|^2+|g_{AA}^{(+)}+i\epsilon g_{AA}^{(-)}|^2)\,, \quad (9)$$

$$B(K^+\to\pi^+\mu^+e^-)=B(K^-\to\pi^-\mu^-e^+)=\frac{1}{\sin^2\theta_C}B(K^+\to\pi^0\mu^+\nu_\mu)(|g_{VV}|^2+|g_{AV}|^2)$$
$$\simeq 0.66(|g_{VV}|^2+|g_{AV}|^2)\,, \quad (10)$$

and

$$B(K^+\to\pi^+\mu^-e^+)=B(K^-\to\pi^-\mu^+e^-)=\frac{1}{\sin^2\theta_C}B(K^+\to\pi^0\mu^+\nu_\mu)(|\tilde{g}_{VV}|^2+|\tilde{g}_{AV}|^2)$$
$$\simeq 0.66(|\tilde{g}_{VV}|^2+|\tilde{g}_{AV}|^2)\,, \quad (11)$$

where (adopting the Wu-Yang phase convention) $K_L\simeq K_2+\epsilon K_1$, $\epsilon\simeq(2.3\times10^{-3})e^{i\pi/4}$ (Ref. 19), $f_K\simeq1.23m_\pi$ is the $K^+\to\mu^+\nu_\mu$ decay constant,[20] and $\theta_C\simeq0.22$ is the Cabibbo angle.[16] The constants $g_{kl}^{(+)}$ and $g_{kl}^{(-)}$ ($kl=VA,AA$) are defined as

$$g_{kl}^{(+)}=\tfrac{1}{2}(g_{kl}+\tilde{g}_{kl})\ \ (kl=VA,AA) \quad (12)$$

and

$$g_{kl}^{(-)}=-\frac{i}{2}(g_{kl}-\tilde{g}_{kl})\ \ (kl=VA,AA)\,. \quad (13)$$

TABLE II. The kinematic factors κ_j^{ba} ($j=V,A,S,P,T$) in Eqs. (3), (4), (43), (44), (86), and (87).

$a\to b$	κ_V^{ba}	κ_A^{ba}	κ_S^{ba}	κ_P^{ba}	κ_T^{ba}
$\Sigma^+\to p$	0.681	2.02	0.672	0.218	8.10
$\Lambda\to n$	0.250	0.747	0.249	0.059	2.99
$\Xi^-\to\Sigma^-$	0.011	0.032	0.011	1.0×10^{-3}	0.130
$\Xi^0\to\Lambda$	0.395	1.18	0.393	0.108	4.72
$\Xi^0\to\Sigma^0$	8.4×10^{-3}	0.025	8.4×10^{-3}	7.4×10^{-4}	0.101

TABLE III. The quantities appearing in Eqs. (7), (8), (51), (52), (89), and (90).

$a\to b$	γ_V^{ba}	γ_A^{ba}	$(m_s-m_d)^2\gamma_S^{ba}$ (MeV^2)	$(m_s+m_d)^2\gamma_P^{ba}$ (MeV^2)	γ_T^{ba}
$\Sigma^+\to p$	1.4×10^{-3}	5.5×10^{-4}	8.4×10	5.2	2.2×10^{-3}
$\Lambda\to n$	4.5×10^{-4}	6.8×10^{-4}	1.4×10	2.0	2.7×10^{-3}
$\Xi^-\to\Sigma^-$	1.6×10^{-6}	7.3×10^{-6}	2.4×10^{-2}	4.0×10^{-3}	2.9×10^{-5}
$\Xi^0\to\Lambda$	1.5×10^{-3}	1.3×10^{-4}	5.9×10	5.8×10^{-1}	5.1×10^{-4}
$\Xi^0\to\Sigma^0$	1.0×10^{-6}	4.7×10^{-6}	1.5×10^{-2}	2.3×10^{-3}	1.9×10^{-5}

The experimental limits

$$B(K_L\to\mu e)_{\rm expt}<2\times10^{-9}\ \ (90\%\ \text{C.L.})\ \ (\text{Ref. 21})\ , \tag{14}$$

$$B(K^+\to\pi^+\mu^+e^-)_{\rm expt}<5\times10^{-9}\ \ (90\%\ \text{C.L.})\ \ (\text{Ref. 22})\ , \tag{15}$$

and

$$B(K^+\to\pi^+\mu^-e^+)_{\rm expt}<7\times10^{-9}\ \ (90\%\ \text{C.L.})\ \ (\text{Ref. 22}) \tag{16}$$

imply

$$|g_{VA}^{(+)}+i\epsilon g_{VA}^{(-)}|^2+|g_{AA}^{(+)}+i\epsilon g_{AA}^{(-)}|^2\lesssim3\times10^{-11}\ , \tag{17}$$

$$|g_{AV}|^2+|g_{VV}|^2\lesssim8\times10^{-9}\ , \tag{18}$$

and

$$|\tilde g_{AV}|^2+|\tilde g_{VV}|^2\lesssim10^{-8}\ , \tag{19}$$

respectively.

Let us consider the contributions of V- and A-type couplings separately.

1. V-type couplings

For $g_{VA}=g_{AA}=\tilde g_{VA}=\tilde g_{AA}=0$ we obtain from Eqs. (7), (8), (10), and (11) the relation

$$B_V(a\to b\mu^\mp e^\pm)\simeq1.5\gamma_V^{ba}B_V(K^+\to\pi^+\mu^\pm e^\mp)\ . \tag{20}$$

Thus,

$$B_V(\Sigma^+\to p\mu^\mp e^\pm)\simeq(2\times10^{-3})B_V(K^+\to\pi^+\mu^\pm e^\mp)\ , \tag{21}$$

$$B_V(\Xi^0\to\Lambda\mu^\mp e^\pm)\simeq(2\times10^{-3})B_V(K^+\to\pi^+\mu^\pm e^\mp)\ . \tag{22}$$

The experimental limits (15) and (16) imply the bounds [ignoring the small difference between the upper limits (15) and (16) and using for both the value (15)]

$$B(a\to b\mu^\mp e^\pm)\leq\Omega_V^{ba}\ . \tag{23}$$

The bounds Ω_V^{ba} are listed in Table IV. For $\Sigma^+\to p\mu e$ and $\Xi^0\to\Lambda\mu e$ they are

$$B(\Sigma^+\to p\mu^\mp e^\pm)\lesssim10^{-11} \tag{24}$$

and

$$B(\Xi^0\to\Lambda\mu^\mp e^\pm)\lesssim10^{-11}\ . \tag{25}$$

2. A-type couplings

Using the identities

$$\tfrac{1}{2}(1+\epsilon)^2|g_{jA}|^2+\tfrac{1}{2}(1-\epsilon)^2|\tilde g_{jA}|^2=|g_{jA}^{(+)}+i\epsilon g_{jA}^{(-)}|^2+|g_{jA}^{(-)}-i\epsilon g_{jA}^{(+)}|^2\quad(j=V,A)\ , \tag{26}$$

we obtain the sum rule

$$\tfrac{1}{2}[B_A(a\to b\mu^-e^+)+B_A(a\to b\mu^+e^-)]\simeq\gamma_A^{ba}[(1.6\times10^{-2})B_A(K_L\to\mu e)+9.1B_A(K_S\to\mu e)]\ , \tag{27}$$

relating the contributions of A-type couplings to the branching ratios of $a\to b\mu^\mp e^\pm$, $K_L\to\mu e$, and $K_S\to\mu e$. The contribution of (2) to $B(K_S\to\mu e)$ is given by[18]

$$B_A(K_S\to\mu e)=\frac{1}{\Gamma(K_S\to\text{all})}\frac{1}{8\pi}G^2m_\mu{}^2m_Kf_K{}^2\left[1-\frac{m_\mu{}^2}{m_K{}^2}\right]^2(|g_{VA}^{(-)}-i\epsilon g_{VA}^{(+)}|^2+|g_{AA}^{(-)}-i\epsilon g_{AA}^{(+)}|^2)$$

$$\simeq0.11(|g_{VA}^{(-)}-i\epsilon g_{VA}^{(+)}|^2+|g_{AA}^{(-)}-i\epsilon g_{AA}^{(+)}|^2)\ . \tag{28}$$

TABLE IV. The upper bounds in Eqs. (23), (31), (37), (39), (70), (74), (80), and (81).

$a\to b$	Ω_V^{ba}	$\Omega_{A,1}^{ba}$	$\Omega_{A,s}^{ba}$	Ω_S^{ba}	$\Omega_{P,1}^{ba}$	$\Omega_{P,s}^{ba}$
$\Sigma^+\to p$	1.0×10^{-11}	1.7×10^{-14}	5.0×10^{-6}	5.7×10^{-12}	2.9×10^{-17}	8.6×10^{-9}
$\Lambda\to n$	3.4×10^{-12}	2.1×10^{-14}	6.2×10^{-6}	9.5×10^{-13}	1.1×10^{-17}	3.3×10^{-9}
$\Xi^-\to\Sigma^-$	1.2×10^{-14}	2.3×10^{-16}	6.7×10^{-8}	1.6×10^{-15}	2.3×10^{-20}	6.6×10^{-12}
$\Xi^0\to\Lambda$	1.1×10^{-11}	4.0×10^{-15}	1.2×10^{-6}	4.0×10^{-12}	3.3×10^{-18}	9.6×10^{-10}
$\Xi^0\to\Sigma^0$	7.7×10^{-15}	1.5×10^{-16}	4.3×10^{-8}	1.0×10^{-15}	1.3×10^{-20}	3.8×10^{-12}

No experimental limit has been set so far on $B(K_S \to \mu e)$. In the following we shall use

$$B(K_S \to \mu e)_{\text{expt}} \lesssim 10^{-3} , \tag{29}$$

which is consistent with the lifetime of K_S and with data on its partial decay rates. The limit (29) implies

$$|g_{VA}^{(-)} - i\epsilon g_{VA}^{(+)}|^2 + |g_{AA}^{(-)} - i\epsilon g_{AA}^{(+)}|^2 \lesssim 9\times 10^{-3} . \tag{30}$$

In models where the contribution of $B(K_S \to \mu e)$ in Eq. (27) can be neglected we have

$$\tfrac{1}{2}[B_A(a\to b\mu^- e^+) + B_A(a\to b\mu^+ e^-)] \le \Omega_{A,l}^{ba} , \tag{31}$$

where $\Omega_{A,l}^{ba}$ are given in Table IV. In particular,[12]

$$\tfrac{1}{2}[B_A(\Sigma^+\to p\mu^- e^+) + B_A(\Sigma^+\to p\mu^+ e^-)] \lesssim 2\times 10^{-14} \tag{32}$$

and

$$\tfrac{1}{2}[B_A(\Xi^0\to \Lambda\mu^- e^+) + B_A(\Xi^0\to \Lambda\mu^+ e^-)] \lesssim 4\times 10^{-15} . \tag{33}$$

An example is the case when $g_{jk} = \tilde{g}_{jk}$ ($jk \simeq VA,AA$), since then

$$B(K_S\to\mu e) = |\epsilon|^2[\Gamma(K_L\to\mu e)/\Gamma(K_S\to\mu e)] \lesssim 2\times 10^{-17} .$$

For $g_{kl} = -\tilde{g}_{kl}$ ($kl = VA,AA$) one would have

$$B(K_L\to\mu e) = |\epsilon|^2\Gamma(K_S\to\mu e)/\Gamma(K_L\to\text{all}) ,$$

implying $B(K_S\to\mu e) \lesssim 7\times 10^{-7}$. Equation (27) yields in this case

$$B_A(a\to b\mu^- e^+) = B_A(a\to b\mu^+ e^-) \lesssim (6\times 10^{-6})\gamma_A^{ba} , \tag{34}$$

so that

$$B_A(\Sigma^+\to p\mu^- e^+) = B_A(\Sigma^+\to p\mu^+ e^-) \lesssim 3\times 10^{-9} \tag{35}$$

and

$$B_A(\Xi^0\to\Lambda\mu^- e^+) = B_A(\Xi^0\to\Lambda\mu^+ e^-) \lesssim 8\times 10^{-10} . \tag{36}$$

In general $B(K_S\to\mu e)$, and consequently $B_A(a\to b\mu e)$, could be larger.[23] Using the limit (29), Eq. (27) yields

$$\tfrac{1}{2}[B_A(a\to b\mu^- e^+) + B_A(a\to b\mu^+ e^-)] \le \Omega_{A,s}^{ba} . \tag{37}$$

The upper bounds $\Omega_{A,s}^{ba}$ are listed in Table IV.

Observing that the limits (14) and (29) imply

$$|B_A(a\to b\mu^- e^+) - B_A(a\to b\mu^+ e^-)| \lesssim (2\times 10^{-6})\gamma_A^{ba} , \tag{38}$$

and also that $\Omega_{A,s}^{ba} \gg (10^{-6})\gamma_A^{ba}$, Eq. (37) can be rewritten as

$$B_A(a\to b\mu^\mp e^\pm) \le \Omega_{A,s}^{ba} . \tag{39}$$

Thus,

$$B_A(\Sigma^+\to p\mu^\mp e^\pm) \lesssim 5\times 10^{-6} \tag{40}$$

and

$$B_A(\Xi^0\to\Lambda\mu^\mp e^\pm) \lesssim 10^{-6} . \tag{41}$$

B. *S*- and *P*-type couplings

The most general coupling is of the form

$$H_{S,P} = \frac{G}{\sqrt{2}}[\bar{e}\mu(g_{SS}\bar{s}d + \tilde{g}_{SS}\bar{d}s) + \bar{e}i\gamma_5\mu(g_{PS}\bar{s}d + \tilde{g}_{PS}\bar{d}s) + \bar{e}\mu(g_{SP}\bar{s}i\gamma_5 d + \tilde{g}_{SP}\bar{d}i\gamma_5 s) + \bar{e}i\gamma_5\mu(g_{PP}\bar{s}i\gamma_5 d + \tilde{g}_{PP}\bar{d}i\gamma_5 s)] + \text{H.c.} \tag{42}$$

The Hamiltonian (42) gives rise to $a\to b\mu^\mp e^\pm$ with rates

$$\Gamma(a\to b\mu^- e^+) = \frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^2 (m_a-m_b)^5\left[\kappa_S^{ba}|F_S^{ba}|^2(|g_{SS}|^2+|g_{PS}|^2) + \kappa_P^{ba}\left[\frac{m_a-m_b}{m_a+m_b}\right]^2|F_P^{ba}|^2(|g_{SP}|^2+|g_{PP}|^2)\right] \tag{43}$$

and

$$\Gamma(a\to b\mu^+ e^-) = \frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^3 (m_a-m_b)^5\left[\kappa_S^{ba}|F_S^{ba}|^2(|\tilde{g}_{SS}|^2+|\tilde{g}_{PS}|^2) + \kappa_P^{ba}\left[\frac{m_a-m_b}{m_a+m_b}\right]^2|F_P^{ba}|^2(|\tilde{g}_{SP}|^2+|\tilde{g}_{PP}|^2)\right] , \tag{44}$$

where κ_S and κ_P are kinematic factors, analogous to κ_V^{ba} and κ_A^{ba}, dependent on $m_\mu/(m_a+m_b)$ and $(m_a-m_b)/(m_a+m_b)$. They are listed in Table II.[24] The form factors F_S^{ba} and F_P^{ba} are defined by

$$\langle b \mid \bar{d}s \mid a \rangle = \bar{u}_b F_S^{ba} u_a \quad (45)$$

and

$$\langle b \mid \bar{d} i\gamma_5 s \mid a \rangle = \bar{u}_b F_P^{ba} i\gamma_5 u_a \ . \quad (46)$$

They can be expressed in terms of F_1^{ba} and G_1^{ba} using the relations

$$\frac{\partial}{\partial x_\lambda}(\bar{d}\gamma_\lambda s) = i(m_d - m_s)\bar{d}s \ , \quad (47)$$

$$\frac{\partial}{\partial x_\lambda}(\bar{d}\gamma_\lambda\gamma_5 s) = (m_s + m_d)\bar{d} i\gamma_5 s \ , \quad (48)$$

where m_s and m_d are the current masses of s and d. One obtains

$$F_s^{ba}(q^2) = \frac{m_a - m_b}{m_s - m_d} F_1^{ba}(q^2)\left[1 - \frac{q^2}{m_a{}^2 - m_b{}^2}\frac{F_3^{ba}(q^2)}{F_1^{ba}(q^2)}\right] \quad (49)$$

and

$$F_P^{ba}(q^2) = \frac{m_a + m_b}{m_s + m_d} G_1^{ba}(q^2)\left[1 + \frac{q^2}{(m_a+m_b)^2}\frac{G_3^{ba}(q^2)}{G_1^{ba}(q^2)}\right]$$
$$\simeq \frac{m_a + m_b}{m_s + m_d} G_1^{ba}(q^2)\left[1 + \frac{q^2}{m_K{}^2 - q^2}\frac{G_1^{ba}(0)}{G_1^{ba}(q^2)}\right] \ . \quad (50)$$

To obtain the last line in Eq. (50) we used the PCAC (partial conservation of axial-vector current) relation

$$G_3^{ba}(q^2) \simeq [(m_a+m_b)^2/(m_K{}^2 - q^2)]G_1^{ba}(0) \ .$$

The branching ratios of $a \to b\mu^\mp e^\pm$ are given by

$$B(a \to b\mu^- e^+) = \gamma_S^{ba}(|g_{SS}|^2 + |g_{PS}|^2) + \gamma_P^{ba}(|g_{PP}|^2 + |g_{SP}|^2) \ , \quad (51)$$

$$B(a \to b\mu^+ e^-) = \gamma_S^{ba}(|\tilde{g}_{SS}|^2 + |\tilde{g}_{PS}|^2) + \gamma_P^{ba}(|\tilde{g}_{PP}|^2 + |\tilde{g}_{SP}|^2) \ . \quad (52)$$

The constants $\gamma_S^{ba}(m_s - m_d)^2$ and $\gamma_P^{ba}(m_s + m_d)^2$, which are independent of the quark masses, are given in Table III. In calculating γ_S^{ba} we have neglected the F_3-dependent term in Eq. (49).[25] As expected, also in this case the branching ratios of $\Xi^{-,0} \to \Sigma^{-,0}\mu e$ are suppressed relative to the others by 2–3 orders of magnitude. The contribution of (42) to $B(K_L \to \mu e)$ and $B(K^\pm \to \pi^\pm \mu e)$ is given by[18]

$$B(K_L \to \mu^+ e^-) = B(K_L \to \mu^- e^+)$$
$$= \frac{1}{\Gamma(K_L \to \text{all})}\frac{1}{8\pi}G^2 m_K{}^3 f_K{}^2\left[1 - \frac{m_\mu{}^2}{m_K{}^2}\right]^2\left[\frac{m_K}{m_s + m_d}\right]^2(|g_{SP}^{(+)} + i\epsilon g_{SP}^{(-)}|^2 + |g_{PP}^{(+)} + i\epsilon g_{PP}^{(-)}|^2)$$
$$\simeq (3.5\times 10^8)(m_s + m_d)^{-2}\ \text{MeV}^2(|g_{SP}^{(+)} + i\epsilon g_{SP}^{(-)}|^2 + |g_{PP}^{(+)} + i\epsilon g_{PP}^{(-)}|^2) \ , \quad (53)$$

$$B(K_S \to \mu^+ e^-) = B(K_S \to \mu^- e^+)$$
$$= \frac{1}{\Gamma(K_S \to \text{all})}\frac{1}{8\pi}G^2 m_K{}^3 f_K{}^2\left[1 - \frac{m_\mu{}^2}{m_K{}^2}\right]^2\left[\frac{m_K}{m_s + m_d}\right]^2(|g_{SP}^{(-)} - i\epsilon g_{SP}^{(+)}|^2 + |g_{PP}^{(-)} - i\epsilon g_{PP}^{(+)}|^2)$$
$$\simeq (6\times 10^5)(m_s + m_d)^{-2}\ \text{MeV}^2(|g_{SP}^{(-)} - i\epsilon g_{SP}^{(+)}|^2 + |g_{PP}^{(-)} - i\epsilon g_{PP}^{(+)}|^2) \ , \quad (54)$$

$$B(K^+ \to \pi^+\mu^+ e^-) = B(K^- \to \pi^-\mu^- e^+) = \frac{1}{\Gamma(K^+ \to \text{all})}\frac{G^2 m_K{}^2}{32\pi^3}|f_S|^2\mathscr{I}(|g_{SS}|^2 + |g_{PS}|^2)$$
$$\simeq (7.4\times 10^4)(m_s - m_d)^{-2}\ \text{MeV}^2(|g_{SS}|^2 + |g_{PS}|^2) \ , \quad (55)$$

and

$$B(K^+ \to \pi^+\mu^- e^+) = B(K^- \to \pi^-\mu^+ e^-) = \frac{1}{\Gamma(K^+ \to \text{all})}\frac{G^2 m_K{}^2}{32\pi^3}|f_S|^2\mathscr{I}(|\tilde{g}_{SS}|^2 + |\tilde{g}_{PS}|^2)$$
$$\simeq (7.4\times 10^4)(m_s - m_d)^{-2}\ \text{MeV}^2(|\tilde{g}_{SS}|^2 + |\tilde{g}_{PS}|^2) \ . \quad (56)$$

The coupling constants $g_{PP}^{(\pm)}$ and $g_{SP}^{(\pm)}$ are defined in the same way as $g_{VA}^{(\pm)}$ and $g_{AA}^{(\pm)}$, i.e.,

$$g_{kl}^{(+)}=\tfrac{1}{2}(g_{kl}+\tilde{g}_{kl}) \quad (kl=SP,PP)\,, \tag{57}$$

$$g_{kl}^{(-)}=\frac{-i}{2}(g_{kl}-\tilde{g}_{kl}) \quad (kl=SP,PP)\,. \tag{58}$$

The quantity f_S in Eqs. (55) and (56) is the form factor associated with the $K^+\to\pi^+$ matrix element of the scalar quark density $[\langle\pi^+|\bar{s}d|K^+\rangle=m_K f_S(q^2)]$ and is given by

$$f_S(q^2)=\frac{m_\pi{}^2-m_K{}^2}{(m_s-m_d)m_K}f_+^{sd}(q^2)\times\left[1-\frac{q^2}{m_\pi{}^2-m_K{}^2}\frac{f_-^{sd}(q^2)}{f_+^{sd}(q^2)}\right]\,, \tag{59}$$

where the form factors $f_+^{sd}(q^2)$ and $f_-^{sd}(q^2)$ are defined as

$$\langle\pi^+(p_\pi)|\bar{s}\gamma_\lambda d|K^+(p_K)\rangle =[f_+^{sd}(q^2)(p_K+p_\pi)_\lambda+f_-^{sd}(q^2)(p_K-p_\pi)_\lambda]\,. \tag{60}$$

We have taken $f_+(q^2)\simeq f_+(0)$, used the SU(3)-symmetric value $f_+(0)=1$, and neglected the $f_-^{sd}(q^2)$-dependent term in Eq. (59).[26]

The quantity $\mathscr{I}$ in Eqs. (55) and (56) is a phase-space integral:

$$\mathscr{I}=\int_{m_\pi}^{E_\pi^{\max}}\frac{(E_\pi^{\max}-x)^2}{E_\pi^{\max}-x+m_\mu{}^2/2m_K}(x^2-m_\pi{}^2)^{1/2}dx \simeq 5.7\times10^5\ \text{MeV}^3 \tag{61}$$

($E_\pi^{\max}\equiv$ maximum energy of the pion; $m_\mu\equiv$ muon mass).

The experimental limits (14), (29), (15), and (16) imply[27]

$$|g_{SP}^{(+)}+i\epsilon g_{SP}^{(-)}|^2+|g_{PP}^{(+)}+i\epsilon g_{PP}^{(-)}|^2 \lesssim 6\times10^{-18}(m_s+m_d)^2\ \text{MeV}^{-2}\,, \tag{62}$$

$$|g_{SP}^{(-)}-i\epsilon g_{SP}^{(+)}|^2+|g_{PP}^{(-)}-i\epsilon g_{PP}^{(+)}|^2 \lesssim 2\times10^{-9}(m_s+m_d)^2\ \text{MeV}^{-2}\,, \tag{63}$$

$$|g_{PS}|^2+|g_{SS}|^2\lesssim 7\times10^{-14}(m_s-m_d)^2\ \text{MeV}^{-2}\,, \tag{64}$$

and

$$|\tilde{g}_{PS}|^2+|\tilde{g}_{SS}|^2\lesssim 10^{-13}(m_s-m_d)^2\ \text{MeV}^{-2}\,. \tag{65}$$

We shall consider now the special cases of pure S-type and pure P-type couplings.

1. *S-type couplings*

From Eqs. (51), (52), (55), and (56) it follows that

$$B_S(a\to b\mu^-e^+)\simeq(1.4\times10^{-5}\ \text{MeV}^{-2})(m_s-m_d)^2 \times\gamma_S^{ba}B_S(K^+\to\pi^+\mu^+e^-)\,, \tag{66}$$

$$B_S(a\to b\mu^+e^-)\simeq(1.4\times10^{-5}\ \text{MeV}^{-2})(m_s-m_d)^2 \times\gamma_S^{ba}B_S(K^+\to\pi^+\mu^-e^+)\,. \tag{67}$$

In particular,

$$B_S(\Sigma^+\to p\mu^\mp e^\pm)\simeq10^{-3}B_S(K^+\to\pi^+\mu^\pm e^\mp) \tag{68}$$

and

$$B_S(\Xi^0\to\Lambda\mu^\mp e^\pm)\simeq(8\times10^{-4})B_S(K^+\to\pi^+\mu^\pm e^\mp)\,. \tag{69}$$

Equations (66) and (67) and the experimental limits (15) and (16) lead to the bounds [ignoring again the small difference between (15) and (16)]

$$B_S(a\to b\mu^\mp e^\pm)\leq\Omega_S^{ba} \tag{70}$$

(cf. Table IV). Thus,

$$B_S(\Sigma^+\to p\mu^\mp e^\pm)\lesssim 6\times10^{-12}\,, \tag{71}$$

$$B_S(\Xi^0\to\Lambda\mu^\mp e^\pm)\lesssim 4\times10^{-12}\,. \tag{72}$$

2. *P-type couplings*

The sum rule analogous to (27) is in this case

$$\tfrac{1}{2}[B_P(a\to b\mu^-e^+)+B_P(a\to b\mu^+e^-)] \simeq\gamma_P^{ba}(m_s+m_d)^2[(2.8\times10^{-9}\ \text{MeV}^{-2})B_P(K_L\to\mu e)+(1.7\times10^{-6}\ \text{MeV}^{-2})B_P(K_S\to\mu e)]\,. \tag{73}$$

If the term in (73) involving $B_P(K_S\to\mu e)$ can be neglected, as is the case, for example, when $g_{kl}=\tilde{g}_{kl}$ $(kl=SP,PP)$, we have

$$\tfrac{1}{2}[B_P(a\to b\mu^-e^+)+B_P(a\to b\mu^+e^-)]\leq\Omega_{P,l}^{ba} \tag{74}$$

(cf. Table IV). For $\Sigma^+\to p\mu e$ and $\Xi^0\to\Lambda\mu e$ Eq. (74) reads

$$\tfrac{1}{2}[B_P(\Sigma^+\to p\mu^-e^+)+B_P(\Sigma^+\to p\mu^+e^-)]\lesssim 3\times10^{-17}\,, \tag{75}$$

$$\tfrac{1}{2}[B_P(\Xi^0\to\Lambda\mu^-e^+)+B_P(\Xi^0\to\Lambda\mu^+e^-)]\lesssim 3\times10^{-18}\,. \tag{76}$$

For $g_{kl}=-\tilde{g}_{kl}$ $(kl=SP,PP)$ the experimental limit (14) implies $B(K_S\to\mu e)\lesssim 7\times10^{-7}$, so that

$$B_P(a\to b\mu^-e^+)=B_P(a\to b\mu^+e^-) \lesssim 10^{-12}\gamma_P(m_s+m_d)^2\,. \tag{77}$$

For $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$ (77) gives

$$B_P(\Sigma^+ \to p\mu^- e^+) = B_P(\Sigma^+ \to p\mu^+ e^-) \lesssim 6\times 10^{-12} \quad (78)$$

and

$$B_P(\Xi^0 \to \Lambda\mu^- e^+) = B_P(\Xi^0 \to \Lambda\mu^+ e^-) \lesssim 6\times 10^{-13} . \quad (79)$$

In general the limit (29) applies, leading to

$$\tfrac{1}{2}[B_P(a \to b\mu^- e^+) + B_P(a \to b\mu^+ e^-)] \le \Omega_{P,s}^{ba} \quad (80)$$

(cf. Table IV), and therefore

$$B_P(a \to b\mu^\mp e^\pm) \le \Omega_{P,s}^{ba} , \quad (81)$$

since the experimental limits (14) and (29) imply

$$|B_P(a \to b\mu^- e^+) - B_P(a \to b\mu^+ e^-)| \lesssim (4\times 10^{-13}\ \mathrm{MeV}^{-2})\gamma_P^{ba}(m_s+m_d)^2 , \quad (82)$$

and because

$$\Omega_{P,s}^{ba} \gg (2\times 10^{-13}\ \mathrm{MeV}^{-2})\gamma_P^{ba}(m_s+m_d)^2 .$$

The bounds for $B_P(\Sigma^+ \to p\mu^\mp e^\pm)$ and $B_P(\Xi^0 \to \Lambda\mu^\mp e^\pm)$ are

$$B_P(\Sigma^+ \to p\mu^\mp e^\pm) \lesssim 9\times 10^{-9} \quad (83)$$

and

$$B_P(\Xi^0 \to \Lambda\mu^\mp e^\pm) \lesssim 10^{-9} . \quad (84)$$

C. T-type couplings

The general effective Hamiltonian is of the form

$$H_T = \frac{G}{\sqrt{2}}[\bar{e}\sigma_{\lambda\eta}\mu(g_{TT}\bar{s}\sigma^{\lambda\eta}d + \tilde{g}_{TT}\bar{d}\sigma^{\lambda\eta}s) + \bar{e}\sigma_{\lambda\eta}i\gamma_5\mu(g'_{TT}\bar{s}\sigma^{\lambda\eta}d + \tilde{g}'_{TT}\bar{d}\sigma^{\lambda\eta}s)] + \mathrm{H.c.} \quad (85)$$

Neglecting the induced form factors, the $a \to b\mu e$ rates corresponding to (85) are

$$\Gamma_T(a \to b\mu^- e^+) = \frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^3 (m_a-m_b)^5 \kappa_T^{ba} |T_1^{ba}|^2 (|g_{TT}|^2 + |g'_{TT}|^2) , \quad (86)$$

$$\Gamma_T(a \to b\mu^+ e^-) = \frac{G^2}{192\pi^3}\left[\frac{m_a+m_b}{2m_a}\right]^3 (m_a-m_b)^5 \kappa_T^{ba} |T_1^{ba}|^2 (|\tilde{g}_{TT}|^2 + |\tilde{g}'_{TT}|^2) . \quad (87)$$

The kinematic factors κ_T^{ba} [dependent on $m_\mu/(m_a+m_b)$ and $(m_a-m_b)/(m_a+m_b)$] are given in Table II.[28] The form factor T_1^{ba} is defined by

$$\langle b(p')|\bar{d}\sigma_{\lambda\eta}s|a(p)\rangle = \bar{u}(p')(T_1^{ab}\sigma_{\lambda\eta} + \cdots)u(p) . \quad (88)$$

The branching ratios can be written as

$$B_T(a \to b\mu^- e^+) = \gamma_T^{ba}(|g_{TT}|^2 + |g'_{TT}|^2) , \quad (89)$$

$$B_T(a \to b\mu^+ e^-) = \gamma_T^{ba}(|\tilde{g}_{TT}|^2 + |\tilde{g}'_{TT}|^2) . \quad (90)$$

An estimate[29] of T_1^{ba} using the MIT bag model gives approximately $T_1^{ba} \simeq G_1^{ba}$. γ_T^{ba} calculated with these values are shown in Table II.

The contribution of the interaction (85) to $B(K \to \pi\mu e)$ is

$$B_T(K^+ \to \pi^+\mu^+ e^-) = B_T(K^- \to \pi^-\mu^- e^+) = (4.6\times 10^{-2})|f_T|^2(|g_{TT}|^2 + |g'_{TT}|^2) , \quad (91)$$

where the form factor f_T is defined as[30]

$$\langle \pi^+|\bar{s}\sigma^{\lambda\eta}d|K^+(p_K)\rangle = \frac{f_T}{m_K}(p_K^\lambda p_\pi^\eta - p_K^\eta p_\pi^\lambda) . \quad (92)$$

$B(K^\pm \to \pi^\pm\mu^\mp e^\pm)$ is given by the same expression, except for $g_{TT} \to \tilde{g}_{TT}$, $g'_{TT} \to \tilde{g}'_{TT}$. It follows that

$$B_T(a \to b\mu^\mp e^\pm) \simeq 22\frac{\gamma_T^{ba}}{|f_T|^2} B_T(K^+ \to \pi^+\mu^\pm e^\mp) . \quad (93)$$

For $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$ we obtain

$$B_T(\Sigma^+ \to p\mu^\mp e^\pm) \simeq \frac{(5\times 10^{-2})B_T(K^+ \to \pi^+\mu^\pm e^\mp)}{|f_T|^2} , \quad (94)$$

$$B_T(\Xi^0 \to \Lambda\mu^\mp e^\pm) \simeq \frac{10^{-2}B_T(K^+ \to \pi^+\mu^\pm e^\mp)}{|f_T|^2} . \quad (95)$$

No estimate is available for f_T. Equations (94) and (95) show that unless $f_T \lesssim 0.2$ ($f_T \lesssim 0.1$), $K \to \pi\mu e$ is more sensitive to a T-type coupling than $\Sigma^+ \to p\mu e$ ($\Xi^0 \to \Lambda\mu e$).

This ends our discussion of the phenomenological aspects of muon-number-violating hyperon decays. The main conclusions can be summarized as follows.

(1) For given coupling constants the branching ratios of $\Sigma^+ \to p\mu e$, $\Xi^0 \to \Lambda\mu e$, and $\Lambda \to n\mu e$ are comparable and about 2—3 orders of magnitude larger than the branching ratios of $\Xi^- \to \Sigma^-\mu e$ and $\Xi^0 \to \Sigma^0\mu e$. Thus, the best candidates for an experimental search appear to be the decays $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$.

(2) For a V-type or an S-type interaction the branching ratios of $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$ could be as large as 10^{-11}. The decays $K \to \pi\mu e$ are, however, more sensitive to these types of couplings by 3 orders of magnitude.

(3) $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ due to an A-type interaction for which $\Gamma(K_S \to \mu e) \lesssim \Gamma(K_L \to \mu e)$ cannot be larger than a few times 10^{-14}. For a general A-type interaction branching ratios of the order of 10^{-6} for $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$, though unlikely,[23] cannot be ruled out. The decay $K_S \to \mu e$ is more sensitive than

$\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$ by a factor of 10^2 and 10^3, respectively, but at a certain level of experimental sensitivity $\Sigma^+ \to p\mu e$ and/or $\Xi^0 \to \Lambda\mu e$ may be more accessible.

(4) For a P-type interaction $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ are negligible if $\Gamma(K_S \to \mu e) \lesssim \Gamma(K_L \to \mu e)$. For a general P-type interaction $B(\Sigma^+ \to p\mu e)$ of the order of 10^{-8} and $B(\Xi^0 \to \Lambda\mu e)$ of the order of 10^{-9}, though unlikely,[23] cannot be ruled out. However, in this case the decay $K_S \to \mu e$ is more sensitive than $\Sigma^+ \to p\mu e$ and $\Xi^0 \to \Lambda\mu e$ by a factor of 10^5 and 10^6, respectively.

(5) For a tensor-type interaction the upper bounds on $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ depend on the unknown ratio of the form factors T_1^{ba} and f_T [cf. Eq. (93)]. With $T_1^{ba} \simeq G_1^{ba}$ (Ref. 29), $B(\Sigma^+ \to p\mu e)$ $[B(\Xi^0 \to \Lambda\mu e)]$ would be more sensitive to these couplings than $B(K^\pm \to \pi^\pm \mu e)$ if $f_T \lesssim 0.2$ $(f_T \lesssim 0.1)$.

A tensor term is expected at the tree level only from spin-0 leptoquark exchange (cf. Sec. III). The corresponding constants g_{TT} and g'_{TT} are related in this case to the coupling constants of scalar-type terms. As a consequence, the upper bounds on $B(a \to b\mu e)$, which follow from $B(K \to \pi\mu e)_{\text{expt}}$, are independent of the form factor f_T.

We note that if A- and P-type couplings appear simultaneously, the constraints on their strength might be weaker [and consequently the upper bounds on $B(a \to b\mu e)$ less stringent] than for pure A-type or pure P-type interactions, in view of the possibility of cancellations in the $K_L \to \mu e$ and/or $K_S \to \mu e$ amplitudes. On the other hand, inspection shows that the upper bounds on the strength of pure S, V, and T couplings from $K^+ \to \pi^+ \mu e$ decays increase by not more than a factor of 5 if these couplings appear simultaneously.

III. THEORETICAL POSSIBILITIES FOR $a \to b\mu e$

A. $a \to b\mu e$ in the minimal standard model

The transition $sd \to \mu e$ is described in lowest order by diagram (a) in Fig. 1. The corresponding effective interaction is of the form (2) with (assuming that m_W is the largest mass in the model[2])

$$g_{VV} \simeq \sum_{j,i=1}^{n} \frac{1}{4\pi^2\sqrt{2}} G m_i^2 U_{ei} U_{\mu i}^* V_{dj} V_{sj}^* I(\epsilon_{vi}, \epsilon_{qj}) , \qquad (96)$$

$$\tilde{g}_{VV} \simeq \sum_{j,i=1}^{n} \frac{1}{4\pi^2\sqrt{2}} G m_i^2 U_{ei} U_{\mu i}^* V_{dj}^* V_{sj} I(\epsilon_{vi}, \epsilon_{qj}) , \qquad (97)$$

$$g_{AA} = -g_{VA} = -g_{AV} = g_{VV} , \qquad (98)$$

$$\tilde{g}_{AA} = -\tilde{g}_{VA} = -\tilde{g}_{AV} = \tilde{g}_{VV} . \qquad (99)$$

In Eqs. (96) and (97), U_{ei}, $U_{\mu i}$ (V_{dj}, V_{sj}) are elements of the lepton (quark) mixing matrix, relating the weak and the mass eigenstates; m_i are the neutrino masses. The quantities $I(\epsilon_{vi}, \epsilon_{qj})$ are functions of $\epsilon_{vi} = m_i^2/m_W^2$ and $\epsilon_{qj} = m_q^2/m_W^2$; n is the number of fermion generations.

In the presence of only the three known fermion generations the largest possible neutrino mass is given by the upper limit of 250 MeV for m_3 (Ref. 4). Using the constraint $|U_{e3} U_{\mu 3}^*| < 10^{-2}$ valid for 35 MeV $\lesssim m_3 \lesssim 300$ MeV (Ref. 31), and taking $m_t \simeq 30$–40 GeV we obtain

$$|\tilde{g}_{VV}| = |g_{VV}| < 10^{-9} , \qquad (100)$$

implying negligible branching ratios

$$B(a \to b\mu^- e^+) = B(a \to b\mu^+ e^-) < (2 \times 10^{-18})(\gamma_V^{ba} + \gamma_A^{ba}) . \qquad (101)$$

$B(a \to b\mu^\mp e^\pm)$ might be larger in the standard model if further generations involving heavier neutrinos are present. However, as the relations (98) and (99) remain valid, the decays $K^\pm \to \pi^\pm \mu e$ are the better probes, Eqs. (7), (8), (10), (11), and (15) implying

$$B(a \to b\mu^+ e^-) = B(a \to b\mu^- e^+) \simeq 1.5(\gamma_V^{ba} + \gamma_A^{ba}) B(K^+ \to \pi^+ \mu^+ e^-) \lesssim (8 \times 10^{-9})(\gamma_V^{ba} + \gamma_A^{ba}) . \qquad (102)$$

Coherent $\mu^- \to e^-$ conversion in nuclei is likely to be an even more sensitive process in this model.[32]

B. Extended electroweak models

An example is a gauge theory of the electroweak interactions based on $SU(2)_L \times SU(2)_R \times U(1)$. In some version of these models muon-number-violating processes may have large rates even without the existence of further families of leptons, due to the presence of heavy right-handed neutrinos.[11] The dominant diagram for $sd \to e\mu$ is expected to be the same as diagram (a) in Fig. 1, but with right-handed rather than left-handed gauge bosons. The resulting effective interaction is then of the form (2), with

$$g_{AA} = g_{VA} = g_{AV} = g_{VV} \qquad (103)$$

and

$$\tilde{g}_{AA} = \tilde{g}_{VA} = \tilde{g}_{AV} = \tilde{g}_{VV} , \qquad (104)$$

so that the bound (102) holds.

C. Flavor-changing neutral-gauge-boson exchange [diagram (b) in Fig. 1]

An important example is flavor-changing gauge bosons associated with possible horizontal gauge symmetries.[5] Horizontal gauge symmetries are invoked to distinguish the different fermion generations and also to attempt to reduce the large number of undetermined parameters (masses, mixing angles, the CP-violating phase angle) in the standard model.

The simplest possibility is horizontal interactions governed by a U(1) gauge group.[33] The associated gauge boson Y_λ is Hermitian and in the general case couples to the lepton and quark mass eigenstates as[34]

$$L_h = g_h(\beta_V \bar{s}\gamma_\lambda d + \beta_A \bar{s}\gamma_\lambda\gamma_5 d + \xi_V \bar{d}\gamma_\lambda d + \cdots + \sigma_V \bar{\mu}\gamma_\lambda e + \sigma_A \bar{\mu}\gamma_\lambda\gamma_5 e + \rho_V \bar{e}\gamma_\lambda e + \cdots) Y^\lambda + \text{H.c.} , \qquad (105)$$

where g_h is the horizontal gauge coupling constant and the quantities β_V, $\beta_A, \ldots,$ depend on the U(1) quantum-number assignments and on the various mixing angles and phase parameters. The interaction (105) conserves flavor in the absence of generation mixing.

The Lagrangian (105) leads to an effective four-fermion interaction [assuming $m_Y{}^2 \gg (m_a - m_b)^2$] of the form (2), with

$$g_{jk} = \frac{\sqrt{2}}{G}\frac{g_h{}^2}{m_h{}^2}\sigma_j^*\beta_k \quad (j=V,A;\ k=V,A)\ , \tag{106}$$

$$\tilde{g}_{jk} = \frac{\sqrt{2}}{G}\frac{g_h{}^2}{m_h{}^2}\sigma_j^*\beta_k^* \quad (j=V,A;\ k=V,A)\ , \tag{107}$$

where m_h is the mass of the horizontal gauge boson.

If the quark-Y couplings conserve *CP* (i.e., if $\beta_V, \beta_A, \ldots,$ are real), we have $g_{jk} = \tilde{g}_{jk}$ and therefore $B_A(a \to b\mu e)$ obeys Eq. (31). $B_V(a \to b\mu e)$, of course, is given by Eq. (20).

In the general case we have from Eqs. (106) and (107) $g_{jk} = e^{2i\phi_k}\tilde{g}_{jk}$, where ϕ_k is the phase of β_k. For $\phi_k = \pi/2, 3\pi/2$ one would have $g_{jA} = -\tilde{g}_{jA}$ $(j=V,A)$, so that the bounds (34)—(36) apply. The constraint on $B(K_S \to \mu e)$ from $B(K_L \to \mu e)_{\rm expt}$ would be the weakest for $\cot\phi = {\rm Im}\epsilon$, in which case the upper bounds for $B_A(a \to b\mu e)$ are twice the values given in (34).

It is important to note that the couplings in (105) required for the existence of $K_{L,S} \to \mu e$ and $K^\pm \to \pi^\pm \mu e$ contribute also to the K_L-K_S mass difference Δm_K, and in general also to the *CP*-violating K_1-K_2 mixing parameter ϵ, resulting in severe constraints on the $K_{L,S} \to \mu e$ and $K^\pm \to \pi^\pm \mu e$ branching ratios.[35] Barring cancellations among various contributions to Δm_K, large rates for these decays are possible only if the ratios of the effective Y-quark to Y-lepton couplings are small. For example, in the *CP*-invariant case $B(K_L \to \mu e)$ and $B(K^\pm \to \pi^\pm \mu e)$ can be as large as the experimental upper limits (14) and (15) only if

$$|\beta_A| \simeq 3\times10^{-3}(|\sigma_V|^2 + |\sigma_A|^2)^{1/2}$$

and

$$|\beta_V| \simeq 10^{-4}(|\sigma_V|^2 + |\sigma_A|^2)^{1/2}\ ,$$

respectively; similarly $B(K_S \to \mu e) \simeq 7\times10^{-7}$ would require

$$|{\rm Im}\beta_A| \simeq 5\times10^{-6}(|\sigma_V|^2 + |\sigma_A|^2)^{1/2}$$

(Ref. 34).

In models based on non-Abelian horizontal gauge groups[36] the horizontal bosons can have flavor-changing couplings even in the absence of generation mixing.[37] In the limit of equal gauge-boson masses and of no generation mixing, the processes $a \to b\mu^- e^+$, $\bar{a} \to \bar{b}\mu^+ e^-$, and $K^\pm \to \pi^\pm \mu^\pm e^\mp$, which are generation-number conserving, are allowed, while generation-number-nonconserving processes, such as $a \to b\mu^+ e^-$, $\bar{a} \to \bar{b}\mu^- e^+$, $K^\pm \to \pi^\pm \mu^\mp e^\pm$, and the $K^0 \to \bar{K}^0$ transition amplitude, are forbidden.[38] Thus, if generation-number conservation is broken only weakly, one expects $\tilde{g}_{jk} \ll g_{jk}$ $(j,k=V,A)$ and consequently $B(a \to b\mu^+ e^-) \ll B(a \to b\mu^- e^+)$ and $g_{jA}^{(+)} \simeq i g_{jA}^{(-)}$ $(j=V,A)$. From the sum rule (27) we then obtain

$$B_A(a \to b\mu^+ e^-) \ll B_A(a \to b\mu^- e^+) \le 4\Omega_{A,1}^{ba}\ . \tag{108}$$

On the other hand, if generation-number conservation is strongly violated, but only by large mixing angles in the leptonic sector, $B(a \to b\mu^+ e^-)$ and $B(a \to b\mu^- e^+)$ could be comparable and at the same time the contribution of the horizontal interactions to the K^0-$\bar{K}^0$ ampiltude would remain suppressed. As a consequence, $B(a \to b\mu e)$ as large as the bounds $\Omega_{A,s}^{ba}$ are then not ruled out.

Flavor-changing neutral color-singlet gauge bosons are present also in extended hypercolor theories.[6] The effective $\Delta S=1$ muon-number-violating interactions they give rise to are of the form (2). Unlike the horizontal gauge bosons discussed above, these gauge bosons are constrained to be light [enough to cause observable effects in $K_{L,S} \to \mu e$, $K \to \pi\mu e$ (Ref. 39)] by the requirement that they generate the masses of ordinary fermions.

D. Flavor-changing neutral-Higgs-boson exchange [diagram (c) in Fig. 1]

In the standard $SU(2)_L \times U(1)$ model with only one Higgs doublet the Higgs meson couples to fermions through scalar densities and the couplings conserve flavor. In the presence of two or more Higgs doublets the neutral Higgs bosons have in general also flavor-changing couplings to fermions,[7] and both scalar and pseudoscalar couplings occur.[40]

The most general coupling of a Hermitian Higgs field ϕ to (sd) and (μe) is of the form

$$L_H = (f_S'\bar{\mu}e + f_P'\bar{\mu}i\gamma_5 e + f_S''\bar{s}d + f_P''\bar{s}i\gamma_5 d)\phi + {\rm H.c.} \tag{109}$$

The exhange of the Higgs boson leads to an effective interaction of the form (42) [assuming $m_H{}^2 \gg (m_a - m_b)^2$], with

$$g_{jk} = \frac{\sqrt{2}}{G}\frac{f_j'^* f_k''}{m_H{}^2} \quad (j,k=S,P)\ , \tag{110}$$

$$\tilde{g}_{jk} = \frac{\sqrt{2}}{G}\frac{f_j'^* f_k''^*}{m_H{}^2} \quad (j,k=S,P)\ . \tag{111}$$

It follows that $g_{jk} = e^{2i\psi}\tilde{g}_{jk}$, where ψ is the phase of f_k''.

The contribution of S-type couplings to $B(a \to b\mu e)$ is constrained by $B(K^\pm \to \pi^\pm \mu e)$ [cf. Eqs. (66) and (67)]. If the Higgs-quark couplings conserve *CP*, we have $g_{jk} = \tilde{g}_{jk}$, and consequently the bound (74) applies for $B_P(a \to b\mu e)$. The weakest constraint on $B_P(K_S \to \mu e)$ and thus on $B_P(a \to b\mu e)$ is obtained for $\cot\psi = {\rm Im}\epsilon$. The corresponding upper bound on $B_P(a \to b\mu e)$ is twice the value of the bound in Eq. (77).

In the presence of several neutral flavor-changing Higgs bosons $B_P(a \to b\mu e)$ could be as large as $\Omega_{P,s}^{ba}$ [cf. Eq. (81)]. However, one must bear in mind that $B_P(K_S \to \mu e)$ is larger than $B_P(\Sigma^+ \to p\mu e)$ and $B_P(\Xi^0 \to \Lambda\mu e)$ by about factors of 10^5 and 10^6, respectively. It should be noted that, as in the case of flavor-changing gauge-boson exchange, the experimental values of Δm_K and ϵ impose severe constraints on the branching ratios of $K_{L,S} \to \mu e$

and $K^{\pm}\rightarrow\pi^{\pm}\mu e$.[18,41] Large rates for these decays are possible only if the ratios of the effective quark—Higgs-boson to lepton—Higgs-boson couplings are small, or if cancellations occur among the various contributions to the K^0-$\bar{K}^0$ amplitude.

Further contributions come in general from charged Higgs bosons [diagram (a) in Fig. 1, with the W's replaced by charged Higgs bosons]. In the standard model with two (or more) Higgs doublets the corresponding effective interaction is of the form (2), with $g_{VV}=g_{AV}=-g_{VA}=-g_{AA}$, $\tilde{g}_{VV}=\tilde{g}_{AV}=-\tilde{g}_{VA}=-\tilde{g}_{AA}$ (Ref. 42). Consequently $B(a\rightarrow b\mu^{\mp}e^{\pm})$ satisfy the bounds (102).

E. Leptoquark exchange [diagram (d) in Fig. 1]

Leptoquarks appear in theories which unify the strong and the flavor interactions, and also in extended hypercolor theories. In extended hypercolor theories[6] and also in some classes of grand unified theories[9] they are sufficiently light to cause observable effects in some rare decays. Strangeness-changing processes mediated by leptoquarks are not constrained significantly by the K_L-K_S mass difference,[43] since in lowest order leptoquark exchange does not generate a nonleptonic interaction. Both spin-1 and spin-0 leptoquarks occur.

1. Spin-1 leptoquarks

The most general four-fermion interaction involving s, d, e, and μ resulting from the exhange of a spin-1 leptoquark (LQ) is of the form (assuming $m_{\rm LQ}{}^2 \gg m_a{}^2$)

$$H_{\rm LQ}=\frac{G}{\sqrt{2}}(f_{VV}\bar{s}\gamma_\lambda\mu\bar{e}\gamma^\lambda d+\tilde{f}_{VV}\bar{d}\gamma_\lambda\mu\bar{e}\gamma^\lambda s+f_{AV}\bar{s}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda d+\tilde{f}_{AV}\bar{d}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda s+f_{VA}\bar{s}\gamma_\lambda\mu\bar{e}\gamma^\lambda\gamma_5 d$$
$$+\tilde{f}_{VA}\bar{d}\gamma_\lambda\mu\bar{e}\gamma^\lambda\gamma_5 s+f_{AA}\bar{s}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda\gamma_5 d+\tilde{f}_{AA}\bar{d}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda\gamma_5 s)+\text{H.c.} \tag{112}$$

Rearrangement of the fermion fields by a Fierz transformation leads to an effective interaction represented by the sum of the Hamiltonians (2) and (42), in which

$$g_{VV}=g_{AA},\ \tilde{g}_{VV}=\tilde{g}_{AA},\ g_{VA}=g_{AV},\ \tilde{g}_{VA}=\tilde{g}_{AV},\ g_{PP}=g_{SS},\ \tilde{g}_{PP}=\tilde{g}_{SS},\ g_{SP}=-g_{PS},\ \tilde{g}_{SP}=-\tilde{g}_{PS}\ . \tag{113}$$

As a consequence, the branching ratio for $a\rightarrow b\mu^- e^+$ is given by

$$B(a\rightarrow b\mu^- e^+)=(\gamma_V^{ba}+\gamma_A^{ba})(|g_{VV}|^2+|g_{AV}|^2)+(\gamma_S^{ba}+\gamma_P^{ba})(|g_{SS}|^2+|g_{PS}|^2)$$
$$+\text{interference terms between } (V,A) \text{ and } (S,P) \text{ couplings}\ . \tag{114}$$

The same expression holds for $a\rightarrow b\mu^+ e^-$ with the replacement $g_{ik}\rightarrow\tilde{g}_{jk}$.

Inspection of the expression for $B(K^{\pm}\rightarrow\pi^{\pm}\mu e)$ shows that the effect of the simultaneous presence of V-type and S-type couplings is only an increase of the upper bounds (18), (19), (64), and (65) by a factor of about 2. As the interference terms in (114) are not likely to change the order of magnitude of $B(a\rightarrow b\mu e)$, we expect from the experimental limit (15) on $B(K^+\rightarrow\pi^+\mu e)$ (using $m_s+m_d \simeq m_s-m_d$)

$$B(\Sigma^+\rightarrow p\mu e)\lesssim 5\times 10^{-11}\ , \tag{115}$$

$$B(\Xi^0\rightarrow\Lambda\mu e)\lesssim 4\times 10^{-11}\ . \tag{116}$$

2. Spin-0 leptoquarks

The effective interaction arising from spin-0 leptoquark exchange is analogous to (112), but constructed from scalar and pseudoscalar, rather than vector and axial-vector densities. The Fierz-transformed form is the sum of the Hamiltonians (2), (42), and (85), with

$$\begin{aligned} &g_{AA}=-g_{VV},\ \tilde{g}_{AA}=-\tilde{g}_{VV}\ ,\\ &g_{VA}=-g_{AV},\ \tilde{g}_{VA}=-\tilde{g}_{AV}\ ,\\ &g_{PP}=g_{SS},\ \tilde{g}_{PP}=\tilde{g}_{SS}\ ,\\ &g_{SP}=g_{PS},\ \tilde{g}_{SP}=\tilde{g}_{PS}\ ,\\ &g_{TT}=g_{SS},\ \tilde{g}_{TT}=\tilde{g}_{SS}\ ,\\ &g'_{TT}=ig_{PS},\ \tilde{g}'_{TT}=i\tilde{g}_{PS}\ . \end{aligned} \tag{117}$$

Hence,

$$B(a\rightarrow b\mu^- e^+)=(\gamma_V^{ba}+\gamma_A^{ba})(|g_{VV}|^2+|g_{AV}|^2)+(\gamma_S^{ba}+\gamma_P^{ba}+\gamma_T^{ba})(|g_{SS}|^2+|g_{PS}|^2)$$
$$+\text{interference terms among } (V,A)\ (S,P), \text{ and } (T) \text{ couplings}\ . \tag{118}$$

$B(a \to b\mu^+ e^-)$ is given by the same expression, but with $g_{jk} \to \tilde{g}_{jk}$. As already mentioned at the end of Sec. II, inspection shows that the upper bounds on $|g_{VV}|^2 + |g_{AV}|^2$, $|\tilde{g}_{VV}|^2 + |\tilde{g}_{AV}|^2$, $|g_{SS}|^2 + |g_{PS}|^2$, and $|\tilde{g}_{SS}|^2 + |\tilde{g}_{PS}|^2$ in the simultaneous presence of S, V, and T couplings are not larger than those for pure S- and V-type couplings by more than a factor of 5. Using $T_1^{ba} \simeq G_1^{ba}$ (cf. Sec. II), $m_s = 150$ MeV and $m_d \simeq 7.5$ MeV (Ref. 27), we obtain

$$B(\Sigma^+ \to p\mu e) \lesssim 10^{-10} , \quad (119)$$

$$B(\Xi^0 \to \Lambda\mu e) \lesssim 9 \times 10^{-11} . \quad (120)$$

IV. CONCLUSIONS

Our results based on phenomenology were summarized at the end of Sec. II. We found that our present knowledge of strangeness-changing muon-number-violating couplings, provided by K decays, does not exclude branching ratios for $\Sigma^+ \to p\mu^\mp e^\pm$ and $\Xi^0 \to \Lambda\mu^\mp e^\pm$ as large as of the order of 10^{-6}. Branching ratios of this size would require an axial-vector-type interaction with $g_{jA} + \tilde{g}_{jA} = O(\epsilon g_{jA})$ ($j = V$ and/or A, $|\epsilon| \simeq 2.3 \times 10^{-3}$), in addition to sufficiently large values of g_{jA} [for $g_{jA} + \tilde{g}_{jA} = O$ $B(K_L \to \mu e)_{\rm expt}$ already constrains $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ to be less than about 10^{-9}]. Among the possible sources of muon-number violation we considered, interactions of this structure could arise only from horizontal interactions governed by non-Abelian gauge groups, or from the exchange of color-singlet gauge bosons of extended hypercolor schemes. The relation $g_{jA} + \tilde{g}_{jA} = O(\epsilon g_{jA})$ does not, of course, follow naturally in any model, and would have to be therefore regarded as accidental. We note also that if $B_A(\Sigma^+ \to p\mu e) \simeq (5 \times 10^{-6})y$ (and therefore $B_A(\Xi^0 \to \Lambda\mu e) \simeq 10^{-6}y$), one would have $B_A(K_S \to \mu e) \simeq 10^{-3}y$ for $10^{-7} \lesssim y \lesssim 1$ [cf. Eq. (27)]. The upper bounds on $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ in models with a U(1)-type horizontal gauge symmetry are of the order of 10^{-8}–10^{-9}.

A pseudoscalar-type interaction with $g_{jP} + \tilde{g}_{jP} = O(\epsilon g_{jP})$, $j = S$ and/or P, which might (in principle) arise in models with several neutral flavor-changing Higgs bosons, could lead to $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ of the order of 10^{-8} and 10^{-9}, respectively. If $B(\Sigma^+ \to p\mu e) \simeq 10^{-8}y$ [and therefore $B(\Xi^0 \to \Lambda\mu e) \simeq 10^{-9}y$], one would have $B_P(K_S \to \mu e) \simeq 10^{-3}y$ for $10^{-8} \lesssim y \lesssim 1$.

In the standard model and in the $SU(2)_L \times SU(2)_R \times U(1)$ model with heavy right-handed neutrinos the V- and A-type couplings in the effective muon-number-violating interaction have the same strength. Among the $\Delta S = 1$ muon-number-violating processes the decays $K^\pm \to \pi^\pm \mu e$ are the best probes in such situations. This is so also for scalar-type couplings, which could arise, e.g., from neutral-Higgs-boson exchange. The decays $K \to \pi\mu e$ are also the best probes of $\Delta S = 1$ muon-number-violating couplings generated by spin-1 or spin-0 leptoquarks.

We note that if both neutral gauge bosons and neutral Higgs bosons would contribute, larger values than quoted above for the contribution of axial-vector-type and pseudoscalar-type couplings to $B(\Sigma^+ \to p\mu e)$ and $B(\Xi^0 \to \Lambda\mu e)$ cannot be excluded, since cancellations might occur in the $K_L \to \mu e$ and/or $K_S \to \mu e$ amplitudes. Finally, in the unlikely event that $a \to b\mu e$ is mediated by a light boson of mass comparable to a value of the magnitude of the four-momentum it can carry, the ratios $B(a \to b\mu e)/\,B(K_{L,S} \to \mu e)$, and possibly also the ratios $B(a \to b\mu e)/\,B(K^\pm \to \pi^\pm \mu e)$, would be enhanced relative to those obtained for local four-fermion couplings.

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$$g_{kl}^{(+)}+i\epsilon g_{kl}^{(-)}=g_{kl}[\beta(\epsilon-1)/2+\epsilon]$$

and

$$g_{kl}^{(-)}-i\epsilon g_{kl}^{(+)}=-ig_{kl}[1+\beta(1-\epsilon)/2]\ .$$

Thus, if $B(K_S\to\mu e)$ is to be of the order of 10^{-3}, β must be of the order of ϵ. More precisely, $B(K_L\to\mu e)=r$ implies

$$|g_{kl}|^2\simeq(6.3\times10^{-2})r/|\beta-2\epsilon|^2\ ,$$

and therefore

$$B(K_S\to\mu e)\simeq(7\times10^{-3})r\ |1+\beta/2|^2/|\beta-2\epsilon|^2\ .$$

Consequently, $B(K_S\to\mu e)\simeq10^{-3}$ if the parameter β satisfies $|\beta-2\epsilon|^2/|1+\beta/2|^2\simeq7r$. For P-type couplings the requirement for $B(K_S\to\mu e)\simeq10^{-3}$ is the same [cf. Eqs. (53) and (54)].

[24]For $m_b=0$, $m_\mu=0$ one would have $\kappa_S=\kappa_P=\frac{1}{2}$.

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[26]In the limit of SU(3) symmetry $f_-^{sd}(q^2)=0$. Experimentally $f_-/f_+=-0.35\pm0.15$ for $K^+\to\pi^0\mu^+\nu_\mu$ and $f_-/f_+=-0.11\pm0.09$ for $K_L\to\pi\mu\nu$ (cf. Ref. 4). As a result, the effect of the f_-^{sd}-dependent term on the $K^\pm\to\pi^\pm\mu e$ rate is less than 35%.

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$$|g_{SP}^{(-)}-i\epsilon g_{SP}^{(+)}|^2+|g_{PP}^{(-)}-i\epsilon g_{PP}^{(+)}|^2\lesssim4\times10^{-5}\ ,$$

$$|g_{SP}^{(-)}-i\epsilon g_{SP}^{(+)}|^2+|g_{PP}^{(-)}-i\epsilon g_{PP}^{(+)}|^2\lesssim4\times10^{-5}\ ,$$

$$|g_{PS}|^2+|g_{SS}|^2\lesssim1.4\times10^{-9}\ ,$$

$$|\tilde{g}_{PS}|^2+|\tilde{g}_{SS}|^2\lesssim2\times10^{-9}\ .$$

For larger quark masses [cf. M. D. Scadron, Rep. Prog. Phys. 44, 213 (1981)] the constraints on the SP and PP terms would be less stringent. Note that the bounds $\Omega_{P,1}^{ba}$, $\Omega_{P,s}^{ba}$, and Ω_S^{ba} are independent of the quark masses.

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$$B(a \to b\mu e) \leq 6\times 10^{-19}(\gamma_V^{ba}+\gamma_A^{ba}) .$$

If $m_3 < 35$ Mev, then

$$B(a \to b\mu e) < 2\times 10^{-18}(\gamma_V^{ba}+\gamma_A^{ba})$$

even for $| U_{e3}U_{\mu 3}^{*} | \lesssim 1$.

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[40]In extended hypercolor schemes the states corresponding to Higgs bosons are the color-singlet pseudo-Goldstone bosons. Some of these are light and their flavor-changing couplings must be suppressed to avoid conflict with the experimental value of the K_S-K_L mass difference (cf. Ref. 6).

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