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New approach to nonleptonic weak interactions. III. Application to hyperon nonleptonic decays

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A new approach to hyperon nonleptonic weak interactions is discussed. It is based on (i) the derived asymptotic constraints on the two-body ground-state-baryon matrix elements of the weak nonleptonic Hamiltonian which satisfy the $|\Delta\vec{I}|=\frac{1}{2}$ rule and its SU(3) counterpart and (ii) the new soft-pion approximation in the infinite-momentum frame of parent particle. In the new extrapolation, a part of the so-called surface term survives in addition to the usual equal-time-commutator term. The result based on the first approximation which keeps only the ground-state-baryon contribution to the surface term is presented. It is shown that the Lee-Sugawara sum rules are satisfied reasonably well for both the S and P waves. Improvement of the approximation should be possible with less ambiguity than the usual approach, by including the contribution of the $L=1$ baryon states to the surface term. Nowhere is the concept of exact SU(3) symmetry used.

I. INTRODUCTION AND SUMMARY

The derivation of the approximate $|\Delta\vec{I}|=\frac{1}{2}$ rule in the strange-particle decays has been attempted by many authors[1] for the past three decades. One of the popular ideas is to suspect that the origin of the rule is rather kinematical, i.e., it is related to the symmetry properties[2] of the quark wave functions of hadrons. The argument, sometimes called Minamikawa-Miura-Pati-Woo theorem, was found to be able to impose the $|\Delta\vec{I}|=\frac{1}{2}$ rule and other constraints, in a nonperturbative way, on the baryon two-body weak vertices. However, the same idea failed to impose any constraint on bosons. Another contrasting idea is to ascribe the origin to some dynamical enhancement of particular diagrams at the level of underlying quarks and gluons. Historically, a similar idea was also entertained earlier in the Sakata model.[3] The recent popular proposal is the assumption of the enhancement of penguin diagram in the QCD-corrected effective Hamiltonian.[4] A difficulty in this type of approach is related to the confinement problem. That is, one has to translate the information obtained at the quark-gluon level into the language of observable hadrons. For example, for penguins one has to rely heavily on the validity of the factorization (or vacuum-insertion) approximation which was also introduced[3] a long time ago. The factorization approximation was not very successful in the Cabibbo-angle-unsuppressed D-meson decays.[5] Attempts, based on the effective Hamiltonian with QCD corrections coupled with the conventional soft-pion technique, have also been discussed[6] recently. In these theories, many meson-pole amplitudes are added in a rather *ad hoc* way to the calculation, suspecting that the soft-pion extrapolation washes away these contributions. Overall, the situation is still rather confused.

In this paper, we show that the hyperon nonleptonic decays, although more complicated to treat than the K-meson decays discussed in the preceding paper[7] II, can also be approached in exactly the same way. In a recent letter,[8] we have shown that in the theoretical framework proposed, there exist certain asymptotic constraints, which include the celebrated $|\Delta\vec{I}|=\frac{1}{2}$ rule, among the *asymptotic two-body* ground-state-baryon matrix elements of the strangeness-changing nonleptonic weak Hamiltonian H. We relate these asymptotic two-body constraints to the physical hyperon decay amplitudes by using a soft-pion extrapolation.[9] We employ a new (much milder) soft-pion approximation [carried out in the infinite-momentum frame (IMF)[10] of the parent hyperon] developed in paper II. In this new extrapolation a part of the so-called surface term survives, in addition to the usual equal-time-commutator (ETC) term. The surface term can be cast into the form

$$\sum_{n_L}\langle B'|H|n_L\rangle\langle n_L|A_\pi|B\rangle\ ,$$

where B and B' are the baryons in the $B\to B'\pi$ decay, A_π is the SU(2) axial-vector charge, and n_L denotes the baryons belonging to the level L ($L=0,1\ldots$). In this paper, we report the sum rules obtained when we keep, in this new approach, *only* the diagonal term (i.e., the $L=0$ ground states) in the surface term. A reasonable result is obtained. Because of the neglect of higher-L-state contributions, the result is, of course, not perfect. However, the theoretical ambiguity involved is considerably less compared with the corresponding old treatment. Nowhere is the concept of exact SU(3) symmetry used and the sum rules are valid in *broken* SU(3) symmetry. It also gives us a clear indication *where* one has to look for improvement. The inclusion of the next $L=1$ (but not $L\geq 2$) baryon-state contribution to the surface term should be important to remove the bulk of the remaining discrepancy with the experiment. However, the meson-pole contributions, which are often added[6] after carrying out the conventional soft-meson approximation, need *not* be considered *here*, since they are already contained in the present formulation in the ETC term evaluated in IMF. This is discussed in Appendix A.

II. CONSTRAINTS ON THE ASYMPTOTIC TWO-BODY GROUND-STATE-BARYON WEAK NONLEPTONIC MATRIX ELEMENTS

We here summarize the constraints obtained[8] on the *asymptotic two-body* ground-state-baryon ($\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decouplet) matrix elements of the weak Hamiltonian, $\langle B'\,|\,H\,|\,B(\vec{p}\,)\rangle$ with $\vec{p}\to\infty$. The weak Hamiltonian in the standard model contains a sizable 27-plet. However, the requirement of levelwise realization of *asymptotic* SU(3) symmetry in the algebras involving the H and A_π's imposes[8] severe constraints on the asymptotic ground-state baryon matrix elements of H. Namely, $\langle B'_8\,|\,H\,|\,B_8(\vec{p}\,)\rangle$ and $\langle B'_{10}\,|\,H\,|\,B_{10}(\vec{p}\,)\rangle$ are required to satisfy the *strict* $|\Delta\vec{I}\,|=\frac{1}{2}$ rule and also octet rules in the limit $\vec{p}\to\infty$. Furthermore, $\langle B'_8\,|\,H\,|\,B_{10}\rangle=\langle B'_{10}\,|\,H\,|\,B_8\rangle=0$ is required for $\vec{p}\to\infty$ and there arise SU(6)-type constraints which relate $\langle B'_8\,|\,H\,|\,B_8\rangle$ to $\langle B'_{10}\,|\,H\,|\,B_{10}\rangle$ in the same asymptotic limit. As a result, the asymptotic matrix elements of $\frac{1}{2}^+$ baryons, $\langle B'_8\,|\,H\,|\,B_8(\vec{p}\,)\rangle$ with $\vec{p}\to\infty$, can be parametrized in terms of just one coupling constant and they are of pure f type. In contrast, in the framework of the Minamikawa-Miura-Pati-Woo theorem $\langle B'_8\,|\,H\,|\,B_8\rangle$'s are parametrized with the d/f ratio -1. The parametrizations of $\langle B'_8\,|\,H\,|\,B_8(\vec{p}\,)\rangle$'s with $\vec{p}\to\infty$ for both the parity-conserving and parity-violating Hamiltonian $H=H^{\rm PC}$ or $H=H^{\rm PV}$ are thus given by[8]

$$
\begin{aligned}
&\langle p\,|\,H^{\rm PC}\,|\,\Sigma^+\rangle=-w_1^{\rm PC}, && \langle p\,|\,H^{\rm PV}\,|\,\Sigma^+\rangle=-w_1^{\rm PV}\,,\\
&\langle n\,|\,H^{\rm PC}\,|\,\Sigma^0\rangle=(1/\sqrt{2})w_1^{\rm PC}, && \langle n\,|\,H^{\rm PV}\,|\,\Sigma^0\rangle=(1/\sqrt{2})w_1^{\rm PV}\,,\\
&\langle n\,|\,H^{\rm PC}\,|\,\Lambda^0\rangle=-(\tfrac{3}{2})^{1/2}w_1^{\rm PC}, && \langle n\,|\,H^{\rm PV}\,|\,\Lambda^0\rangle=-(\tfrac{3}{2})^{1/2}w_1^{\rm PV}\,,\\
&\langle \Xi^-\,|\,H^{\rm PC}\,|\,\Sigma^-\rangle=w_1^{\rm PC}, && \langle \Xi\,|\,H^{\rm PV}\,|\,\Sigma^-\rangle=w_1^{\rm PV}\,,\\
&\langle \Xi^0\,|\,H^{\rm PC}\,|\,\Sigma^0\rangle=-(1/\sqrt{2})w_1^{\rm PC}, && \langle \Xi^0\,|\,H^{\rm PV}\,|\,\Sigma^0\rangle=-(1/\sqrt{2})w_1^{\rm PV}\,,\\
&\langle \Xi^0\,|\,H^{\rm PC}\,|\,\Lambda^0\rangle=(\tfrac{3}{2})^{1/2}w_1^{\rm PC}, && \langle \Xi^0\,|\,H^{\rm PV}\,|\,\Lambda^0\rangle=(\tfrac{3}{2})^{1/2}w_1^{\rm PV}\,.
\end{aligned}
\tag{2.1}
$$

III. APPLICATION OF THE NEW SOFT-PION TECHNIQUE

We consider the hyperon decay $B(p_1)\to B'(p_2)+\pi(q)$ and write the amplitude as

$$M(B\to B'\pi;q)\equiv(2q_0E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\pi(q)\,|\,H(0)\,|\,B(p_1)\rangle\,. \tag{3.1}$$

Using PCAC (partial conservation of axial-vector current), $\partial_\mu A_\mu^\pi(x)=f_\pi m_\pi^2\phi_\pi(x)$, and the Lehman-Symanzik-Zimmermann reduction formula, M can be cast into the form

$$M(B\to B'\pi;q)=i(f_\pi m_\pi^2)^{-1}(q^2+m_\pi^2)\left[q_\mu T_\mu(q)+\int d^4x\,e^{-iqx}\delta(x_0)\langle B'(p_2)\,|\,[A_0^{\bar\pi}(x),H(0)]\,|\,B(p_1)\rangle \times(E_{B'}E_B/m_{B'}m_B)^{1/2}\right],$$

where T_μ is given by

$$T_\mu(q)=i\int d^4x\,e^{iqx}(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,T[A_\mu^{\bar\pi}(x),H(0)]\,|\,B(p_1)\rangle\,. \tag{3.2}$$

$A_\mu^{\bar\pi}(x)$ is the conjugate complex of $A_\mu^\pi(x)$.

We now consider a new soft-pion approximation[7] $\vec{q}\to 0$, instead of the more drastic extrapolation $q_\mu\to 0$, in the IMF, i.e., $\vec{p}_1\to\infty$ and approximate Eq. (3.1) by

$$M(B\to B'\pi;q)\sim M(B\to B'\pi;\vec{q}=0)_{\vec{p}_1\to\infty}\equiv M^{\rm ETC}+M^{\rm S}\,, \tag{3.3}$$

where

$$M^{\rm ETC}\equiv-f_\pi^{-1}(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,[A_{\bar\pi}(0),H(0)]\,|\,B(p_1)\rangle_{\vec{p}_1\to\infty}\,, \tag{3.4}$$

$$M^{\rm S}\equiv if_{\bar\pi}^{-1}[q_\mu T_\mu(\vec{q}\to 0)]_{\vec{p}_1\to\infty}\,. \tag{3.5}$$

In Eq. (3.4) $A_{\bar{\pi}}(0)$ is the axial-vector charge defined by $A_{\bar{\pi}}(0)=-i\int d^3x\, A_0^{\bar{\pi}}(\vec{x},0)$. $M^{\rm ETC}$ has the same form as the usual equal-time-commutator (ETC) term in the conventional soft-pion extrapolation, except for one important difference that the term should now be evaluated in the IMF, i.e., $\vec{p}_1=\vec{p}_2\to\infty$. This is, in principle, important, since this permits us to compute the ETC term without using exact flavor symmetry. $M^{\rm S}$ denotes the so-called surface term. In the conventional soft-pion extrapolation,[9] T_μ is known to involve terms (baryon pole terms) which become singular for $q_\mu\to 0$. To avoid this difficulty one usually subtracts the Born amplitudes corresponding to the process $B\to B'\pi$ from both sides of Eq. (3.1) and then applies the $q_\mu\to 0$ limit to the resulting expressions. Even after this subtraction, some doubts still remain as to whether the extrapolation is as smooth as one wishes and whether the terms which are formally dropped are not producing a significant contribution. As a possible correction, Gronau,[11] for example, took the K^*-pole contribution. As mentioned in Sec. I, several recent works[6] also propose to add various meson-pole (scalar-meson, etc.) contributions as a possible remedy for the extrapolation involved. In the present soft-pion procedure which involves a much milder extrapolation, as shown in Appendix A of Ref. 7, the situation is less ambiguous.

We decompose $T_\mu(q)$ as $T_\mu=T_\mu^{(+)}(q)+T_\mu^{(-)}(q)$,

$$T_\mu^{(+)}(q)=i\int d^4x\, e^{-iqx}\theta(x_0)(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,A_\mu^{\bar{\pi}}(x)H(0)\,|\,B(p_1)\rangle\ , \tag{3.6}$$

$$T_\mu^{(-)}(q)=i\int d^4x\, e^{-iqx}\theta(-x_0)(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,H(0)A_\mu^{\bar{\pi}}(x)\,|\,B(p_1)\rangle\ . \tag{3.7}$$

We now insert a complete set of single-particle *on-mass-shell* baryon intermediate states (which, in our theoretical framework, should be the qqq baryon states with level excitations) between the factors $A_\mu^{\bar{\pi}}(x)$ and $H(0)$ in Eqs. (3.6) and (3.7). As in Ref. 7, we decompose the intermediate states in terms of levels $\sum_{n_L}|\,n_L\rangle\langle n_L\,|$ $(L=0,1,\ldots)$, in the same way as we do in the level realization of the algebras discussed in I. In this paper we keep *only* the "diagonal" term, i.e., the $L=0$ ground-state baryons (i.e., $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet), which gives the leading (though not completely dominant) contribution. After the integration over d^4x and the momenta involved and the spin summation over intermediate states, n and l, we obtain,

$$T_\mu^{(+)}(q)=\sum_n \bar{u}_{B'}(p_2)\{A_u^{\bar{\pi}}\}_{B'n}\frac{(-i\gamma\cdot p_n+m_n)}{2E_n(E_n-E_B)}g^{\rm PC,PV}(B\to n)\begin{bmatrix}1\\ \gamma_5\end{bmatrix}u_B(p_1)\,|_{\vec{p}_n=\vec{p}_1}+\cdots\ , \tag{3.8}$$

$$T_\mu^{(-)}(q)=\sum_l \bar{u}_{B'}(p_2)g^{\rm PC,PV}(l\to B)\begin{bmatrix}1\\ \gamma_5\end{bmatrix}\frac{(-i\gamma\cdot p_l+m_l)}{2E_l(E_l-E_{B'})}\{A_\mu^{\bar{\pi}}\}_{lB}u_B(p_1)\,|_{\vec{p}_l=\vec{p}_1}+\cdots\ . \tag{3.9}$$

Here PC and PV refers to the parity-conserving and -violating Hamiltonian, respectively, and the 1 (γ_5) refers to the PC (PV) case. n and l denote the ground-state $\frac{1}{2}^+$ hyperons. As shown in Sec. II, $\langle B'_{10}\,|\,H\,|\,B_8\rangle$ and $\langle B'_8\,|\,H\,|\,B_{10}\rangle$ are constrained to vanish in the asymptotic limit. In anticipation of taking the asymptotic limit $\vec{p}_1=\vec{p}_2\to\infty$ later, we have omitted the ground-state $\frac{3}{2}^+$ decuplet from the intermediate states n and l. The dots denote the nondiagonal higher-level $(L=1,2,\ldots)$ contribution. $\{A_\mu^{\bar{\pi}}\}_{B'n}$ is defined, for example, as follows and is expressed in terms of the well-known axial-vector form factors $G(q^2)$ and $F(q^2)$ as

$$\begin{aligned}\bar{u}_{B'}(p_2)\{A_\mu^{\bar{\pi}}\}_{B'n}u_n(p_n)&\equiv(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,A_\mu^{\bar{\pi}}(0)\,|\,n(p_n)\rangle\\ &=\bar{u}_{B'}(p_2)\{i\gamma_\mu\gamma_5 G_{B'n}({q_n}^2)+(q_n)_\mu\gamma_5F_{B'n}({q_n}^2)\}u_n(p_n),\ (q_n=p_n-p_2)\ .\end{aligned} \tag{3.10}$$

The on-mass-shell weak couplings g are defined by

$$\langle n(p_n)\,|\,H^{\rm PC,PV}(0)\,|\,B(p_1)\rangle=(m_nm_B/E_nE_B)^{1/2}g^{\rm PC,PV}(B\to n)\bar{u}_n(p_n)\begin{bmatrix}1\\ \gamma_5\end{bmatrix}u_B(p_1)\ . \tag{3.11}$$

To compute Eq. (3.5) we now evaluate the invariant quantity $if_\pi^{-1}\sum_\mu q_\mu T_\mu^{(+)}(q)$, etc., in the IMF ($\vec{p}_1\to\infty$) and make the soft-pion approximation $\vec{q}\to 0$. We then obtain [upper (lower) form refers to PC (PV) case]

$$[if_\pi^{-1}q_\mu T_\mu^{(+)}(\vec{q}\to 0)]_{\vec{p}_1\to\infty}=f_\pi^{-1}(m_{B'}\pm m_B)\sum_n\frac{g^{\rm PC,PV}(B\to n)}{m_n\mp m_B}G_{B'n}(0)i\bar{u}_{B'}(p_2)\gamma_5\begin{bmatrix}1\\ \gamma_5\end{bmatrix}u_B(p_1)\,|_{\vec{p}_1=\vec{p}_2\to\infty}+\cdots\ , \tag{3.12}$$

$$[if_\pi^{-1}q_u T_\mu^{(-)}(\vec{q}\to 0)]_{\vec{p}_1\to\infty}=f_\pi^{-1}(m_B\pm m_{B'})\sum_l\frac{g^{\rm PC,PV}(l\to B')}{m_l\mp m_{B'}}G_{lB}(0)i\bar{u}_{B'}(p_2)\begin{bmatrix}1\\ \gamma_5\end{bmatrix}\gamma_5u_B(p_1)\,|_{\vec{p}_1=\vec{p}_2\to\infty}+\cdots\ . \tag{3.13}$$

In deriving Eq. (3.12) we have used ${q_n}^2\to O(1/|\vec{p}_1|^2)$, $(q_n\cdot q)\to O(1/|\vec{p}_1|^2)$, $2E_n(E_n-E_B)\to({m_n}^2-{m_B}^2)+O(1/|\vec{p}_1|^2)$, $i\gamma\cdot p_n\to i\gamma\cdot p_1+O(1/|\vec{p}_1|)$, etc., in the limit $\vec{p}_1\to\infty$.

The extrapolation involved is from the physical point $q^2=-{m_\pi}^2$ to $q^2=0$. Therefore, if the hyperon decay amplitude is a smooth function of q^2, we obtain, for the surface term $M^{\rm S}$ in Eq. (3.5)

$$M^{S}=f_\pi^{-1}\left[(m_{B'}\pm m_B)\sum_n \frac{g^{\text{PC,PV}}(B\to n)}{(m_n\mp m_B)}G_{B'n}(0)+(m_B\pm m_{B'})\sum_l \frac{g^{\text{PC,PV}}(l\to B')}{(m_l\mp m_{B'})}G_{lB}(0)\right] i\bar{u}_{B'}(p_2)\gamma_5\begin{bmatrix}1\\ \gamma_5\end{bmatrix}u_B(p_1)+\cdots \ . \tag{3.14}$$

Here the upper (lower) form refers to the case of $H=H^{\text{PC}}$ (H^{PV}), and the dots denote the neglected nondiagonal ($L=1,2,\ldots$ intermediate states) terms. Using the remarkable relation $[A_\pi(0),H^{\text{PC,PV}}]=[V_\pi(0),H^{\text{PV,PC}}]$, the ETC term in Eq. (3.4) then becomes

$$M^{\text{ETC}}=f_\pi^{-1}(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2)\,|\,[V_{\mp},H^{\text{PC,PV}}]\,|\,B(p_1)\rangle \ . \tag{3.15}$$

We evaluate Eqs. (3.14) and (3.15) in the next section. Before doing this we add here several important comments.

(1) In the present calculation, the intermediate states n and l are on the mass shell. $G_{B'n}(0)$'s are the axial-vector couplings at zero-four-momentum-transfer-squared limit of hyperon semileptonic decays, $n\to B'+e+\bar{\nu}$. By using the Goldberger-Treiman relation,[12] obtained by inserting the PCAC relation between the states $\langle B'(p_2)|$ and $|n(p_n)\rangle$, see Eq. (3.10), one can replace $G_{B'B}(0)$ in Eq. (3.14) by the slightly off-shell ($m_\pi^2\to 0$) B-B'-π coupling. Then the (explicitly given) *diagonal* term in M^S, Eq. (3.14), *reproduces* the baryon-pole amplitudes in the conventional theory of hyperon decays based on current algebras and the soft-pion technique. There, these amplitudes were subtracted from the hyperon-decay matrix elements *before* the application of soft-pion approximation, in order to avoid the appearance of awkward singularity in the $q_u\to 0$ limit. On the contrary, in the present approach, they are automatically contained in the diagonal term of M^S. A subtle but potentially important difference is that in the present approach all the quantities appearing in M^S are on the mass shell (except for the extrapolation $q^2=-m_\pi^2\to 0$), whereas in the conventional approach the (added) pole terms have to be treated as the Feynman diagrams.

(2) M^S contains nondiagonal ($L=1,2,\ldots$) terms. The diagonal term ($L=0$) considered give the leading contribution. However, the next $L=1$ intermediate states in M^S will be fairly important. The higher level contribution ($L=2,3,\ldots$) can be neglected. This statement is in line with the usual expectation that the pole contributions from the higher resonances ($L=2,3,\ldots$) become small from the argument of wave function overlapping at the vertex. One can give a more instructive argument as follows. We define the asymptotic matrix elements of the axial-vector charge A_π, $\langle B',\alpha\,|\,A_\pi\,|\,B,\alpha\rangle$ (α is the helicity), and the invariant matrix element of H, $\langle B',\alpha\,|\,H\,|\,B,\alpha\rangle$, in the IMF by

$$\langle B',\alpha\,|\,A_\pi\,|\,B,\alpha\rangle(2\pi)^3\delta^3(\vec{k}_1-\vec{k}_2)\equiv(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2,\alpha)\,|\,A_\pi\,|\,B(p_1,\alpha)\rangle_{\vec{p}_1=\vec{p}_2\to\infty}\ , \tag{3.16}$$

$$\langle B',\alpha\,|\,H\,|\,B,\alpha\rangle\equiv(E_{B'}E_B/m_{B'}m_B)^{1/2}\langle B'(p_2,\alpha)\,|\,H\,|\,B(p_1,\alpha)\rangle_{\vec{p}_1=\vec{p}_2\to\infty}\ . \tag{3.17}$$

The off-shell ($m_\pi^2=0$) hyperon-pion $B\to B'\pi$ coupling constant $G(B\to B'\pi)$ is related to $\langle B',\alpha\,|\,A_\pi\,|\,B,\alpha\rangle$ as follows:

$$G(B\to B'\pi)=-f_\pi^{-1}(m_B+m_{B'})\langle B',\alpha\,|\,A_\pi\,|\,B,\alpha\rangle\ , \tag{3.18}$$

where $\alpha=\frac{1}{2}$. One can also work out the relation between $\langle B',\alpha\,|\,H\,|\,B,\alpha\rangle$ and the weak coupling constant g (at zero four-momentum transfer squared) defined in (3.11) as follows:

$$g^{\text{PC}}(B\to B')=2(m_B+m_B')^{-1}\langle B',\alpha\,|\,H^{\text{PC}}\,|\,B,\alpha\rangle\ ,\quad \alpha=\tfrac{1}{2}\ , \tag{3.19}$$

$$g^{\text{PV}}(B\to B')=2(m_B-m_{B'})^{-1}\langle B',\alpha\,|\,H^{\text{PV}}\,|\,B,\alpha\rangle\ ,\quad \alpha=\tfrac{1}{2}\ . \tag{3.20}$$

The appearance of various masses in Eqs. (3.18)—(3.20) are the explicit manifestation of the application of *asymptotic* SU(3) symmetry. In this formulation, $\langle B',\alpha\,|\,A_\pi\,|\,B,\alpha\rangle$ can be parametrized according to the usual SU(3) parametrization, i.e., exact SU(3) plus mixing. However, the SU(3) parametrization of $G(B\to B'\pi)$ exhibits explicit deviation from exact SU(3) according to Eq. (3.18). In the same way, although the *asymptotic* two-body hyperon weak matrix elements, $\langle B',\alpha\,|\,H\,|\,B,\alpha\rangle$, satisfy the $|\Delta\vec{I}\,|=\frac{1}{2}$ rule and octet rule, etc., as discussed in I, $g(B\to B')$'s satisfy more complicated relations according to Eqs. (3.19) and (3.20). Equation (3.20) also explicitly demonstrates that in the SU(3)-symmetry limit

$$\langle B',\alpha\,|\,H^{\text{PV}}\,|\,B,\alpha\rangle=0\ . \tag{3.21}$$

With the help of Eqs. (3.18)—(3.20) and the corresponding relations involving higher ($L=1,2,\ldots$) states, one can always cast M^S into the more instructive form, in terms of the asymptotic matrix elements of the axial-vector charges and H, i.e.,

$$M^{S}(\mathrm{PC})=-\frac{2}{f_\pi}\left[\sum_n \frac{(m_B+m_{B'})}{(m_n-m_B)(m_n+m_B)}\langle B'|A_\pi|n\rangle\langle n|H^{\mathrm{PC}}|B\rangle + \sum_l \frac{(m_B+m_{B'})}{(m_l-m_{B'})(m_l+m_B)}\langle B'|H^{\mathrm{PC}}|l\rangle\langle l|A_\pi|B\rangle\right] i\bar{u}_{B'}(p_2)\gamma_5 u_B(p_1)+\cdots , \tag{3.22}$$

$$M^{S}(\mathrm{PV})=-\frac{2}{f_\pi}\left[\sum_n \frac{(m_B-m_{B'})}{(m_n-m_B)(m_n+m_B)}\langle B'|A_\pi|n\rangle\langle n|H^{\mathrm{PV}}|B\rangle + \sum_l \frac{(m_B-m_{B'})}{(m_l-m_{B'})(m_l+m_B)}\langle B'|H^{\mathrm{PV}}|l\rangle\langle l|A_\pi|B\rangle\right] i\bar{u}_{B'}(p_2) u_B(p_1)+\cdots . \tag{3.23}$$

The nondiagonal term denoted by the dots in Eqs. (3.22) and (3.23) also takes the same form as the diagonal term explicitly written. The forms of Eqs. (3.22) and (3.23) are instructive for the estimate of the nondiagonal contribution. They explicitly show that as the masses of intermediate states (n and l) increases their contributions to M^S decreases like $1/m_n^2$ and $1/m_l^2$. Also the asymptotic matrix elements, $\langle L'|A_\pi|L\rangle$, will assume appreciable values only for the case of $L'=L$ and $L'=L\pm 1$. Therefore, the most important nondiagonal contribution should come from the $L=1$ states.[13]

(3) As long as we keep *only* the diagonal term in M^S, present calculation in its appearance does not look very different from the old current-algebra calculation with soft-pion approximation. Of course, on the asymptotic two-body baryon weak vertices we use the new constraints rather than the Minamikawa-Miura-Pati-Woo constraints. In the conventional soft-pion extrapolation, many authors add many boson- and baryon-pole amplitudes with the purpose of compensating the possible effect of extrapolation. In the present milder extrapolation, one needs to evaluate the nondiagonal term in M^S to increase the accuracy of the computation. However, *only* the excited baryon states give the contributions there. Therefore, the effect of meson pole terms, such as the popular K^*-meson term,[11] etc., should already be contained in the ETC term, M^{ETC}, evaluated in IMF. We give an argument that this is indeed the case in the present extrapolation in Appendix A. In the old pole model of hyperon decays, the S-wave amplitudes are known[14] to be described well by the K^*-pole model. On the other hand, it is known[14] that in the current-algebra calculation, the S-wave amplitudes are also well described in terms of a pure ETC term with predominantly pure f-type coupling ($d_w/f_w\sim -0.3$) in SU(3) symmetry. The fact that the ETC term in the present theory is a pure f coupling (in contrast with the $d_w/f_w=-1$ in the Minamikawa-Miura-Pati-Woo theorem) is, therefore, compatible with the claim that the K^*-pole effect is already included in the ETC term and need not be added by hand. Instead one has to worry about the $L=1$ baryon contribution to M^S.

IV. SUM RULES FOR HYPERON NONLEPTONIC DECAYS

In Sec. III, we have expressed the hyperon nonleptonic decay amplitudes, M^S and M^{ETC}, in terms of g^{PC} and g^{PV} and the axial-vector semileptonic coupling constants as in Eqs. (3.14) and (3.15), or equivalently in terms of the *asymptotic* two-body hyperon weak matrix elements $\langle B'|H|B\rangle$ and the *asymptotic* axial-vector matrix elements $\langle B'|A_\pi|B\rangle$ as in Eqs. (3.22), (3.23), and (3.15). The purpose of this section is to study, by using the asymptotic constraints on $\langle B'|H|B\rangle$ summarized[8] in Sec. II, how well these constraints can produce the sum rules (for example, the Lee-Sugawara sum rules)[15] which explain phenomenologically the experiment well.

However, we have to be aware of the following two limitations imposed by the approximation adopted in this paper, although the same method with the same approximation seems to produce a more successful result for the $K\to 2\pi$ decays (see paper II).

(1) In this paper, we keep only the diagonal term in M^S. In the case of $K\to 2\pi$ amplitudes, M^{ETC} turned out[7] to be more important than M^S and M^{ETC} consists of only the diagonal matrix elements of H. However, for the P-wave hyperon decay amplitudes, M^{ETC} is small and M^S thus has to play a more prominant role. Therefore, the importance of the role of the neglected nondiagonal term, especially the $\langle L=1|H|L=0\rangle$ term, in M^S is enhanced for the hyperon decays. Therefore, one has to expect a not completely satisfactory result under the present approximation, especially for the P-wave decays.

(2) Although we use the concept of asymptotic SU(3) symmetry, the treatment of broken SU(3) symmetry made in this paper is known to have some deficiency, if we confine our attention only to the ground-state baryons. There is a leakage due to symmetry breaking. As was noted a long time ago,[16] the presence of the exotic charge commutators such as $[\dot{V}_{K^0}, V_{K^0}]=0$ [which are valid, if the SU(3)-breaking interaction belongs to an octet] requires, with asymptotic SU(3) symmetry, the presence of quadratic Gell-Mann—Okubo mass formula, such as

$$3m_\Lambda^{\,2}+m_\Sigma^{\,2}=2(m_n^{\,2}+m_\Xi^{\,2}) . \tag{4.1}$$

However, if we consider further the presence of exotic commutator such as $[\dot{V}_{K^0}, A_{\pi^-}]=0$ (which should also hold in the usual model of flavor symmetry breaking) we encounter the Σ-Λ degeneracy, i.e.,

$$m_\Lambda=m_\Sigma\equiv(m_Y) . \tag{4.2}$$

Note that Y is not the $I=1$ decouplet. It only describes the fictitious degenerate mass of the Λ and Σ inherent in the present approximation. In the treatment of hyperon decays, especially the P-wave amplitudes which become singular [see Eq. (3.22)] in the SU(3) degenerate mass limit, the presence of internal inconsistency due to the approximation such as Eq. (4.2) will be keenly felt, since the masses of baryons play a crucial role in the sum rules, reflecting broken SU(3) symmetry. In view of Eq. (4.2), the more realistic hyperon mass relation *in the present approximation* is

$$(m_Y+m_n)\Delta_1=(m_\Xi+m_Y)\Delta_2$$
$$(\Delta_1=m_Y-m_N \text{ and } \Delta_2=m_\Xi-m_Y)\ . \quad (4.3)$$

The above mass degeneracy can only be removed,[3] if we consider the SU(3) mixings between the ground-state baryons and their radially excited states, for example. These mixings should exist in principle and it has already been shown[17] that an inclusion, for example, of the ninth $I=0$ $J^P=\frac{1}{2}^+$ baryon Λ' not only removes the Σ-Λ degeneracy in Eq. (4.2) improving the agreement of the Gell-Mann—Okubo mass formula with experiment, but also removes the sometimes rather considerable discrepancy between the experimentally known values of the hyperon axial-vector semileptonic coupling constants and their theoretical values given in Eq. (4.12). Therefore, we have to include, eventually, the effect of the mixings discussed above in the treatment of hyperon decays. For the boson cases, we do *not* have the analog of Σ-Λ degeneracy as long as we treat the $q\bar{q}$ nonet mesons properly.

We define the invariant physical amplitudes of the hyperon decays by

$$M(B\to B'\pi)=\bar{u}_{B'}(p_2)[a(B\to B'\pi)+b(B\to B'\pi)\gamma_5]u_B(p_1)\ , \quad (4.4)$$

where a and b denote the S- and P-wave amplitudes, respectively. From Eqs. (3.14) and (3.15) we have for the S-wave (parity-violating) decays, denoting $G(0)$ by G,

$$a(\Lambda\to p\pi^-)=\frac{\Delta_1}{f_\pi(m_n+m_Y)}[-g^{PV}(\Lambda\to n)G_{pn}+g^{PV}(\Sigma^+\to p)G_{\Sigma^+\Lambda}]-\frac{g^{PC}(\Lambda\to n)}{f_\pi}\ , \quad (4.5a)$$

$$a(\Lambda\to n\pi^0)=\frac{\sqrt{2}\Delta_1}{f_\pi(m_n+m_Y)}[-g^{PV}(\Lambda\to n)G_{nn}+g^{PV}(\Sigma^0\to n)G_{\Sigma^0\Lambda}]+\frac{g^{PC}(\Lambda\to n)}{\sqrt{2}f_\pi}\ , \quad (4.6a)$$

$$a(\Sigma^+\to n\pi^+)=\frac{\Delta_1}{f_\pi(m_n+m_Y)}[-g^{PV}(\Sigma^+\to p)G_{np}+g^{PV}(\Lambda\to n)G_{\Lambda\Sigma^+}+g^{PV}(\Sigma^0\to n)G_{\Sigma^0\Sigma^+}]$$
$$-\frac{1}{f_\pi}[g^{PC}(\Sigma^+\to p)+\sqrt{2}g^{PC}(\Sigma^0\to n)]\ , \quad (4.7a)$$

$$a(\Sigma^+\to p\pi^0)=\frac{\sqrt{2}\Delta_1}{f_\pi(m_n+m_Y)}[-g^{PV}(\Sigma^+\to p)G_{pp}+g^{PV}(\Sigma^+\to p)G_{\Sigma^+\Sigma^+}]+\frac{g^{PC}(\Sigma^+\to p)}{\sqrt{2}f_\pi}\ , \quad (4.8a)$$

$$a(\Sigma^-\to n\pi^-)=\frac{\Delta_1}{f_\pi(m_n+m_Y)}[g^{PV}(\Sigma^0\to n)G_{\Sigma^0\Sigma^-}+g^{PV}(\Lambda\to n)G_{\Lambda\Sigma^-}]+\frac{\sqrt{2}g^{PC}(\Sigma^0\to n)}{f_\pi}\ , \quad (4.9a)$$

$$a(\Xi^-\to\Lambda\pi^-)=\frac{\Delta_2}{f_\pi(m_Y+m_\Xi)}[-g^{PV}(\Xi^-\to\Sigma^-)G_{\Lambda\Sigma^-}+g^{PV}(\Xi^0\to\Lambda)G_{\Xi^0\Xi^-}]-\frac{g^{PC}(\Xi^0\to\Lambda)}{f_\pi}\ , \quad (4.10a)$$

$$a(\Xi^0\to\Lambda\pi^0)=\frac{\sqrt{2}\Delta_2}{f_\pi(m_Y+m_\Xi)}[-g^{PV}(\Xi^0\to\Sigma^0)G_{\Lambda\Sigma^0}+g^{PV}(\Xi^0\to\Lambda)G_{\Xi^0\Xi^-}]+\frac{g^{PC}(\Xi^0\to\Lambda)}{\sqrt{2}f_\pi}\ . \quad (4.11a)$$

The last term of each equation corresponds to the ETC term.

For P-wave (parity-conserving) decays, we obtain similarly,

$$b(\Lambda\to p\pi^-)=\frac{(m_Y+m_n)}{f_\pi\Delta_1}[-g^{PC}(\Lambda\to n)G_{pn}+g^{PC}(\Sigma^+\to p)G_{\Sigma^+\Lambda}]+\frac{g^{PV}(\Lambda\to n)}{f_\pi}\ , \quad (4.5b)$$

$$b(\Lambda\to n\pi^0)=\frac{\sqrt{2}(m_Y+m_n)}{f_\pi\Delta_1}[-g^{PC}(\Lambda\to n)G_{nn}+g^{PC}(\Sigma^0\to n)G_{\Sigma^0\Lambda}]-\frac{g^{PV}(\Lambda\to n)}{\sqrt{2}f_\pi}\ , \quad (4.6b)$$

$$b(\Sigma^+\to n\pi^+)=\frac{(m_Y+m_n)}{f_\pi\Delta_1}[-g^{PC}(\Sigma^+\to p)G_{np}+g^{PC}(\Sigma^0\to n)G_{\Sigma^0\Sigma^+}+g^{PC}(\Lambda\to n)G_{\Lambda\Sigma^+}]$$
$$+\frac{1}{f_\pi}[g^{PV}(\Sigma^+\to p)+\sqrt{2}g^{PV}(\Sigma^0\to n)]\ , \quad (4.7b)$$

$$b(\Sigma^+\to p\pi^0)=\frac{\sqrt{2}(m_Y+m_n)}{f_\pi\Delta_1}[-g^{PC}(\Sigma^+\to p)G_{PP}+g^{PC}(\Sigma^+\to p)G_{\Sigma^+\Sigma^+}]-\frac{g^{PV}(\Sigma^+\to p)}{\sqrt{2}f_\pi}\ , \quad (4.8b)$$

$$b(\Sigma^+\to n\pi^-)=\frac{(m_Y+m_n)}{f_\pi\Delta_1}[g^{PC}(\Sigma^0\to n)G_{\Sigma^0\Sigma^-}+g^{PC}(\Lambda\to n)G_{\Lambda\Sigma^-}]-\frac{\sqrt{2}g^{PV}(\Sigma^0\to n)}{f_\pi}, \tag{4.9b}$$

$$b(\Xi^-\to\Lambda\pi^-)=\frac{(m_\Xi+m_Y)}{f_\pi\Delta_2}[-g^{PC}(\Xi^-\to\Sigma^-)G_{\Lambda\Sigma^-}+g^{PC}(\Xi^0\to\Lambda)G_{\Xi^0\Xi^-}]+\frac{g^{PV}(\Xi^0\to\Lambda)}{f_\pi}, \tag{4.10b}$$

$$b(\Xi^0\to\Lambda\pi^0)=\frac{(m_\Xi+m_Y)}{f_\pi\Delta_2}[-g^{PC}(\Xi^0\to\Sigma^0)G_{\Lambda\Sigma^0}+g^{PC}(\Sigma^0\to\Lambda)G_{\Xi^0\Xi^0}]-\frac{g^{PV}(\Xi^0\to n)}{\sqrt{2}f_\pi}. \tag{4.11b}$$

Here $g^{PC}(B\to B')$ and $g^{PV}(B\to B')$ are related to the asymptotic two-body weak baryon matrix elements, $\langle B'|H^{PC}|B\rangle$ and $\langle B'|H^{PV}|B\rangle$ by Eqs. (3.19) and (3.20), respectively, and the relations among $\langle B'|H|B\rangle$'s are summarized in Sec. II. For the axial-vector semileptonic coupling constant $G_{B'B}(0)$ defined in Eq. (3.10), the following sum rules have been obtained a long time ago[18] by realizing the algebra, $[A_{\pi^+},A_{\pi^-}]=2V_3$, among the ground-state $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. k denotes the fractional contribution of the ground-state baryons to the algebra ($k\simeq0.6$). Writing $G_{B'B}(0)$ as $G_{B'B}$,

$$\begin{aligned}
&G_{pn}=G_{np}=\tfrac{5}{3}\sqrt{k},\quad G_{pp}=-G_{nn}=\tfrac{5}{6}\sqrt{k},\\
&G_{\Sigma^+\Sigma^0}=G_{\Sigma^0\Sigma^+}=-G_{\Sigma^0\Sigma^-}=-G_{\Sigma^-\Sigma^0}=-\tfrac{2}{3}\sqrt{2}\sqrt{k},\\
&G_{\Lambda\Sigma^+}=G_{\Sigma^+\Lambda}=G_{\Sigma^-\Lambda}=G_{\Lambda\Sigma^-}=\sqrt{\tfrac{2}{3}}\sqrt{k},\quad G_{\Lambda\Sigma^0}=G_{\Sigma^0\Lambda}=\frac{1}{\sqrt{3}}\sqrt{k},\\
&G_{\Xi^0\Xi^-}=-G_{\Xi^-\Xi^0}=\tfrac{1}{3}\sqrt{k},\quad G_{\Xi^0\Xi^0}=-G_{\Xi^-\Xi^-}=-\tfrac{1}{6}\sqrt{k}.
\end{aligned} \tag{4.12}$$

The above parametrization of G's corresponds to the d/f ratio $\frac{3}{2}$, same as the SU(6) result. However, the value of $g_A(0)=G_{pn}$ is not $\frac{5}{3}$ but $(\frac{5}{3})\sqrt{k}$.

We now discuss the sum rules of hyperon decays. From Eqs. (4.5a)–(4.11b), the $|\Delta\vec{I}|=\frac{1}{2}$ is always satisfied exactly, as long as we keep *only* the diagonal term in M^S. The most prominent feature of the S-wave decays is that $a(\Sigma^+\to n\pi^+)_{\rm expt}$ is very small compared with other amplitudes, i.e., $a(\Sigma^+\to n\pi^+)_{\rm expt}\simeq0$. This can be understood easily. From Eqs. (4.7a), (2.1), and (4.12), we see immediately that our sum rules yield

$$a(\Sigma^+\to n\pi^+)=0, \tag{4.13}$$

without referring to special symmetry limit, etc. Another feature of the S-wave decays is that experimentally they satisfy the Lee-Sugawara (LS) sum rules[15] very well. We define δ^S (LS) by

$$\delta^S({\rm LS})\equiv a(\Lambda\to p\pi^-)+2a(\Xi^-\to\Lambda\pi^-)-\sqrt{3}a(\Sigma^+\to p\pi^0). \tag{4.14}$$

δ^S (LS)=0, if the LS relation is satisfied. Experimentally, it is known that

$$|\delta^S({\rm LS})/a(\Lambda^0\to p\pi^-)|_{\rm expt}\simeq0.03. \tag{4.15}$$

By using Eqs. (2.1) and (4.12) in Eqs. (4.5a), (4.8a), and (4.10a), we obtain

$$\frac{\delta^S({\rm LS})}{a(\Lambda\to p\pi^-)}=\frac{(\Delta_1+\Delta_2)}{(m_Y+m_\Xi)}\left[\frac{2\sqrt{k}\,w_1^{PV}+6w_1^{PC}}{3\sqrt{k}\,w_1^{PV}+3w_1^{PC}}\right]. \tag{4.16}$$

In the SU(3) limit $\Delta_1=\Delta_2=0$, Eq. (4.16) implies that our S-wave amplitudes satisfy the LS sum rules in the SU(3) *degenerate* mass limit. If we set $|w_1^{PV}|\ll|w_1^{PC}|$ [in the SU(3) limit $w_1^{PV}=0$] in Eq. (4.16), we then obtain from Eq. (4.3)

$$|\delta^S({\rm LS})/a(\Lambda\to p\pi^-)|\simeq0.3. \tag{4.17}$$

This shows that the present sum rules reproduce the LS sum rule *fairly* well for the S waves. It is known that a good agreement is realized for the value of $d_W/f_W\simeq-(0.3–0.4)$ for the ETC term, whereas our ETC term is pure f type. We may note that in any theory the S-wave amplitudes (rather than the P wave) should first be accounted for[1] by the basic approximation procedure.

The P-wave amplitudes involve terms proportional to Δ_1^{-1} and Δ_2^{-1} which are singular in the degenerate mass limit. They are thus *much more* sensitive to SU(3)-symmetry breaking. Corresponding to Eq. (4.14), we define

$$\delta^P({\rm LS})\equiv b(\Lambda\to p\pi^-)+2b(\Xi^-\to\Lambda\pi^-)-\sqrt{3}b(\Sigma^+\to p\pi^0). \tag{4.18}$$

In place of Eq. (4.16), we then obtain

$$\frac{\delta^P({\rm LS})}{b(\Lambda\to p\pi^-)}=\frac{(\Delta_2-\Delta_1)}{\Delta_2}\left[\frac{2\sqrt{k}\,w_1^{PC}+6w_1^{PV}}{3\sqrt{k}\,w_1^{PC}+3w_1^{PV}}\right]. \tag{4.19}$$

Therefore, if $\Delta_2=\Delta_1$ [equal mass spacing but *not* the SU(3)-symmetry limit], the P-wave amplitudes also *satisfy* the LS sum rules. Since, from Eq. (4.3) $|(\Delta_2-\Delta_1)/\Delta_2|=(m_\Xi-m_n)/(m_Y+m_n)\simeq0.18$, we obtain again assuming $w_1^{PV}\ll w_1^{PC}$, $\delta^P({\rm LS})/b(\Lambda\to p\pi^-)\simeq0.12$. This value should be compared with the experimental value $|\delta^P({\rm LS})/b(\Lambda\to p\pi^-)|_{\rm expt}\simeq0.3$, which shows that the P-wave amplitudes do not satisfy the LS sum rules so well. However, the agreement is not as disastrous either. It can even be said that in the approximation involved, the result obtained is rather a reasonable one. It is in fact interesting to notice that the discrepancies between the theory and experiments for both the S- and P-wave decays are of the same order of magnitude. As mentioned before, to remedy the situation we have to improve not only our soft-meson approximation by including the $L=1$ baryons in the nondiagonal term of M^S but also our asymptotic-SU(3)-symmetry calculation by removing the inherent Σ-Λ degeneracy. The drawback of the approximations used appears most prominently in the fact that the sum rules lead to $b(\Sigma^+\to n\pi^+)=0$ in the contradiction with experiment. The inclusion of $L=1$ baryons, especially the $\frac{1}{2}^-$ baryons, will remove this difficulty. The SU(3)-singlet baryons such as $Y^{*0}(1405)$ may play an interesting role there. To obtain a definitive handle of this problem, we have to first solve the level-realization constraints including the $L=1$ baryon states.

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APPENDIX A: MESON POLE AMPLITUDES IN THE NEW SOFT-PION APPROXIMATION

For illustration, we take up the K^*-meson pole amplitudes in the hyperon decays in the light of new soft-pion approximation introduced.

The *invariant* K^*-pole matrix elements are written as

$$M^{K^*}(B\to B'\pi;q)=f_W(\bar{K}^*\to\pi)G(B\to B'\bar{K}^*)\times q_\mu \bar{u}_{B'}(p_2)\gamma_\mu u_B(p_1)\ , \quad \text{(A1)}$$

where f_W denotes the $\bar{K}^*\to\pi$ vertex and $q_\mu=(p_1-p_2)_\mu$. In the conventional soft-pion approximation $q_\mu\to 0$, (A1) is dropped because it involves q_μ. However, (A1) can also be written, if the pion is on the mass shell, as

$$M^{K^*}(B\to B'\pi;q)=(m_B-m_{B'})f_W(\bar{K}^*\to\pi)G(B\to B'\bar{K}^*)\times i\bar{u}_{B'}(p_2)u_B(p_1)\ . \quad \text{(A2)}$$

In the new soft-pion approximation $\vec{q}\to 0$ in the $\vec{p}_1=\vec{p}_2\to\infty$ frame, we never take $q_0\to 0$ limit. Therefore, one can still use the energy-momentum conservation. The only extrapolation involved is $q^2\to 0$ instead of $q^2=-m_\pi^2$. Therefore, the (extrapolated) *invariant* amplitude can still be written in the same form as Eq. (A2), i.e.,

$$M^{K^*}(B\to B'\pi;q)_{\vec{p}_1=\vec{p}_2\to\infty}\sim(m_B-m_{B'})f_W(\bar{K}^*\to\pi)G(B\to B'\bar{K}^*)i\bar{u}_{B'}(p_2)u_B(p_1)_{\vec{p}_1=\vec{p}_2\to\infty}\ . \quad \text{(A3)}$$

The only difference between Eqs. (A2) and (A3) is that for the vertices in (A2) $q^2=-m_\pi^2$, whereas for the same vertices in (A3) the extrapolation $q^2\to 0$ is involved. This shows that under the new milder soft-pion approximation, the K^*-pole amplitudes will not receive a drastic change. The same argument also holds for the other meson-pole amplitudes.

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