PHYSICAL REVIEW D VOLUME 29, NUMBER 3 1 FEBRUARY 1984

New approach to nonleptonic weak interactions. II. Application to strange- and charm-meson decays

K. Terasaki
*Research Institute for Theoretical Physics, Hiroshima University, Takehara, Hiroshima-ken, 725, Japan**
and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742

S. Oneda and T. Tanuma
Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 31 January 1983)

The constraints obtained in I for the asymptotic two-body meson matrix elements of the weak nonleptonic Hamiltonian are related to the physical $K\rightarrow\pi\pi$ and $D\rightarrow\bar{K}\pi$ decay amplitudes by using a soft-pion approximation in the infinite-momentum frame of the parent particle. In this new milder extrapolation, a part of the so-called surface terms survives in addition to the usual equal-time-commutator term and plays a significant role, especially for the $D\rightarrow\bar{K}\pi$ decays. However, to the approximation in which only the ground-state-meson contribution is kept in the surface terms, the $|\Delta\vec{I}|=\frac{1}{2}$ rule for the $K\rightarrow 2\pi$ decays and the $6\oplus 6^*$ rule [in exact SU(3) symmetry] for the $D\rightarrow\bar{K}\pi$ decays are shown to hold exactly and the amplitudes predict a reasonable value for the ratio of the rates, $\Gamma(K_S^0\rightarrow\pi^+\pi^-)/\Gamma(D^0\rightarrow K^-\pi^+)$. A semiquantitative scenario is also drawn, which suggests that the $L=1$ meson contribution to the surface terms plays an important role for the violation of these selection rules and yields the right order of magnitude for the violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule in $K\rightarrow 2\pi$ decays. For the $D\rightarrow\bar{K}\pi$ decays it leads to a more significant violation of the $6\oplus 6^*$ rule.

I. INTRODUCTION AND SUMMARY

The strange- and charm-meson nonleptonic decay is one of the unresolved and persistent problems of particle physics.[1] In perturbative QCD, the popular prescription to explain the $|\Delta\vec{I}|=\frac{1}{2}$ rule is to assume the enhancement of the penguin diagram.[2] The enhancement may be justified, if the factorization (or vacuum-insertion) approximation, which was introduced[3] a long time ago and provides a way to translate the information at the quark-gluon level into that of hadronic level, is a good approximation. However, if one applies the same method to the Cabibbo-angle-favored charm-meson decays (to which penguins cannot contribute), one finds[4] that the $D^0\rightarrow\bar{K}^0\pi^0$ and $\bar{K}^{*0}\pi^0$ modes are much suppressed compared with other two-body or quasi-two-body modes such as $D^0\rightarrow K^-\pi^+$, $K^{*-}\pi^+$, $K^-\rho^+$ and $D^+\rightarrow\bar{K}^0\pi^+$, $\bar{K}^{*0}\pi^+$, $\bar{K}^0\rho^+$. This is in sharp contradiction with experiment. There are also other (more qualitative) propositions based, for example, on the hypothesis of conservation of total quark numbers[5] or the dominance of W-exchange diagrams,[6] etc. In view of these situations, a new approach,[7,8] which pays a proper attention to the long-distance-physics aspects of the problem, may be in order.

The task of this paper is to give a unified description of the characteristics of the strange- and charmed-meson nonleptonic interactions, by relating the *physical* amplitudes to the asymptotic two-body ground-state-meson matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$ already discussed in the preceding paper called I, using a new soft-meson approximation. The soft-pion technique[9] has provided a powerful nonperturbative approximation in low-energy physics, although it sometimes produced ambiguous results. One relates, for example, the $K_S^0\rightarrow 2\pi$ decay amplitude to the two-body matrix element $\langle\pi|H^{(0,-)}|K\rangle$ via the soft-pion approximation, retaining only the equal-time-commutator (ETC) term and discarding the so-called surface term in the limit $q_\mu\rightarrow 0$. However, for the hyperon non-leptonic decays one is forced to retain[9] a part of the surface term (the baryon-pole terms) because of the appearance (in the soft-pion limit) of the singularity due to the mass degeneracy. In fact, these pole terms do play a significant role in the current-algebra calculation of the hyperon decays using the soft-pion approximation. This seems to suggest that the dropping of the surface term in the $q_\mu\rightarrow 0$ limit is a rather tricky procedure and the extrapolation may not be as smooth as one wishes.

In this paper, we are led naturally to use a new "soft"-meson approximation which is more accurate (see especially, Appendix A for comparison) than the usual $q_\mu\rightarrow 0$ method. Tomozawa[10] has noted that one can achieve *effectively* the $q_\mu\rightarrow 0$ limit, if we consider the extrapolation $\vec{q}\rightarrow 0$ in the infinite-momentum frame of the incident momentum $\vec{p}_1$. In this frame, such a quantity as $(q\cdot p_1)$ remains finite even in the limit $\vec{q}\rightarrow 0$, so that a part of the surface term survives in the limit $\vec{q}\rightarrow 0$. Since the constraints on the two-body ground-state weak matrix elements (derived in I) are valid only in the asymptotic limit, we are naturally led to work in the infinite-momentum frame of the parent particle ($\vec{p}_1\rightarrow\infty$) and a hard-pion extrapolation $q^2\rightarrow 0$ can then be realized by taking a limit $\vec{q}\rightarrow 0$ in this frame. By using this method, we can relate the $K\to 2\pi$ and $D\to \bar{K}\pi$ amplitudes to the two-body asymptotic matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$.

We then show that, *to the extent* that the ground-state-meson contribution dominates over that of the higher levels in the nonvanishing surface term, we obtain $|\Delta \vec{I}|=\frac{1}{2}$ rule for the $K\to 2\pi$ decays and its charm counterparts for the $D\to\bar{K}\pi$ decays. We then compute the ratio of the amplitude of $K_S^0\to\pi^+\pi^-$ decay to that of $D^0\to K^-\pi^+$ decay and show that the prediction is in reasonable agreement with the present experimental ratio $\Gamma(D^0\to K^-\pi^+)/\Gamma(K_S^0\to\pi^+\pi^-)$. Therefore, a unified description of strange- and charm-meson decays seems feasible in the present approach. We also give a semiquantitative estimate of the violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule and its charm counterpart, which arises mainly from the contributions of the first excited meson states to the surface term. We show that the magnitudes of the violation are of the right order of magnitude and are *larger* for the D decays.

II. CONSTRAINTS ON THE ASYMPTOTIC TWO-BODY WEAK NONLEPTONIC MATRIX ELEMENTS

We here summarize the constraints obtained in I on the *asymptotic two-body* ground-state meson matrix elements of the relevant weak nonleptonic effective Hamiltonian, $H_W=(H^{(0,-)}$ and $H^{(-,-)})$. In I, we have obtained the $|\Delta\vec{I}|=\frac{1}{2}$ rule and octet rule for the asymptotic *two-body ground-state-meson* matrix elements of $H^{(0,-)}$ and its charm counterpart, i.e., the 20 dominance [or $6\oplus 6^*$ rule in exact SU(3) symmetry] for the similar asymptotic matrix elements of $H^{(-,-)}$. The $|\Delta\vec{I}|=\frac{1}{2}$ selection rule obtained is *asymptotic* ($\vec{p}\to\infty$) and is given by ($H=H^{(0,-)}$), for example,

$$-\sqrt{2}\langle\pi^0|H|K^0(\vec{p})\rangle=\langle\pi^+|H|K^+(\vec{p})\rangle\ , \tag{2.1}$$

$$-\sqrt{2}\langle\rho^0|H|K^{*0}(\vec{p})\rangle=\langle\rho^+|H|K^{*+}(\vec{p})\rangle\ , \tag{2.2}$$

$$-\sqrt{2}\langle\pi^0|H|K^{*0}(\vec{p})\rangle=\langle\pi^+|H|K^{*+}(\vec{p})\rangle\ . \tag{2.3}$$

In addition, we also obtained[11] SU(6)-type asymptotic constraints such as ($\vec{p}\to\infty$),

$$\langle\pi^+|H|K^+(\vec{p})\rangle=\langle\rho^+|H|K^{*+}(\vec{p})\rangle$$
$$=\pm\langle\pi^+|H|K^{*+}(\vec{p})\rangle\ . \tag{2.4}$$

The charm counterpart of the above asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule is the asymptotic $\Delta V=0$ rule given, for example, by

$$\langle\bar{K}^0|H^{(-,-)}|D^0(\vec{p})\rangle$$
$$=-\langle\pi^+|H^{(-,-)}|F^+(\vec{p})\rangle,\ \ \vec{p}\to\infty\ . \tag{2.5}$$

Only asymptotic SU(3) [not SU(4)] symmetry was needed in deriving Eq. (2.5). Altogether, we obtained in I ($H\equiv H^{(-,-)}$),

$$\langle\bar{K}^0|H|D^0(\vec{p})\rangle=-\langle\pi^+|H|F^+(\vec{p})\rangle=-\cot\theta_C\langle\pi^+|H^{(0,-)}|K^+(\vec{p})\rangle\ , \tag{2.6}$$

$$\langle\bar{K}^{*0}|H|D^{*0}(\vec{p})\rangle=-\langle\rho^+|H|F^{*+}(\vec{p})\rangle=-\cot\theta_C\langle\rho^+|H^{(0,-)}|K^{*+}(\vec{p})\rangle\ , \tag{2.7}$$

$$\langle\bar{K}^0|H|D^{*0}(\vec{p})\rangle=-\langle\pi^+|H|F^{*+}(\vec{p})\rangle=-\cot\theta_C\langle\pi^+|H^{(0,-)}|K^{*+}(\vec{p})\rangle\ , \tag{2.8}$$

$$\langle\bar{K}^0|H|D^0(\vec{p})\rangle=\langle\bar{K}^{*0}|H|D^{*0}(\vec{p})\rangle=\pm\langle\bar{K}^0|H|D^{*0}(\vec{p})\rangle\ ,\ \ \vec{p}\to\infty\ . \tag{2.9}$$

However, the last equality of each of the above sum rules was obtained by using the algebra (θ_C is the Cabibbo angle)

$$[H^{(-,-)},V_{D^0}]=\cot\theta_C H^{(0,-)}\ , \tag{2.10}$$

and asymptotic SU(4) symmetry for the SU(4) charge V_{D^0}. Therefore, they are more susceptible to the effect of SU(4) symmetry breaking but will still be of great value in relating the strange-meson decays to the charm-meson decays.

III. NEW SOFT-MESON APPROXIMATION IN THE INFINITE-MOMENTUM FRAME

We consider the decay of pseudoscalar meson $P_1(p_1)$ into two pseudoscalar mesons $P_2(p_2)$ and $P_3(q)$, $P_1(p_1)\to P_2(p_2)+P_3(q)$, where p_1, p_2, and q denote the four-momenta with $p_1=p_2+q$. (P_2,P_3) is either (π,π) or (K,π) system.

In order to take full advantage of the asymptotic constraints already obtained and also to minimize the effect of flavor symmetry breaking by using the notion of asymptotic flavor symmetry [i.e., asymptotic SU(2) and SU(3) symmetries for the (π,π) and (K,π) systems, respectively], we evaluate the invariant amplitude proportional to $\langle P_2(p_2)P_3(q)|H_W|P_1(p)\rangle$ in the frame in which all the participating particles have infinite momenta, i.e., $\vec{p}_2\to\infty$ and $\vec{q}\to\infty$ with $p_1=p_2+q$. It then follows[12] that the *invariant* amplitude of the physical matrix element $\langle P_2(p_2)P_3(q)|H_W|P_1(p_1)\rangle$ must be symmetric, even in *broken* flavor symmetry, with respect to the exchange of four-momenta of the final two pseudoscalar mesons, $p_2\leftrightarrow q$. [In exact SU(3) symmetry, in which P_2 and P_3 belong to the same flavor multiplet with the same mass, this symmetry must, of course, be satisfied.] Therefore, in applying the soft-meson approximation, we have to pay close attention to this inherent constraint[12] which exists even in broken symmetry. We, thus, always consider the amplitude which is symmetrized with respect to the two final pseudoscalar mesons. We thus define

$$M(P_1 \to P_2P_3)_{\rm sym} \equiv \tfrac{1}{2}\langle P_2(p_2)P_3(q)+P_3(p_2)P_2(q)\,|\,H_W\,|\,P_1(p_1)\rangle\sqrt{2q_0}\ . \tag{3.1}$$

In anticipation of the use of Lehmann-Symanzik-Zimmermann (LSZ) reduction formulas, we define a hypothetical amplitude

$$T_\mu^{(j)}(P_1 \to P_2P_3;q) \equiv i\int d^4x\, e^{-iqx}\langle P_k(p_2)\,|\,T[A_\mu^{\bar{j}}(x),H_W(0)]\,|\,P_1(p_1)\rangle\ . \tag{3.2}$$

$A_\mu^j(x)$ is the axial-vector current and (j,k) takes either $j=2$ and $k=3$ or $j=3$ and $k=2$. (Note that the index j represents the *flavor* index of the meson P_j.) The standard reduction and the use of PCAC (partial conservation of axial-vector current)

$$\partial_\mu A_\mu^j(x) = f_j m_j^2 \phi_j(x) \tag{3.3}$$

leads to [$\phi_j(x)$ denotes the field of P_j with mass m_j and decay constant f_j]

$$M(P_1 \to P_2P_3)_{\rm sym} = \frac{i}{2}\sum_{j=2,3}\left[\frac{q^2+m_j^2}{f_j m_j^2}\right] q_\mu T_\mu^{(j)} + \sum_{j,k}\left[\frac{q^2+m_j^2}{f_j m_j^2}\right]\int d^4x\, e^{-iqx}\delta(x_0)\langle P_k(p_2)\,|\,[A_0^{\bar{j}}(x),H_W(0)]\,|\,P_1(p_1)\rangle\ . \tag{3.4}$$

Here $\sum_{j,k}$ implies the summation over $j=2$, $k=3$ and $j=3$, $k=2$. We now consider the *invariant* amplitude $M^{\rm phys} \equiv (2p_{10}2p_{20})^{1/2} M(P_1 \to P_2P_3)_{\rm sym}$, evaluated in the infinite-momentum frame (IMF) with $\vec{p}_1 \to \infty$, and carry out a soft-meson approximation $\vec{q} \to 0$ in the IMF, i.e.,

$$M^{\rm phys} \simeq [(2p_{10}2p_{20})^{1/2} M(P_1 \to P_2P_3)_{\rm sym}]_{\vec{q}\to 0,\,\vec{p}_1=\vec{p}_2\to\infty} \equiv M^{\rm S} + M^{\rm ETC}\ , \tag{3.5}$$

where

$$M^{\rm S} = \lim_{\vec{q}\to 0,\,\vec{p}_1\to\infty}\ \sum_{j=2,3}(2p_{10}2p_{20})^{1/2}(2f_j)^{-1} i q_\mu T_\mu^{(j)}\ , \tag{3.6}$$

$$M^{\rm ETC} = \lim_{\vec{q}\to 0,\,p_1\to\infty} -i\sum_{j,k}(2f_j)^{-1}(2p_{10}2p_{20})^{1/2}\langle P_k(p_2)\,|\,[A_{\bar{j}}(0),H_W(0)]\,|\,P_1(p_1)\rangle\ . \tag{3.7}$$

Here $M^{\rm S}$ is the surface term and $M^{\rm ETC}$ corresponds to the usual equal-time-commutator term. $A_j(0) \equiv -i\int d^3x\, A_0^j(\vec{x},0)$ is the axial-vector charge and the index j denotes the flavor index of P_j. In the IMF with $\vec{q}=0$, $\vec{p}_1=\vec{p}_2\to\infty$. Therefore, $q_0=(\vec{p}_1^{\,2}+m_1^2)^{1/2}-(\vec{p}_2^{\,2}+m_2^2)^{1/2}\to 0$ like $(m_1^2-m_2^2)/2\,|\vec{p}_1|$ as $|\vec{p}_1|\to\infty$. This implies that in the IMF one can effectively achieve $q_\mu\to 0$, without assuming the *masslessness* of the pseudoscalar meson. In the conventional soft-meson approximation, it is hoped that the surface term (3.6) can be dropped in the limit $q_\mu\to 0$. However, if the extrapolation $q_\mu\to 0$ is not as smooth as we would wish, the neglect of $M^{\rm S}$ is not warranted, as exemplified by the calculations of the hyperon nonleptonic interactions. In the proposed $\vec{q}\to 0$ extrapolation in the IMF, the surface term does give a contribution. The extrapolation involved is much less severe than the $q_\mu\to 0$, as will be demonstrated in Appendix A, but we now have to retain some parts of $M^{\rm S}$ in addition to $M^{\rm ETC}$.

We decompose $T_\mu^{(j)}$ as $T_\mu^{(j)}=T_\mu^{(j,+)}+T_\mu^{(j,-)}$, where

$$T_\mu^{(j,+)} = i\int d^4x\, e^{-iqx}\theta(x_0)\langle P_k(p_2)\,|\,A_\mu^{\bar{j}}(x)H_W(0)\,|\,P_1(p_1)\rangle\ , \tag{3.8}$$

$$T_\mu^{(j,-)} = i\int d^4x\, e^{-iqx}\theta(-x_0)\langle P_k(p_2)\,|\,H_W(0)A_\mu^{\bar{j}}(x)\,|\,P_1(p_1)\rangle\ . \tag{3.9}$$

We now insert a complete set of single-particle boson intermediate states (which, in our theoretical framework, should be the $q\bar{q}$ meson states) between the factors $A_\mu^{\bar{j}}(x)$ and $H_W(0)$ in Eqs. (3.8) and (3.9). We then *decompose* the intermediate states in terms of *levels*, exactly in the same way as we do in the level realization of algebras discussed in I. Namely, we may write the sum over the intermediate states as, for example,

$$\sum_{n_L}\langle P_k(p_2)\,|\,A_\mu^{\bar{j}}(x)\,|\,n_L\rangle\langle n_L\,|\,H_W(0)\,|\,P_1(p_1)\rangle\ . \tag{3.10}$$

n_L denotes the mesons belonging to the level L. In this decomposition we keep only the "diagonal" term, i.e., the ground-state-meson intermediate states, since the external mesons P belong to the ground-state mesons. We show later that the nondiagonal higher-intermediate-meson-state contribution is relatively small. We thus see that the concept of levels is also useful for evaluating $M^{\rm S}$. Among the ground-state mesons, only the vector mesons contribute to the intermediate states of Eq. (3.10). We denote the vector mesons appearing in $T_\mu^{(j,+)}$ and $T_\mu^{(j,-)}$ as V_n and V_l, respectively, and their four-momenta are denoted as $(\vec{p}_n, p_{n0}=(\vec{p}_n^{\,2}+m_n^2)^{1/2})$ and $(\vec{p}_l, p_{l0}=(\vec{p}_l^{\,2}+m_l^2)^{1/2})$. After performing the integration over d^4x and over the momenta of the intermediate states in Eqs. (3.8) and (3.9) we obtain

$$(2p_{10}2p_{20})^{1/2}T_\mu^{(j,+)} = \sum_n \frac{g_W(P_1\to V_n)}{2p_{n0}(p_{n0}-p_{10})}\{A_\mu^{\bar{j}}\}_{P_kV_n}(k_n e_n^*)\,|_{\vec{p}_n=\vec{p}_1} + \cdots\ , \quad \cdot\ , \tag{3.11}$$

$$(2p_{10}2p_{20})^{1/2}T_\mu^{(j,-)}=\sum_l \frac{g_W(V_l\to P_k)}{2p_{l0}(p_{l0}-p_{20})}(k_l e_l)\{A_\mu^{\bar{j}}\}_{V_l P_1}\,|_{\vec{p}_l=\vec{p}_2}+\cdots\ . \tag{3.12}$$

Here $k_n=p_1-p_n$ and $k_l=p_l-p_2$, e_n and e_l are the polarization four-vectors of V_n, and V_l. $g_W(P_1\to V_n)$ is the weak coupling constant defined by

$$(2p_{n0}2p_{10})^{1/2}\langle V_n(p_n)\,|\,H_W\,|\,P_1(p_1)\rangle=g_W(P_1\to V_n)(k_n\cdot e_n^*)\ . \tag{3.13}$$

$\{A_\mu^j\}_{P_kV_n}$ is the invariant matrix element of $A_\mu^j(0)$ ($j=2$ or 3) and is defined in terms of the form factors,

$$\begin{aligned}\{A_\mu^j\}_{P_kV_n}&\equiv(2p_{20}2p_{n0})^{1/2}\langle P_k(p_2)\,|\,A_\mu^j(0)\,|\,V_n(p_n)\rangle\\ &=F^{(1)}(q_n{}^2)(e_n)_\mu+F^{(2)}(q_n{}^2)(p_n+p_2)_\mu(q_ne_n)+F^{(3)}(q_n{}^2)q_{n\mu}(q_ne_n)\ .\end{aligned} \tag{3.14}$$

Here q_n is $q_n=p_n-p_2$. In Eqs. (3.11) and (3.12), $\sum_n$ and $\sum_l$ involve also the spin summation and the dots denote the higher-level contributions.

If we insert the PCAC relation Eq. (3.3) between the states $\langle P_k(p_2)\,|$ and $|\,V_n(p_n)\rangle$ and use Eq. (3.14), we then obtain, at $q_n{}^2=0$, the Goldberger-Treiman (GT) relation[13]

$$F^{(1)}(0)+(m_k{}^2-m_n{}^2)F^{(2)}(0)=-f_jG(V_n\to P_k\bar{P}_j)\ , \tag{3.15}$$

where $G(V_n\to P_k\bar{P}_j)$ is the off-shell ($m_j{}^2=0$) coupling constant for the interaction $V_n\to P_k+\bar{P}_j$ defined by

$$(2p_{20}2p_{n0})^{1/2}\langle P_k(p_2)\,|\,J_j(0)\,|\,V_n(p_n)\rangle\equiv G(V_n\to P_k\bar{P}_j)K(V_n\to P_k\bar{P}_j;q_n{}^2)(q_ne_n)\ . \tag{3.16}$$

$J_j(x)$ is the source function of $\phi_j(x)$ and we will assume that $K(0)\simeq1$. In order to compute the surface term in Eq. (3.6), we now evaluate, for example, $q_\mu\{A_\mu^{(j)}\}_{P_kV_n}$ in the limit $\vec{q}\to0$ and $\vec{p}_1\to\infty$. We note $2p_{n0}(p_{n0}-p_{10})\to(m_n{}^2-m_1{}^2)+O(1/|\vec{p}_1|^2)$, $q\cdot(p_n-p_2)\to O(1/|\vec{p}_1|^2)$, $q\cdot(p_n+p_2)\to(m_k{}^2-m_1{}^2)+O(1/|\vec{p}_1|^2)$, and $(q\cdot e_n)\to(m_k{}^2-m_1{}^2)(m_k{}^2-m_2{}^2)^{-1}(q_n\cdot e_n)+O(1/|\vec{p}_1|^2)$, etc. Then using the GT relation, Eq. (3.15), we obtain

$$q_\mu\{A_\mu^j\}_{P_kV_n}\xrightarrow[\vec{q}\to0,\,\vec{p}_1\to\infty]{}(-)\left[\frac{m_k{}^2-m_1{}^2}{m_k{}^2-m_n{}^2}\right]f_jG(V_n\to P_k\bar{P}_j)(q_n\cdot e_n)\,|_{\vec{q}\to0,\,\vec{p}_1\to\infty}\ .$$

Substituting the above result into Eq. (3.6) and carrying out the summation over the spin states of V's, we finally obtain for the surface term which is explicitly symmetric in indices 2 and 3 and corresponds to Fig. 1,

$$\begin{aligned}M^S=&-i\sum_n\left[\left[\frac{m_2{}^2-m_1{}^2}{8m_n{}^2}\right]g_W(P_1\to V_n)G(V_n\to P_2P_3)+\left[\frac{m_3{}^2-m_1{}^2}{8m_n{}^2}\right]g_W(P_1\to V_n)G(V_n\to P_3P_2)\right]\\ &+i\sum_l\left[\left[\frac{m_2{}^2-m_1{}^2}{8m_l{}^2}\right]G(P_1\to V_lP_3)g_W(V_l\to P_2)+\left[\frac{m_3{}^2-m_1{}^2}{8m_l{}^2}\right]G(P_1\to V_lP_2)g_W(V_l\to P_3)\right]+\cdots\ .\end{aligned} \tag{3.17}$$

We denote the invariant matrix element of H_W and axial-vector charge A_j in the IMF by $\langle V\,|\,H_W\,|\,P\rangle$ and $\langle P\,|\,A_j\,|\,V\rangle$, etc., i.e.,

$$\langle V\,|\,H_W\,|\,P\rangle\equiv(2k_{10}2k_{20})^{1/2}\langle V(k_2)\,|\,H_W\,|\,P(k_1)\rangle_{\vec{k}_1=\vec{k}_2\to\infty}\ , \tag{3.18}$$

$$\langle P\,|\,A_j\,|\,V\rangle(2\pi)^3\delta^3(\vec{k}_1-\vec{k}_2)\equiv(2k_{10}2k_{20})^{1/2}\langle P(k_2)\,|\,A_j\,|\,V(k_1)\rangle_{\vec{k}_1=\vec{k}_2\to\infty}\ . \tag{3.19}$$

Then Eq. (3.17) can also be cast (see Sec. IV for derivation) into an *alternative* instructive form involving the axial-vector charges,

$$\begin{aligned}M^S=&\,i\sum_n\left[\frac{1}{f_3}\left[\frac{m_2{}^2-m_1{}^2}{m_n{}^2-m_1{}^2}\right]\langle P_2\,|\,A_3\,|\,V_n\rangle\langle V_n\,|\,H_W\,|\,P_1\rangle+\frac{1}{f_2}\left[\frac{m_3{}^2-m_1{}^2}{m_n{}^2-m_1{}^2}\right]\langle P_3\,|\,A_2\,|\,V_n\rangle\langle V_n\,|\,H_W\,|\,P_1\rangle\right]\\ &+i\sum_l\left[\frac{1}{f_3}\left[\frac{m_2{}^2-m_1{}^2}{m_l{}^2-m_2{}^2}\right]\langle P_2\,|\,H_W\,|\,V_l\rangle\langle V_l\,|\,A_3\,|\,P_1\rangle+\frac{1}{f_2}\left[\frac{m_3{}^2-m_1{}^2}{m_l{}^2-m_3{}^2}\right]\langle P_3\,|\,H_W\,|\,V_l\rangle\langle V_l\,|\,A_2\,|\,P_1\rangle\right]+\cdots\ .\end{aligned} \tag{3.20}$$

We stress that all the intermediate states are on the mass shell. The terms which involve higher-level contributions n_L ($L\ge1$) denoted by the dots can also be cast into a similar form, i.e., for example, $\sum\langle P_2\,|\,A_3\,|\,n_L\rangle\langle n_L\,|\,H_W\,|\,P_1\rangle$, explicitly demonstrating the role played by the axial-vector charge in *selecting out* the intermediate-state contributions. M^{ETC} can be evaluated by using the remarkable relation $[A_j(0),H_W(0)]=[V_j(0),H_W(0)]$ in our IMF, utilizing asymptotic flavor symmetry (instead of exact symmetry),

$$M^{\rm ETC}=-(2f_3)^{-1}i(2p_{10}2p_{20})^{1/2}\langle P_2(p_2)\,|\,[V_3,H_W(0)]\,|\,P_1(p_1)\rangle_{\vec{p}_1=\vec{p}_2\to\infty}$$
$$-(2f_2)^{-1}i(2p_{10}2p_{20})^{1/2}\langle P_3(p_2)\,|\,[V_2,H_W(0)]\,|\,P_1(p_1)\rangle_{\vec{p}_1=\vec{p}_2\to\infty}\ . \tag{3.21}$$

In the IMF, V_i ($i=2$ and 3) acting on the states $|P_j\rangle$ ($j=1,2,3$) produces *only* the ground-state-nonet pseudoscalar-meson states. From Eqs. (3.17) or (3.20) and (3.21), we see that the decay amplitudes are now expressed in terms of the *two-body asymptotic* matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$ which allows us to predict the rates of the charmed meson decays from the strange meson decay rates.

IV. PREDICTION ON THE RATIO $\Gamma(D^0\to K^-\pi^+)/\Gamma(K_s^0\to\pi^+\pi^-)$

We first compute the ratio of the rates of the typical *unsuppressed* strange- and charm-meson decays to obtain a quantitative idea about the method used. In Sec. V we discuss the approximate selection rules (the $|\Delta\vec{\rm I}|=\frac{1}{2}$ rule and its charmed counterparts). We choose the $K_S^0\to\pi^+\pi^-$ as the model strange-meson decay. For the charm-meson decays, the branching ratios are now fairly well known,[14] although their lifetimes are not yet firmly established. For $\tau(D^0)$, we adopt[15] tentatively the value $\tau(D^0)=(4.8^{+2.4}_{-1.5})\times10^{-13}$ sec which is presently cited. We choose the $D^0\to K^-\pi^+$ as the typical D-meson decay, since no selection rule is found for this decay in the present theory. The branching ratio of this decay is reported to be about 3%. Using these numbers, we tentatively estimate

$$R_{\rm expt}=\frac{\Gamma(D^0\to K^-\pi^+)}{\Gamma(K_S^0\to\pi^+\pi^-)}\simeq3.6-11\ . \tag{4.1}$$

From Eq. (3.5), we write for the invariant amplitudes

$$M^{\rm phys}(K_S^0\to\pi^+\pi^-)\sim M^{\rm ETC}(K_S^0)+M^{\rm S}(K_S^0)\ , \tag{4.2}$$

$$M^{\rm phys}(D^0\to K^-\pi^+)\sim M^{\rm ETC}(D^0)+M^{\rm ETC}(D^0)\ , \tag{4.3}$$

where $\sim$ implies that the soft-meson approximation in the IMF developed in Sec. III has been used. From Eq. (3.21), $M^{\rm ETC}$ are given by

$$M^{\rm ETC}(K_S^0)=-(f_\pi)^{-1}[(2p_{10}2p_{20})^{1/2}\langle\pi^0(p_2)\,|\,H_W\,|\,K^0(p_1)\rangle]_{\vec{p}_1=\vec{p}_2\to\infty}\ , \tag{4.4}$$

$$M^{\rm ETC}(D^0)=(2f_\pi)^{-1}[(2p_{10}2p_{20})^{1/2}\langle\bar K^0(p_2)\,|\,H_W\,|\,D^0(p_1)\rangle]_{\vec{p}_1=\vec{p}_2\to\infty}\ . \tag{4.5}$$

In deriving Eqs. (4.4) and (4.5), we have already used the asymptotic $|\Delta\vec{\rm I}|=\frac{1}{2}$ rule, Eq. (2.1), and its charmed counterpart, Eq. (2.6), obtained in I for the asymptotic two-body matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$. For $M^{\rm S}$ in Eqs. (4.2) and (4.3), we keep *only* the diagonal terms. From an intuitive (overlapping of wave functions, etc.) as well as the more sophisticated argument which will be given in Sec. V, the nondiagonal term will certainly not produce a leading contribution. Corresponding to Fig. 1, the contribution of the intermediate ground-state mesons (i.e., the vector mesons) to M_S can be written schematically as follows ($\vec{p}_1=\vec{p}_2\to\infty$);

$$M^{\rm S}(K_S^0)=(4\sqrt{2})^{-1}(2p_{10}2p_{20})^{1/2}[M(K^0(p_1)\to K^{*+}\pi^-\to\pi^+(p_2)\pi^-(q);q^2=0)$$
$$+M(\bar K^0(p_1)\to K^{*-}\pi^+\to\pi^-(p_2)\pi^+(q);q^2=0)]\ , \tag{4.6}$$

$$M^{\rm S}(D^0)=(4)^{-1}(2p_{10}2p_{20})^{1/2}[M(D^0(p_1)\to K^{*0}\to K^-(p_2)\pi^+(q);q^2=0)+M(D^0(p_1)\to\bar K^{*0}\to\pi^+(p_2)K^-(q);q^2=0)$$
$$+M(D^0(p_1)\to F^{*+}K^-\to\pi^+(p_2)K^-(q);q^2=0)]\ . \tag{4.7}$$

In terms of the strong VPP couplings G and the weak two-body VP couplings g_W, the general forms of the amplitudes M which appear in Eqs. (4.6) and (4.7) are given by, as in Eq. (3.17),

$$(2p_{10}2p_{20})^{1/2}M(P_1\to V\to P_2(p_2)P_3(q);q^2=0)_{\vec{p}_1=\vec{p}_2\to\infty}=i(2m_V{}^2)^{-1}(m_1{}^2-m_2{}^2)g_W(P_1\to V)G(V\to P_2P_3)\ , \tag{4.8}$$

$$(2p_{10}2p_{20})^{1/2}M(P_1\to V\to P_3(p_2)P_2(q);q^2=0)_{\vec{p}_1=\vec{p}_2\to\infty}=i(2m_V{}^2)^{-1}(m_1{}^2-m_3{}^2)g_W(P_1\to V)G(V\to P_3P_2)\ , \tag{4.9}$$

$$(2p_{10}2p_{20})^{1/2}M(P_1\to VP_3\to P_2(p_2)P_3(q);q^2=0)_{\vec{p}_1=\vec{p}_2\to\infty}=-i(2m_V{}^2)^{-1}(m_1{}^2-m_2{}^2)G(P_1\to VP_3)g_W(V\to P_2)\ , \tag{4.10}$$

$$(2p_{10}2p_{20})^{1/2}M(P_1\to VP_2\to P_3(p_2)P_2(q);q^2=0)_{\vec{p}_1=\vec{p}_2\to\infty}=-i(2m_V{}^2)^{-1}(m_1{}^2-m_3{}^2)G(P_1\to VP_2)g_W(V\to P_3)\ . \tag{4.11}$$

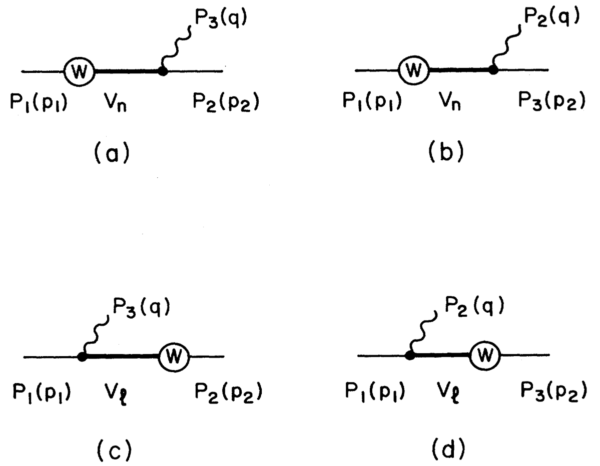

FIG. 1. General schematic diagrams of the contributions of the on-mass-shell intermediate states V_n and V_l to the surface term M^S. W denotes the weak vertex and the wiggly line represents the pseudoscalar meson with $q^2=0$ instead of $q^2=-m^2$ in the reference frame with $\vec{p}_1 \to \infty$.

The ratio of the physical amplitudes, $r \equiv M^{\text{phys}}(D^0 \to K^-\pi^+)/M^{\text{phys}}(K_S^0 \to \pi^+\pi^-)$, is then given by

$$r \simeq r^{\text{ETC}} \left[\frac{1+s(D^0)}{1+s(K_S^0)} \right] . \tag{4.12}$$

r^{ETC} is the ratio of the ETC term of the two decay amplitudes and $s(D^0)$ and $s(K_S^0)$ denote the ratios of M^S/M^{ETC} for each amplitude, i.e.,

$$r^{\text{ETC}} \equiv M^{\text{ETC}}(D^0)/M^{\text{ETC}}(K_S^0) , \tag{4.13}$$

$$s(K_S^0) \equiv M^S(K_S^0)/M^{\text{ETC}}(K_S^0)$$

and (4.14)

$$s(D^0) \equiv M^S(D^0)/M^{\text{ETC}}(D^0) .$$

The off-shell (i.e., $q^2=0$) strong coupling constants [see Eq. (3.16)], for example, $G(P_1 \to V\bar{P}_j)$, is related to the asymptotic axial-vector matrix element $\langle V \,|\, A_j \,|\, P_1 \rangle$ defined in Eq. (3.19), through the important general asymptotic formula based on PCAC,

$$\langle B'(p_2) \,|\, A_j \,|\, B(p_1) \rangle_{\vec{p}_1 \to \infty} = -i(2\pi)^3 \delta^3(\vec{p}_1 - \vec{p}_2) \left[\frac{f_j}{m_B{}^2 - m_{B'}{}^2} \right] (2p_{10} 2p_{20})^{1/2} \, \langle B'(p_2) \,|\, J_j(0) \,|\, B(p_1) \rangle . \tag{4.15}$$

We have, for example,

$$G(P_i \to V_k \bar{P}_j) = (f_j)^{-1} (2m_{V_k}) \langle V_k \,|\, A_j \,|\, P_i \rangle , \tag{4.16}$$

$$G(V_k \to P_i \bar{P}_j) = (f_j)^{-1} (2m_{V_k}) \langle P_i \,|\, A_j \,|\, V_k \rangle , \text{ etc .} \tag{4.17}$$

Equations (4.16) and (4.17) illustrate the importance of the use of the concept of *asymptotic* flavor SU(N) symmetry. In this theoretical framework, the asymptotic axial-vector matrix elements, such as $\langle V_k \,|\, A_j \,|\, P_i \rangle$, can be parametrized (when we vary the flavor indices i, j, and k), in *broken* flavor symmetry, by the conventional "exact SU(N) symmetry plus mixing" prescription. However, this simple prescription cannot be used for the strong coupling constants G, since Eqs. (4.16) and (4.17) involve the physical masses m_{V_k} and f_j. The weak coupling constant $g_W(q^2)$ defined by Eq. (3.13) is related, at $q^2=0$, to the asymptotic invariant weak matrix element $\langle P \,|\, H_W \,|\, V \rangle$ defined by Eq. (3.18) as follows:

$$g_W(V \to P) = -\left[\frac{2m_V}{m_V{}^2 - m_P{}^2} \right] \langle P \,|\, H_W \,|\, V \rangle . \tag{4.18}$$

This also demonstrates that, although we have in the present theory simple *asymptotic* flavor-SU(N)-symmetry relations among the *asymptotic* weak matrix elements $\langle P \,|\, H_W \,|\, V \rangle$ as obtained in I, the corresponding relations for the actual weak coupling constants g_W at $q^2=0$ limit are more complicated because of the mass factor $m_V(m_V{}^2 - m_P{}^2)^{-1}$ in Eq. (4.18). One thus sees that a naive treatment of flavor symmetry [especially in SU(4), SU(5), $\cdots$) symmetry] could lead to a misleading result. The present method copes with the problem of broken symmetry in a more serious way, apart from the fact that it does not rely on the dominance of particular diagrams at the quark-gluon level. In Sec. III we have cast Eq. (3.17) into the more illuminating form, Eq. (3.20), by using Eqs. (4.16)–(4.18) derived above.

By using the relation between $\langle \pi^0 \,|\, H^{(0,-)} \,|\, K^0 \rangle$ and $\langle \bar{K}^0 \,|\, H^{(-,-)} \,|\, D^0 \rangle$ given in Eq. (2.6), we obtain $r^{\text{ETC}} \sim -3.14$.

In $s(K_S^0)$ and $s(D^0)$ the weak couplings will disappear using the relations given in Eq. (2.8) and the s's depend only on the values of strong coupling constants G or the asymptotic axial-vector matrix elements. Since all G's are related by asymptotic flavor symmetry, it is sufficient to determine, for example, $G(\rho^0 \to \pi^+\pi^-) = (f_\pi)^{-1} 2m_\rho \times \langle \pi^+ \,|\, A_{\pi^+} \,|\, \rho^0 \rangle$. From the rate $\Gamma(\rho^0 \to \pi^+\pi^-) = 158 \pm 5$ MeV, we obtain $|\, G(\rho^0 \to \pi^+\pi^-) \,| \sim 12.4$. By using this value, we find $s(K_S^0) \sim 0.21$. In deriving this value of s we have chosen the relevant signs, i.e., the positive sign in Eq. (2.4) and $f_\pi G(\rho^0 \to \pi^+\pi^-) > 0$. The value of $s(K_S^0)$ implies that for the $K_S^0 \to 2\pi$ decay, the ETC term is more *important* than the surface term. On the other hand, the same calculation [using asymptotic SU(4) symmetry] yields, $s(D^0) \sim 0.49$, i.e., the surface term plays a more important role for the $D^0 \to \bar{K}\pi$ decays.[16] Using the values of r^{ETC}, $s(K_S^0)$ and $s(D^0)$, we finally obtain

$$r \equiv \frac{M^{\text{phys}}(D^0 \to K^-\pi^+)}{M^{\text{phys}}(K_S^0 \to \pi^+\pi^-)} \simeq -3.9 . \tag{4.19}$$

From the ratio of the rates $R \equiv \Gamma(D^0 \to K^-\pi^+)/\Gamma(K_S^0 \to \pi^+\pi^-) = (m_K/m_D)^2 (\,|\, \vec{q}_D \,|\, / \,|\, \vec{q}_K \,|\,) \,|\, r \,|^2$, where $\vec{q}_D$ and $\vec{q}_K$ are the c.m. momenta in the D and K decays, we thus obtain $R_{th} \sim 4.4$ which may be compared with the present preliminary experimental value given by Eq. (4.1). The agreement seems reasonable. As will be briefly discussed in the next section, the most important correction to the prediction Eq. (4.19) will come, among others,[17] from the nondiagonal term in $M^S(D^0)$ involving the 0^{++} and 2^{++} mesons.

V. APPROXIMATE $|\Delta\vec{I}|=\frac{1}{2}$ RULE AND ITS CHARM COUNTERPART

For the decay modes $K^+\to\pi^+\pi^0$ and $D^+\to\bar{K}^0\pi^+$, for which *approximate* selection rules are at work in the present theory, we write analogs to Eqs. (4.2) and (4.3)

$$M^{\text{phys}}(K^+\to\pi^+\pi^0)\sim M^{\text{ETC}}(K^+)+M^S(K^+)\,, \tag{5.1}$$

$$M^{\text{phys}}(D^+\to\bar{K}^0\pi^+)\sim M^{\text{ETC}}(D^+)+M^S(D^+)\,. \tag{5.2}$$

Since $\langle\pi|H_W|K\rangle$'s satisfy exact $|\Delta\vec{I}|=\frac{1}{2}$ rule, $M^{\text{ETC}}(K^+)=0$ as shown previously.[7] Using the same method as applied to $M^S(K_S^0)$ in Eq. (4.6) we find that the "diagonal" term of $M^S(K^+)$ is proportional to

$$-\frac{2}{f_\pi}\left[\frac{m_K{}^2-m_\pi{}^2}{m_{K^*}^2-m_\pi{}^2}\right](\sqrt{2}\langle\pi^+|H^W|K^{*+}\rangle\langle K^{*+}|A_{\pi^0}|K^+\rangle+\langle\pi^0|H_W|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^+\rangle)\,. \tag{5.3}$$

Equation (5.3) vanishes, since $\langle\pi|H_W|K^*\rangle$'s satisfy the strict $|\Delta\vec{I}|=\frac{1}{2}$ rule and for the asymptotic axial-vector matrix elements we find $2\langle K^{*+}|A_{\pi^0}|K^+\rangle=\langle K^{*0}|A_{\pi^-}|K^+\rangle$. Therefore, to the approximation that we keep *only* the "diagonal" term, M_D^S, in M^S the $K^+\to\pi^+\pi^-$ decay is *strictly* forbidden [of course, in exact SU(2) symmetry] and $\Gamma(K_S^0\to\pi^0\pi^0)/\Gamma(K_S^0\to\pi^+\pi^-)=\frac{1}{2}$ for the K_S^0. The observed small violation of the rule must come from the "*nondiagonal*" term, $M_{\text{ND}}^S(K)$. As shown before,[7] for the amplitude of $D^+\to\bar{K}^0\pi^+$ decay $M^{\text{ETC}}(D^+)$ is proportional to $(H=H^{(-,-)})$,

$$f_K{}^{-1}\langle\pi^+|H|F^+\rangle+f_\pi{}^{-1}\langle\bar{K}^0|H|D^0\rangle\,, \tag{5.4}$$

which vanishes in the SU(3) symmetry limit $f_K=f_\pi$, using the asymptotic selection rule obtained before, i.e., Eq. (2.6). The "diagonal" term of $M^S(D^+)$ is proportional to [again using Eq. (2.8)],

$$-\frac{2}{f_\pi}\langle\bar{K}^0|H_W|D^{*0}\rangle\langle D^{*0}|A_{\pi^-}|D^+\rangle \times\left[\frac{(m_D{}^2-m_K{}^2)}{(m_{D^*}{}^2-m_K{}^2)}-\left[\frac{f_\pi}{f_K}\right]\frac{(m_D{}^2-m_\pi{}^2)}{(m_{F^*}{}^2-m_\pi{}^2)}\right]. \tag{5.5}$$

This expression vanishes in the SU(3) symmetry limit, i.e., $m_{D^*}=m_{F^*}$, $m_K=m_\pi$, and $f_K=f_\pi$, demonstrating explicitly the working of the asymptotic $6\oplus6^*$ rule of exact SU(3) symmetry in *broken* SU(4) symmetry. Since we have never used the concept of exact flavor symmetry, Eqs. (5.4) and (5.5) are valid in broken SU(3) and SU(4) symmetry. For D^0 we obtain from ETC term alone,[7] $\Gamma(D^0\to K^-\pi^+)/\Gamma(D^0\to\bar{K}^0\pi^0)=|\sqrt{2}(f_K/f_\pi)^2/[2(f_K/f_\pi)-1]|^2\simeq2$ compared with current experimental value[15] 1.6 ± 0.9. However, we see that the above selection rule [which is strict in exact SU(3) symmetry] is obeyed *fairly strictly* even in broken SU(4) symmetry, as long as we keep *only* the "diagonal" term in M^S. The ratio $R\equiv\Gamma(D^+\to\bar{K}^0\pi^+)/\Gamma(D^0\to K^-\pi^+)$ computed from Eqs. (4.5), (4.7), (5.4), and (5.5), using the physical values of f_K, f_π and the masses $(m_{D^*}, m_{F^*}, \cdots)$ involved, is of the order of $\sim\frac{1}{10}$, whereas the present preliminary experiments seem to suggest the value of R in the range of $\frac{1}{3}-\frac{1}{10}$.

We now discuss a *semiquantitative* estimate of the "nondiagonal" contribution to M^S. This term should account for the *small* violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule in the $K\to2\pi$ decays and will also explain the possibly *larger* violation of the selection rule observed in the $D^+\to\bar{K}^0\pi^+$ decay. As discussed in Sec. VI of I, in the present theory, all the *diagonal two-body* asymptotic weak matrix elements $\langle L|H_W|L\rangle$ are always constrained to satisfy the *strict* $|\Delta\vec{I}|=\frac{1}{2}$ rule, octet rule, and its charm counterpart. However, this is not necessarily the case for the nondiagonal ones, $\langle L|H_W|L'\rangle$ with $L\neq L'$. The M^S can always be written in the instructive form [see Eq. (3.20)] such as, for example,

$$\Sigma_{n_L}\langle P_2|A_3|n_L\rangle\langle n_L|H_W|P_1\rangle\quad(L=0,1,\ldots)\,. \tag{5.6}$$

Because of the nonet structure of $q\bar{q}$ mesons, for the $K^+\to\pi^+\pi^0$ and $D^+\to\bar{K}^0\pi^+$ decays only the terms described by Fig. 1(c) and 1(d) (i.e., the V_l term or the u-channel intermediate states) contribute to M^S, whereas for the $K^0\to\pi\pi$ and $D^0\to\bar{K}\pi$ decays both the V_n (or s-channel intermediate states) term described by Fig. 1(a) and 1(b) and the V_l term appear in M^S.

For the relative importance of the contributions from each level ($L=0,1,2,\ldots$) in Eq. (5.6), we may develop the following scenario. The leading contribution, of course, comes from the ground-state ($L=0$) mesons in the diagonal term but the $L=1$ states contributes appreciably in the nondiagonal term. The contributions of the $L\geq2$ states will not be very important as illustrated below. The contribution of the $L=1$ 0^{++} state to $M_{\text{ND}}^S(K^+)$ is, analogous to Eq. (5.3), proportional to

$$-\frac{2}{f_\pi}\left[\frac{m_K{}^2-m_\pi{}^2}{m_\kappa{}^2-m_\pi{}^2}\right](\sqrt{2}\langle\pi^+|H_W|\kappa^+\rangle\langle\kappa^+|A_{\pi^0}|K^+\rangle+\langle\pi^0|H_W|\kappa^0\rangle\langle\kappa^0|A_{\pi^-}|K^+\rangle)\,, \tag{5.7}$$

where κ is the $I=\frac{1}{2}$ 0^+ meson. The $L=1$ 2^{++} state also gives the contribution through $K^{**}(1420)$ which is proportional to

$$\sqrt{2}\langle\pi^+|H_W|K^{**+}\rangle\langle K^{**+}|A_{\pi^0}|K^+\rangle+\langle\pi^0|H_W|K^{**0}\rangle\langle K^{**0}|A_{\pi^-}|K^+\rangle\,.$$

If the weak κ and K^{**} vertices satisfy exact $|\Delta\vec{I}|=\frac{1}{2}$

rule, the above contributions of κ and K^{**} to $M^{\rm S}_{\rm ND}(K^+)$, of course, vanish. From the result discussed in Sec. VI of I, we also have simple asymptotic constraints relating the relative strength of the weak asymptotic matrix elements to that of asymptotic axial-vector matrix elements, i.e.,

$$\frac{\langle L=1\,|\,H_W\,|\,L=0\rangle}{\langle L=0\,|\,H_W\,|\,L=0\rangle}\simeq\frac{\langle L=1\,|\,A_\pi\,|\,L=0\rangle}{\langle L=0\,|\,A_\pi\,|\,L=0\rangle}\;. \tag{5.8}$$

Therefore, we have an interesting relation useful for the estimate of the relative importance of the level contributions to $M^{\rm S}$, i.e.,

$$\frac{\langle L=0\,|\,A\;\,|\,L=1\rangle\langle L=1\,|\,H_W\,|\,L=0\rangle}{\langle L=0\,|\,A\;\,|\,L=0\rangle\langle L=0\,|\,H_W\,|\,L=0\rangle}$$

$$\simeq\left|\frac{\langle L=1\,|\,A\;\,|\,L=0\rangle}{\langle L=0\,|\,A\;\,|\,L=0\rangle}\right|^2\;. \tag{5.9}$$

Equation (5.9) will also hold for $L=2,\ldots$. Then we see that the diagonal term of $M^{\rm S}_D(K^0_S)$ and the κ, $K^{**},\ldots$ contributions to the $M^{\rm S}_{\rm ND}(K^+)$ (we know that the κ, $K^{**},\ldots$ weak vertices *violate* the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule) are proportional to $|\langle K^{*0}|A_{\pi^-}|K^+\rangle|^2$, $|\langle\kappa^0|A_{\pi^-}|K^+\rangle|^2$, $|\langle K^{**0}|A_{\pi^-}|K^+\rangle^2,\ldots,$ respectively.

It is instructive to consider the saturation of the algebra $[A_{\pi^+},A_{\pi^-}]=2V_{\pi^0}$, sandwiched between the states $\langle K^+(\vec{\rm p})|$ and $|K^+(\vec{\rm p})\rangle$ with $\vec{\rm p}\to\infty$. Inserting the intermediate states ($L=0,1,\ldots$) between the charges A_{π^+} and A_{π^-}, we obtain in the limit $\vec{\rm p}\to\infty$,

$$|\langle K^{*0}|A_{\pi^-}|K^+\rangle|^2+|\langle\kappa^0|A_{\pi^-}|K^+\rangle|^2$$
$$+|\langle K^{**0}|A_{\pi^-}|K^+\rangle|^2+\cdots=1\;. \tag{5.10}$$

The dots denote the contribution of $L\geq 2$ states (including the radially excited states such as $K^{*'}$, etc). By using PCAC and the rate of $K^*\to K\pi$ decay we obtain $|\langle K^{*0}|A_{\pi^-}|K^+\rangle|^2\simeq 0.5$, i.e., the asymptotic fractional contribution of the ground state to the algebra is around 50%. From the rate of $K^{**}\to K\pi$ decay we get $|\langle K^{**0}|A_{\pi^-}|K^+\rangle|^2\sim 0.07$ which is significantly smaller. This suggests that $|\langle K^0(L)|A_{\pi^-}|K^+\rangle|^2$ is small for the higher-spin states with $L\geq 2$, so that $L\geq 2$ states will not play an important role for the violation of the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule. However, the fact that $|\langle K^{*0}|A_{\pi^-}|K^+\rangle|^2\simeq 0.5$ in Eq. (5.10) suggests that $|\langle\kappa^0|A_{\pi^-}|K^+\rangle|^2$ is relatively large[18] and it is probably reasonable to estimate $|\langle\kappa^0|A_{\pi^-}|K^+\rangle|^2\lesssim 0.2-0.4$.

We now write, *assuming* that the weak κ vertex *violates*[19] the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule and keeping only the κ term in $M^{\rm S}_{\rm ND}(K^+)$,

$$\begin{aligned}T&\equiv\frac{M(K^+\to\pi^+\pi^0)}{M(K^0_S\to\pi^+\pi^-)}\\&\simeq\frac{M^{\rm S}_{\rm ND}(K^+)}{M^{\rm ETC}(K^0_S)+M^{\rm S}_D(K^0_S)+M^{\rm S}_{\rm ND}(K^0_S)}\\&\simeq\frac{M^{\rm S}_{\rm ND}(K^+)/M^{\rm ETC}(K^0_S)}{1+M^{\rm S}_D(K^0_S)/M^{\rm ETC}(K^0_S)}\\&\simeq\frac{0.2[(M^{\rm S}_{\rm ND}(K^+)/M^{\rm S}_D(K^0_S)]}{1+0.21}\;.\end{aligned} \tag{5.11}$$

For $M^{\rm S}_D(K^0_S)/M^{\rm ETC}(K^0_S)$ we have used the value 0.21 obtained in Sec. IV and we have neglected $M^{\rm S}_{\rm ND}(K^0_S)$ without a serious error, since $M^{\rm S}_{\rm ND}(K^0_S)\lesssim M^{\rm S}_D(K^0_S)$. Crudely we estimate

$$M^{\rm S}_{\rm ND}(K^+)/M^{\rm S}_D(K^0_S)\simeq\left[\frac{0.2-0.4}{0.5}\right]\frac{1}{2}\simeq 0.2-0.4\;. \tag{5.12}$$

The factor $\simeq\frac{1}{2}$ comes from the kinematical mass factor appearing in Eq. (5.7) and the corresponding one for $M^{\rm S}_D(K^0_S)$. It reflects the fact that the κ is heavier than the K^*. We then obtain $T\sim\frac{1}{30}-\frac{1}{15}$. *If* the nondiagonal weak vertex $\langle\pi\,|\,H_W\,|\,\kappa\rangle$ *violates*[19] the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule, it will then produce the nonvanishing $M^{\rm S}_{\rm ND}(K^+)$ term in Eq. (5.11) and the resulting value of T is thus of the right order of magnitude to explain the violation of the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule in the $K\to 2\pi$ decays. It may be noted that in Eq. (5.11), the ratio of the surface versus the ETC term ~ 0.21 plays an important role for the *suppression* of the $|\Delta\vec{\rm I}\,|=\frac{3}{2}$ contribution. Exactly similar argument can be repeated for the $D^+\to\bar{K}^0\pi^+$ amplitude. A main difference is that for the D^0-decay, the ratio of the surface term to the ETC term is 0.49 compared with the value of the same ratio in the K^0_S decay 0.21. Therefore, the surface term, which involves the selection-rule-violating term, plays a more important role for the D-meson decays. Therefore, the violation of the selection rule in the D^+ decay can be considerably larger than the case of the $|\Delta\vec{\rm I}\,|=\frac{1}{2}$ rule. Very crudely, $T'\equiv M(D^+\to\bar{K}^0\pi^+)/M(D^0\to K^-\pi^+)$ is estimated to be larger than T by a factor ~ 7. We hope that we can, in the future, present a more rigorous version of the scenario described above by solving the algebraic constraints involving the $L=1$ states. However, we feel that the argument already involves a very new element toward the understanding of the problem of approximate dynamical selection rules of the $K\to 2\pi$ and $D\to\bar{K}\pi$ decays.

The inclusion of the neglected nondiagonal term in the surface term of the K^0_S and D^0 decays, especially the scalar-meson contribution to the D^0 decay, will also modify the value of $R\equiv\Gamma(D^0\to K^-\pi^+)/\Gamma(K^0_S\to\pi^+\pi^-)\simeq 4.4$ obtained in Sec. IV probably upward, to be more consistent with Eq. (4.1). With the future determination of more precise values of the lifetimes of the D mesons, the study of the level realization involving the $L=1$ level becomes interesting.

ACKNOWLEDGMENTS

The authors thank Professor R. Brandt for useful discussions on the soft-pion extrapolation. They are also grateful to Torray Science Foundation and Suga Gijitsu Sinko Foundation for the support of this collaboration. One of us (K.T.) wishes to thank the members of the Elementary Particle physics group at the University of Maryland for the hospitality extended to him during his stay at Maryland.

APPENDIX A: THE MERIT OF THE NEW SOFT-MESON TECHNIQUE

We consider the decay process $\alpha(p_1)\to\beta(p_2)+\pi_k(q)$, where π denotes the pion or other pseudoscalar mesons for which we use a soft-meson approximation. Conventional soft-meson approximation $q_\mu\to 0$ involves a rather drastic extrapolation, since, by four-momentum conservation, we have to approximate the amplitude at the *unphysical* point, $m_\alpha=m_\beta$ or $p_1=p_2$. By using the LSZ reduction formula, we write the invariant amplitude[20]

$$R=i\int d^4x\, e^{-iqx}(m_\pi^2-\Box)\theta(x_0)\times\langle\beta(p_2)\,|\,[\pi_k(x),H_W(0)]\,|\,\alpha(p_1)\rangle\ . \tag{A1}$$

We may regard R as describing a fictitious scattering amplitude, $F(s,t,k^2)$, for the process, $S(k)+\alpha(p_1)\to\beta(p_2)+\pi_k(q)$, where S denotes the fictitious spurion. s, t, and u are the usual scattering variables, $s=-(p_1+k)^2$, $t=-(p_1-p_2)^2$, and $u=-(p_1-q)^2$ with $p_1+k=p_2+q$. Since $s+t+u=m_\alpha{}^2+m_\beta{}^2-k^2-q^2$, we use k^2 in place of u. For physical amplitude ($k=0$), we have $s=m_\alpha{}^2$, $t=m_\pi{}^2$, and $k^2=0$ i.e., $R_{\rm phys}=F(m_\alpha{}^2,m_\pi{}^2,0)\simeq F(m_\alpha{}^2,0,0)$. In the last of the above equations we have assumed that $t=m_\pi{}^2\to 0$ is a good extrapolation.

Let us now assume that $F(s)$ satisfies a once subtracted dispersion relation,[21]

$$F(s,t,k^2)=F(s_0,t,k^2)+\frac{s-s_0}{\pi}\int_{-\infty}^{\infty}\frac{{\rm Im}F(s',t,k^2)}{(s'-s_0)(s'-s-i\epsilon)}ds'\ . \tag{A2}$$

In the usual soft-meson approximation $q_\mu\to 0$, $s=-(p_2+q)^2\to m_\beta{}^2$. So we can write Eq. (A2) for the case $k^2=0$ and $t=m_\pi{}^2\simeq 0$,

$$F(s,0,0)=F(m_\beta{}^2,0,0)+\frac{s-m_\beta{}^2}{\pi}\int_{-\infty}^{\infty}\frac{{\rm Im}F(s',0,0)}{(s'-m_\beta{}^2)(s'-s-i\epsilon)}ds'\ . \tag{A3}$$

Here $F(m_\beta{}^2,0,0)$ is the amplitude computed by the *usual* soft-meson approximation i.e., $F(m_\beta{}^2,0,0)=\lim_{q_\mu\to 0}R$, which is given by [compare with Eq. (3.4)],

$$F(m_\beta{}^2,0,0)=-\frac{\sqrt{2}i}{f_\pi}\langle\beta\,|\,[A_k(0),H_W(0)]\,|\,\alpha\rangle+\frac{\sqrt{2}}{f_\pi}\lim_{q_\mu\to 0}q_\mu T_\mu^k\ . \tag{A4}$$

Therefore, we obtain

$$R_{\rm phys}\sim F(m_\alpha{}^2,0,0)=F(m_\beta{}^2,0,0)+\frac{(m_\alpha{}^2-m_\beta{}^2)}{\pi}\times\int_{-\infty}^{\infty}\frac{{\rm Im}F(s',0,0)}{(s'-m_\beta{}^2)(s'-m_\alpha{}^2-i\epsilon)}ds'\ . \tag{A5}$$

The dispersion integral in (A5) corresponds explicitly to the term *neglected* in the usual soft-meson approximation and it is not an easy task to make a reliable estimate of this term.

In the *new* soft-meson approximation, i.e, $\vec{q}\to 0$ in the frame $\vec{p}_1=\vec{p}_2\to\infty$, $s\to m_\alpha{}^2$ (not $m_\beta{}^2$). Therefore, we have instead of (A3),

$$F(s,0,0)=F(m_\alpha{}^2,0,0)+\frac{s-m_\alpha{}^2}{\pi}\times\int_{-\infty}^{\infty}\frac{{\rm Im}F(s',0,0)}{(s'-m_\alpha{}^2)(s'-s-i\epsilon)}ds'\ . \tag{A6}$$

Therefore, for the physical amplitude of $\alpha\to\beta+\pi$, we have $R_{\rm phys}\sim F(m_\alpha{}^20,0)$, where $F(m_\alpha{}^2,0,0)$ can be evaluated by the *new* soft-meson approximation

$$F(m_\alpha{}^2,0,0)=\lim_{\vec{q}\to 0,\,\vec{p}_1=\vec{p}_2\to\infty}R\ ,$$

$$F(m_\alpha{}^2,0,0)=-\frac{i}{f_\pi}\langle\beta\,|\,[A_k(0),H_W(0)]\alpha\rangle+\frac{i}{f_\pi}\lim_{\vec{q}\to 0,\,\vec{p}_1=\vec{p}_2\to\infty}q_\mu T_\mu^k\ . \tag{A7}$$

Therefore, in the new soft-meson approximation, the problem reduces to the evaluation of the nonvanishing contribution of the surface term of (A7) in the limit considered.

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*Present address.

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[11]The unique choice of the signs appearing in Eqs. (2.4) and (2.9) has not yet been achieved from the realization of algebras so far considered.

[12]From the asymptotic flavor parametrization of the matrix elements of H_W mentioned above, we obtain

$$\langle P_2(\vec{p}_2)P_3(\vec{q})\,|\,H_W\,|\,P_1(\vec{p}_1)\rangle = \langle P_3(\vec{p}_2)P_2(\vec{q})\,|\,H_W\,|\,P_1(\vec{p}_1)\rangle$$

in the limit $\vec{p}_2 \to \infty$ and $\vec{q} \to \infty\,(\vec{p}_1 = \vec{p}_2 + \vec{q})$. This implies that the on-mass-shell invariant amplitude,

$$(2E_1 2E_2 2E_3)^{1/2}\langle P_2(\vec{p}_2)P_3(\vec{q})\,|\,H_W\,|\,P_1(\vec{p}_1)\rangle\ ,$$

which is a scalar function of $p_2{}^2 = -m_2{}^2$, $q^2 = -m_3{}^2$, and $(p_2 \cdot q)$, is symmetric with respect to the exchange $p_2 \leftrightarrow q$. This implies that the amplitude must be a symmetric function with respect to the physical masses m_2 and m_3 of P_2 and P_3. The presence of this constraint, is a natural one, since one can then smoothly reach the symmetry limit.

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[16]This important difference arises essentially from the mass factors appearing, for example, in Eq. (3.20). Roughly, $s(D^0)$ is proportional to $(m_D/m_{D^*})^2$ and $s(K_S^0)$ to $(m_K/m_{K^*})^2$. For heavy mesons, the surface term thus becomes *important and is comparable with* the ETC term.

[17]For example, we have neglected the effect of particle mixings between the ground-state mesons considered and their radially excited states. This could produce a 10% error in the amplitude.

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[19]For the sake of the present argument, we assume a maximum violation for the vertex.

[20]The normalization of the states in (A1) is different from the one used in the text.

[21]See, for example, Marshak, Riazuddin, and Ryan (Ref. 9).