New approach to nonleptonic weak interactions. I. Derivation of asymptotic selection rules for the two-particle weak ground-state-hadron matrix elements

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A new approach to nonleptonic weak interactions is presented. It is argued that the presence and violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule as well as those of the quark-line selection rules can be explained in a unified way, along with other fundamental physical quantities [such as the value of $g_A(0)$ and the smallness of the isoscalar nucleon magnetic moments], in terms of a single dynamical asymptotic ansatz imposed at the level of observable hadrons. The ansatz prescribes a way in which asymptotic flavor SU(N) symmetry is secured levelwise for a certain class of chiral algebras in the standard QCD model. It yields severe asymptotic constraints upon the two-particle hadronic matrix elements of nonleptonic weak Hamiltonians as well as QCD currents and their charges. It produces for weak matrix elements the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule and its charm counterpart for the ground-state hadrons, while for strong matrix elements quark-line-like approximate selection rules. However, for the less important weak two-particle vertices involving higher excited states, the $|\Delta\vec{I}|=\frac{1}{2}$ rule and its charm counterpart are in general violated, providing us with an explicit source of the violation of these selection rules in physical processes.

I. INTRODUCTION AND SUMMARY

The origin of the approximate $|\Delta\vec{I}|=\frac{1}{2}$ rule[1] has long been one of the persistent problems of particle physics. The success of the standard model[2] of weak interactions suggests that the rule is *dynamical* in origin. In the sixties, current algebra, PCAC (partial conservation of axial-vector current), and the soft-meson technique provided a new powerful method and achieved reasonable success[3] for the problem of strangeness-changing weak nonleptonic processes. However, the very origin of the $|\Delta\vec{I}|=\frac{1}{2}$ rule remained unresolved.

With the advent of QCD, recent study[3] dealt with the underlying quarks and gluons directly. The leading QCD short-distance correction to the W-exchange diagram was found to be insufficient to enhance the $|\Delta\vec{I}|=\frac{1}{2}$ contribution and the role of penguin operators, which appear with quite small coefficients among the effective Hamiltonians, has been promoted. A drawback of this quark-gluon-diagram approach is that one needs a reliable tool to translate the result obtained at the quark-gluon level into information about the relevant hadronic matrix elements. Therefore, the factorization (or vacuum-insertion) approximation, which was introduced a long time ago[4] and was instrumental in demonstrating the dominance of the hadronic matrix elements of the penguin operators over other amplitudes, must undergo close scrutiny.[5] The hadronic matrix elements of the QCD-corrected effective Hamiltonians have thus been estimated by using various models:[6] MIT bag model, relativistic quark model, harmonic-oscillator model, etc. It is also generally agreed[3,7] that the perturbative QCD short-distance correction to the W-exchange diagram is unable to explain *simultaneously* the $|\Delta\vec{I}|=\frac{1}{2}$ rule and the Cabibbo-angle-unsuppressed charmed-meson decays. There is even a claim that nonleptonic physics is essentially determined by long-distance dynamics.[8]

It has long been recognized[9] that one can impose $|\Delta\vec{I}|=\frac{1}{2}$ constraints on the two-body baryon strangeness-changing weak nonleptonic vertices, by incorporating the color-singlet nature of hadrons into the quark-model wave functions. This result (sometimes called the Mimura-Minamikawa-Pati-Woo theorem) is indicative of a close link between the possible origin of the $|\Delta\vec{I}|=\frac{1}{2}$ rule and long-distance physics. However, though this result is often used as one of the important inputs in the current-algebra approach to nonleptonic hyperon decays,[3] the same argument unfortunately *fails* to yield the $|\Delta\vec{I}|=\frac{1}{2}$ rule for the two-body boson weak vertices.

Under these circumstances we attack the problem from a slightly different angle, paying close attention to the long-distance dynamics. We use current-algebra, PCAC, and an improved soft-meson extrapolation. However, most importantly, for the asymptotic two-particle hadron weak vertices which play a crucial role, we derive an *alternative* to the Mimura-Minamikawa-Pati-Woo theorem which now works for ground-state bosons as well as for baryons. The derivation is based on one dynamical ansatz[10] called *level realization of asymptotic flavor SU(N) symmetry*. The same ansatz has already successfully produced[10] in other places a correct value of $g_A(0)$, a good prediction[11] for the nucleon anomalous magnetic moments $k_p = -k_n$, the Okubo-Zweig-Iizuka (OZI) rule[12] for the asymptotic two-particle hadronic matrix elements, etc. Hence, we claim that a number of seemingly unrelated phenomena, i.e., the $|\Delta\vec{I}| = \frac{1}{2}$ rule, the well-known smallness of the isoscalar components of nucleon anomalous magnetic moments, the selection rules for the $\phi \to \rho\pi$, $f' \to \pi\pi$ decays, etc., as well as the value of $g_A(0)$, may all have the *same* theoretical origin. In this picture, they are the patterns by which hadrons respond to the constraints imposed upon them by the underlying quarks and gluons.

In a recent Letter, one of us (S.O.) pointed out[13] that the dynamical $|\Delta\vec{I}| = \frac{1}{2}$ rule can emerge as a result of constraints imposed upon certain asymptotic two-particle weak matrix elements, exactly in the same fashion as the OZI-type rule emerges, in the same theoretical framework, among the two-body hadronic matrix elements of the vector and axial-vector currents and their charges.[10] It was then argued, by using a soft-pion extrapolation, that the observed $|\Delta\vec{I}| = \frac{1}{2}$ rule in the *K*-meson decays is the result of the existence of an exact $|\Delta\vec{I}| = \frac{1}{2}$ rule for the asymptotic two-body weak vertices involving ground-state mesons. In a subsequent Letter,[14] we pointed out that the same method could also be extended to the charm-meson decays, thereby providing a unified description of the charm- and strange-meson weak nonleptonic processes.

The purpose of this (called I) and the subsequent (called II and III) papers is to give, with a substantial technical improvement, a more complete account of hadronic weak nonleptonic interactions. The scenario is as follows: In I, we make a comprehensive study of the *asymptotic weak two-body ground-state hadron* matrix elements of the strangeness-changing Hamiltonian $H^{(0,-)} \equiv H(\Delta C = 0, \Delta S = -1)$ and the strangeness- and charm-changing Hamiltonian $H^{(-,-)} \equiv H(\Delta C = \Delta S = -1)$. We show that the requirement that asymptotic SU(2) and SU(3) symmetry be realized *levelwise* among the chiral algebras involving $H^{(0,-)}$ and $H^{(-,-)}$ does impose severe constraints upon the asymptotic hadronic matrix elements. Namely, we see that the $|\Delta\vec{I}| = \frac{1}{2}$ rule or the octet rule in SU(3) and also the $\Delta V = 0$ rule (V is V spin) or $6 \oplus 6^*$ dominance in SU(4) do emerge (but only *asymptotically)* for the ground-state hadron two-body matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$ and further that the weak matrix elements satisfy certain SU(6)- or SU(8)-type relations also asymptotically. In some particular cases, these selection rules do emerge hand in hand with the OZI rules for the *strong* hadronic asymptotic matrix elements involving the axial-vector charges, corroborating the claim that these two seemingly different selection rules indeed share the same dynamical origin. However, for other less important two-particle weak hadron vertices involving excited states, the above asymptotic $|\Delta\vec{I}| = \frac{1}{2}$ rule, etc., are violated, providing us with an explicit source of the violation of the rules.

In papers II and III, we relate the asymptotic two-body weak matrix elements obtained in I to the physical amplitudes. We develop a new soft-meson technique especially suited to the present approach which uses the concept of asymptotic flavor symmetry. We use $\vec{q}_\pi \to 0$ instead of $(q_\pi)_\mu \to 0$ in the infinite-momentum frame of the parent particle, which, in effect, produces a *hard-meson* extrapolation. A part of the usually neglected surface term in the soft-meson approximation then turns out to give a *nonvanishing* contribution. However, we may again use the concept of levels of hadrons in evaluating the surface term. Keeping, for the moment, *only* the ground-state contribution in the surface term, we relate the rate of $K^0 \to 2\pi$ decays to that of the $D^0 \to \bar{K}\pi$ decays and obtain a reasonable numerical result in paper II. Hyperon nonleptonic interactions are treated in paper III.

II. THEORETICAL FRAMEWORK AND METHOD

We work in the theoretical framework[15] which deals directly only with observable hadrons and not with (confined) quarks and gluons. Nevertheless, the underlying quarks and gluons do control the world of hadrons firmly in the following two respects: (i) Hadrons are severely constrained by the presence of the QCD algebras, involving especially the vector $[V_\alpha^\mu(x)]$ and axial-vector $[A_\alpha^\mu(x)]$ currents and their charges which enter into the world of hadrons as observable weak currents. The successful calculation of $g_A(0)$ by Alder and Weisberger[16] was indeed carried out in this theoretical framework: (ii) Hadrons have to obey a certain level scheme of (mainly $q\bar{q}$ and qqq) constituent model. This may be the simplest abstraction of long-distance physics of the underlying quark-gluon dynamics. The $q\bar{q}$ and qqq level scheme immediately fixes the flavor multiplicities of hadrons associated with each level, without using the notion of broken higher symmetry. Furthermore, the concept of levels can, at present, be rather flexible and need not be tied to nonrelativistic model.[17]

In this theoretical framework, one proceeds as follows. The various chiral quark algebras which we are going to use are valid in *broken* SU(N) flavor symmetry. This provides a remarkable opportunity to deal with broken SU(N) flavor symmetry in a nonperturbative way, by introducing a dynamical ansatz called asymptotic SU(N) flavor symmetry.[10] It states that in the asymptotic limit the flavor SU(N) transformation does maintain its *linearity* [including the possibility of SU(N) particle mixing] in the presence of the SU(N) flavor symmetry breaking and that the nonlinear terms vanish sufficiently fast in the asymptotic limit. It has been shown[18] that this hypothesis can be made in the presence of Gell-Mann—Okubo mass splittings with SU(N) particle mixings. The use of this asymptotic ansatz in realizing the algebras involving SU(N) charge V_α permits us to derive *broken*- SU(N)-symmetry sum rules as asymptotic constraints. The most useful consequences is that one can parametrize, in broken symmetry, the asymptotic matrix elements of important operators (charges, currents, weak Hamiltonians, etc.) in terms of the usual prescription of exact symmetry plus general SU(N) mixing. What we are going to derive is not a strict $|\Delta\vec{I}| = \frac{1}{2}$ rule but rather an *asymptotic*

$|\Delta\vec{I}|=\frac{1}{2}$ rule valid in broken SU(3) symmetry.

Another (nonperturbative) dynamical ansatz made at the level of hadrons, which has been found to work remarkably well, is called level realization of asymptotic flavor SU(N) symmetry in the class of chiral algebras involving the axial-vector charges A_α. For example, one may consider the algebraic constraints given by the charge-charge or charge-current chiral $SU(2)_L \otimes SU(2)_R$-type algebras[10,11] such as

$$[A_{\pi^+},A_{\pi^-}]=2V_3\ ,$$
$$[[A_3,A_{\pi^+}],A_{\pi^-}]=2A_3 \tag{1}$$

and

$$[[A_3,A_{\pi^-}],A_{\pi^+}]=2A_3\ , \tag{2}$$

$$[[V_3^\mu(0)(A_3^\mu(0)),A_{\pi^+}],A_{\pi^-}]=2V_3^\mu(0)(A_3^\mu(0)) \text{ etc.}, \tag{3}$$

$$[[J_{\rm EM}^\mu(0)(A_N^\mu(0)),A_{\pi^+}],A_{\pi^-}]=2V_3^\mu(0)(A_3^\mu(0)) \text{ etc.} \tag{4}$$

Here $J_{\rm EM}^\mu(x)$ and $A_N^\mu(x)$ are the electromagnetic and the weak axial-vector neutral currents, respectively; $A_{\pi^+}=A_1+iA_2$, etc.

The ansatz requires that the realization of the asymptotic flavor SU(N) symmetry in these algebras should be *levelwise*. We sandwich these algebras between the hadron states, $\langle B_M(\alpha,\vec{p},\lambda)|$ and $|B_M(\beta,\vec{p}\,',\lambda')\rangle$, belonging to the *same* level M with infinite momenta, $\vec{p}\to\infty$ and $\vec{p}\,'\to\infty$. α and β denote the physical SU(N) indices $(\pi,K,D,\ldots)$ and (λ,λ') the helicities. The right-hand side (RHS) of this equation is denoted by $C(\alpha,\lambda;\beta,\lambda')$. On the left-hand side (LHS) we insert a complete set of single particle hadron intermediate states[19] among the factors of the equal-time commutation relations. For single commutators such as Eq. (1), we distinguish, among the set of complete intermediate states, the *fractional* contribution $f^L(\alpha,\lambda;\beta,\lambda')$ which sums the *entire* contributions coming from *all the single particles (hadrons) belonging to a particular level* L $(L=0,1,2,\ldots)$. Thus, the equation takes the form,

$$C(\alpha,\lambda;\beta,\lambda')(f^0+f^1+f^2+\cdots)=C(\alpha,\lambda;\beta,\lambda')\ , \tag{5}$$

where $f^0+f^1+f^2+\cdots=1$, *unless* all $C(\alpha,\lambda;\beta,\lambda')$'s vanish under the SU($N$) rotation of physical indices α and β. For Eq. (5), we now study the possible variations of the SU(N) induces α and β which produce meaningful (i.e., *not trivially* zero) values of C's. On the LHS the intermediate states also undergo a corresponding SU(N) rotation. The ansatz states that the (asymptotic) fraction f^L of level L could depend on the choice of the helicity states (λ,λ') but *not* on the SU(N) indices α and β, i.e., *it is invariant under the SU(N) rotation in the asymptotic limit.* In the case of double commutators such as Eqs. (2)–(4), there appear two sets of intermediate states so that the f's will be characterized by two indices, $f^{LL'}$. As the special realization pattern of Eq. (5), we can also have $0+0+0+\cdots=0$, i.e., all the C's related by SU(N) rotation are zero and on the LHS these are then realized by having a vanishing contribution from each level L $(L=0,1,\ldots)$.

The idea of level realization is not as radical as it sounds. For example, let us consider the level realization of SU(2) symmetry in the algebras Eqs. (1) and (2) among the SU(2) boson multiplets with u and d quarks assuming *exact* SU(2) symmetry. We then find that the realization is automatically satisfied. If we extend the procedure from SU(2) to SU(3) multiplets by introducing s quark, the realization now implies[10] that the $\phi\to\rho\pi$, $f'\to\pi\pi$ decays, etc., are *forbidden,* if the ϕ and f' mesons were *pure* $s\bar{s}$ states. Actually, if all the $q\bar{q}$ meson nonets are ideal, the realization of Eqs. (1) and (2) are automatic,[10] which suggests that the near *ideal* character of most of the $q\bar{q}$ nonets is closely related[20] to the working of the ansatz. The realization of the algebras Eqs. (1)–(4) among ground-state baryons produced,[10,11] among others, the correct value of $g_A(0),k_p=-k_n$, etc., mentioned before. The task of this and subsequent papers is to study whether we can accommodate the $|\Delta\vec{I}|=\frac{1}{2}$ rule and the related problems in this pattern recognition of dynamical constraints in the hadronic world.

III. ALGEBRAIC CONSTRAINTS INVOLVING NONLEPTONIC WEAK HAMILTONIANS

The nonleptonic Hamiltonian (in the local limit) of the standard four-quark scheme is described by

$$H_0=(G/2\sqrt{2})\int d^3x\,:j_\mu^{(+)}(x)j_\mu^{(-)}(x):\ , \tag{6}$$

with the current $j_\mu(x)$,

$$j_\mu^{(+)}=\cos\theta_C\bar{u}\gamma_{\mu L}d+\sin\theta_C\bar{u}\gamma_{\mu L}s+(-\sin\theta_C)\bar{c}\gamma_{\mu L}d+\cos\theta_C\bar{c}\gamma_{\mu L}s\ ,$$
$$j_\mu^{(-)}=(j_\mu^{(+)})^\dagger\ ,\quad \gamma_{\mu L}=\gamma_\mu\tfrac{1}{2}(1+\gamma_5)\ . \tag{7}$$

As far as the transformation property of the quark fields under SU(4) in flavor space is concerned, the current may be conveniently denoted by

$$j^{(+)}\propto\cos\theta_C\pi^+ +\sin\theta_C K^+ +(-\sin\theta_C)D^+ +\cos\theta_C F^+\ .$$

The charm-conserving and strangeness-changing ($\Delta S=-1$) Hamiltonian $H_0^{(0,-)}$ and the Cabibbo-angle-favored strangeness- and charm-changing ($\Delta C=\Delta S=-1$) Hamiltonian $H_0^{(-,-)}$ are given symbolically by

$$H_0^{(0,-)}=\sin\theta_C\cos\theta_C(\pi^+K^- -D^+F^-)$$

and

$$H_0^{(-,-)}=\cos^2\theta_C\pi^+F^-\ ,$$

respectively.

We now demand that the *effective* Hamiltonian (in the local limit) H responsible for the $\Delta C=0$, $|\Delta S|=1$ process commutes with the generators of the chiral group $SU_f(2)_R$, i.e.,

$$[H(|\Delta S|=1),V_\alpha-A_\alpha]=0\quad(\alpha=1,2,3)\ , \tag{8}$$

where α denotes the isotopic-spin indices and V_α and A_α are the vector and axial-vector charges appearing in Sec.

II. In the current-algebra approach to nonleptonic weak interactions, Eq. (8) played a crucial role.[21] The bare Hamiltonian, Eq. (6), of course, satisfies Eq. (8), since the Glashow-Iliopoulos-Maiani currents contain only the left-handed components of quarks. However, Eq. (8) allows, more generally, the right-handed components of u and d quarks to appear in H as long as they form an isotopic spin singlet, while both the right- and left-handed components of s and c quarks can enter into Eq. (8) freely, since they are isospin singlet. Equation (8) can be cast into the following forms[22] for the *effective* Hamiltonian $H^{(0,-)}$ for the processes with $\Delta C=0$ and $\Delta S=-1$:

$$[[H^{(0,-)},A_{\pi^-}],A_{\pi^+}]=[[H^{(0,-)},V_{\pi^-}],V_{\pi^+}]\ , \quad (9a)$$

$$[[H^{(0,-)},A_{\pi^+}],A_{\pi^-}]=[[H^{(0,-)},V_{\pi^+}],V_{\pi^-}]\ . \quad (9b)$$

The validity of Eqs. (9a) and (9b) in the world of hadrons, is instrumental in deriving selection rules in this paper. Since they involve two axial charges, the parity-conserving and -violating parts of H, $H^{\rm PC}$, and $H^{\rm PV}$, satisfy these equations *separately,* whereas the single commutator Eq. (8) connects $H^{\rm PV}$ to $H^{\rm PC}$ such as $[H^{\rm PC},A_\alpha]=[H^{\rm PV},V_\alpha]$. Therefore, the algebras Eqs. (9a) and (9b) provide us with a *weaker* version[23] of the current-algebra constraint Eq. (8) and, in fact, they are satisfied by the QCD-corrected Hamiltonians, even including the penguin operators. When we discuss possible selection rules associated with the charm mesons, it is natural to relax Eq. (8) so that the effective Hamiltonian for the Cabibbo-angle-favored charm- and strangeness-changing processes $H^{(-,-)}$ commutes with the generators of the chiral group $\mathrm{SU}_f(3)_R$. Corresponding to Eqs. (9a) and (9b), we have

$$[[H^{(-,-)},A_{K^-}],A_{\pi^+}]=[[H^{(-,-)},V_{K^-}],V_{\pi^+}]\ , \quad (10a)$$

$$[[H^{(-,-)},A_{\pi^+}],A_{K^-}]=[[H^{(-,-)},V_{\pi^+}],V_{K^-}]\ . \quad (10b)$$

We emphasize that the validity of Eqs. (9a)—(10b) is independent of chiral symmetry breaking.

IV. DERIVATION OF THE ASYMPTOTIC $|\Delta\vec{I}\,|=\frac{1}{2}$ RULE AND THE OZI RULE FOR MESONS

In order to avoid the unnecessary complications caused by the proliferation of quark flavors, we, in this section, restrict ourselves within the framework of asymptotic SU(3) symmetry.[24]

We now consider the level realization of asymptotic SU(3) symmetry in the chiral $\mathrm{SU}_f(2)_L\otimes \mathrm{SU}_f(2)_R$ type algebras, Eqs. (9a) and (9b). We insert these algebras between the *ground-state* mesons, $\langle B(\alpha,\vec{p},\lambda)|$ and $|B(\beta,\vec{p}\,',\lambda')\rangle$, with $\alpha=\pi^{+,0},\eta,\eta',\rho^{+,0},\phi,\omega$ and $\beta=K^{+,0},K^{*+,0}$. We take both $\vec{p}$ and $\vec{p}\,'$ along the z axis (actually, $\vec{p}\,'=\vec{p}$ in the present case) and let $\vec{p}\to\infty$, in order to cope with broken SU(3) symmetry using the ansatz of asymptotic SU(3) symmetry. On the LHS of these equations we look at the fractional contribution coming from the ground-state (0^{-+} and 1^{--}) mesons.

A. $\lambda=1$ sum rules

To demonstrate the working of the remarkable interplay mentioned in Sec. I (especially between the weak vertices and the strong couplings), we start with the sum rules involving the matrix elements of $H^{(0,-)}$ inserted between the ground-state mesons with helicity ±1, i.e., $\langle B(\alpha,\vec{p},\lambda=1)|H^{(0,-)}|B(\beta,\vec{p}\,';\lambda'=1)\rangle$ with $\vec{p}\,'=\vec{p}\to\infty$. For simplicity, we write simply $H^{(0,-)}\equiv H$ in Sec. IV. For the parity-conserving Hamiltonian $H^{\rm PC}$ the matrix element

$$[2E(\vec{p})]^{1/2}[2E(\vec{p}\,')]^{1/2} \times\langle B(\alpha,\vec{p},\lambda=1)|H^{\rm PC}|B(\beta,\vec{p},\lambda'=1)\rangle$$

will take the form proportional to $e_\mu^{*\alpha}(\vec{p},\lambda)e_\mu^{\beta}(\vec{p}\,',\lambda)$, while it is proportional to $\epsilon_{\mu\nu\sigma\rho}e_\mu^{*\alpha}(\vec{p},\lambda)\ e_\nu^{\beta}(\vec{p}\,',\lambda')p_\sigma p'_\rho$ for the parity-violating Hamiltonian $H^{\rm PV}$. $e_\mu^{\alpha}(\vec{p},\lambda)$ is the polarization four-vector of the vector meson $B(\alpha,\vec{p})$. The latter coupling thus vanishes in the exact-SU(3)-symmetry limit, i.e., $p'_\mu\to p_\mu$. However, to our approximation the *asymptotic* matrix elements of $H^{\rm PV}$ involving helicity $\lambda=\pm1$ states remain to be zero even in broken SU(3) symmetry. This is the consequence of asymptotic SU(3) symmetry and CP invariance. We now insert Eqs. (9a) and (9b) between the $\lambda=1$ vector-meson states ($\vec{p}\,'=\vec{p}\to\infty$), i.e.,: (i) between $\langle\rho^+(\vec{p},\lambda=1)|$ and $|K^{*+}(\vec{p}\,',\lambda=1)\rangle$; (ii) $\langle\rho^0|$ and $|K^{*0}\rangle$; (iii) $\langle\omega|$ and $|K^{*0}\rangle$; and (iv) $\langle\phi|$ and $|K^{*0}\rangle$. Then, in the intermediate states on the LHS of these equations, the contribution of the ground-state mesons (0^{-+} and 1^{--} mesons) consists *solely* of the 1^{--} mesons with $\lambda=1$. Writing out explicitly this ground-state contribution for the case (i)—(iv) [assuming exact $\mathrm{SU}_f(2)$ symmetry], we obtain for the realization of Eq. (9a) ($H\equiv H^{\rm PC}$ in this subsection),

$$-\langle\rho^+(\vec{p})|A_{\pi^+}|\omega,\phi\rangle\langle\omega,\phi|H|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^{*+}(\vec{p}\,')\rangle+\langle\rho^+(\vec{p})|A_{\pi^+}|\omega,\phi\rangle\langle\omega,\phi|A_{\pi^-}|\rho^+\rangle\langle\rho^+|H|K^{*+}(\vec{p}\,')\rangle$$
$$+\text{ higher-level contributions }=\sqrt{2}\langle\rho^0|H|K^{*0}\rangle+2\langle\rho^+|H|K^{*+}\rangle\ , \quad (11)$$

$$\langle\rho^0|H|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^{*+}\rangle\langle K^{*+}|A_{\pi^+}|K^{*0}\rangle+\text{ higher-level contributions}$$
$$=3\langle\rho^0|H|K^{*0}\rangle+\sqrt{2}\langle\rho^+|H|K^{*+}\rangle\ , \quad (12)$$

$$\langle\omega|H|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^{*+}\rangle\langle K^{*+}|A_{\pi^+}|K^{*0}\rangle-\langle\omega|A_{\pi^-}|\rho^+\rangle\langle\rho^+|H|K^{*+}\rangle\langle K^{*+}|A_{\pi^+}|K^{*0}\rangle$$
$$+\langle\omega|A_{\pi^+}|\rho^-\rangle\langle\rho^-|A_{\pi^-}|\omega,\phi\rangle\langle\omega,\phi|H|K^{*0}\rangle+\text{higher-level contributions}=\langle\omega|H|K^{*0}\rangle\ , \quad (13)$$

[replace ω by ϕ and ϕ by ω in Eq.(13)] . (14)

In the same way, the realization of Eq. (9b) for cases (i)—(iv) is

$$\langle\rho^+|H|K^{*+}\rangle\langle K^{*+}|A_{\pi^+}|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^{*+}\rangle-\langle\rho^+|A_{\pi^+}|\omega,\phi\rangle\langle\omega,\phi|H|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|K^{*+}\rangle$$
$$+\text{higher-level contributions}=\sqrt{2}\langle\rho^0|H|K^{*0}\rangle+\langle\rho^+|H|K^{*+}\rangle\ , \quad (15)$$

$$(\text{zero, i.e., no ground-state contribution})+\text{higher-level contributions}=2\langle\rho^0|H|K^{*0}\rangle+\sqrt{2}\langle\rho^+|H|K^{*+}\rangle\ , \quad (16)$$

$$-\langle\omega|A_{\pi^-}|\rho^+\rangle\langle\rho^+|H|K^{*+}\rangle\langle K^{*+}|A_{\pi^+}|K^{*0}\rangle+\langle\omega|A_{\pi^-}|\rho^+\rangle\langle\rho^+|A_{\pi^+}|\omega,\phi\rangle\langle\omega,\phi|H|K^{*0}\rangle$$
$$+\text{higher-level contributions}=\langle\omega|[[H,V_{\pi^+}],V_{\pi^-}]|K^{*0}\rangle=0\ , \quad (17)$$

[replace ω by ϕ and ϕ by ω in *Eq.*(17)] . (18)

On the RHS of Eqs. (11)—(18) we have already used exact $SU_f(2)$ symmetry.

The zeros which appear in Eqs. (16), (17), and (18) are due to conservations of the G parity and the strangeness in the strong couplings involved. The structure of Eqs. (15)—(18) are already suggestive of the presence of the $|\Delta\vec{I}|=\frac{1}{2}$ rule. Namely, the ground-state contribution in Eq. (16) vanishes (the strong couplings such as $\langle\rho^0|A_{\pi^-}|\rho^+\rangle$ are G-forbidden), which implies that the asymptotic fraction of the ground state or the RHS of Eq. (16) is zero. Namely, the sufficient condition to satisfy the requirement of the level realization in the algebra (9b) is

$$\sqrt{2}\langle\rho^0(\vec{p},\lambda=1)|H|K^{*0}(\vec{p}\,',\lambda=1)\rangle+\langle\rho^+(\vec{p},\lambda=1)|H|K^{*+}(p',\lambda=1)\rangle=0\ ,$$
$$\vec{p}\,'=\vec{p}\to\infty\ , \quad (19)$$

since the RHS of Eqs. (17) and (18) are also zero. In this case the pattern of the level realization of asymptotic SU(3) symmetry in the algebra Eq. (9b), i.e., Eqs. (15)—(18), takes the form mentioned in Sec. II,

$$0+0+\cdots=0\ . \quad (20)$$

In the process, we also discover that, in addition to the constraints imposed on the weak vertices (which include the $|\Delta\vec{I}|=\frac{1}{2}$ rule), the following constraint on the *strong couplings (the asymptotic axial-vector matrix elements)* must *also* be satisfied *simultaneously,*

$$\frac{\langle\phi(\vec{p},\lambda=1)|A_{\pi^-}|\rho^+(\vec{p},\lambda=1)\rangle}{\langle\omega(\vec{p},\lambda=1)|A_{\pi^-}|\rho^+(\vec{p},\lambda=1)\rangle}=-\tan(\theta_{\omega\phi}-\theta_0)\ ,\ \vec{p}\to\infty\ . \quad (21)$$

Actually this constraint has also been found[25,26] for the level realization of asymptotic SU(3) symmetry in *different* class of chiral algebras, i.e., Eqs. (1)—(4), and it reduces via PCAC to

$$\frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}}=-\tan(\theta_{\omega\phi}-\theta_0)\ , \quad (22)$$

where θ_0 is the ideal angle, $\sin\theta_0=+(\frac{1}{3})^{1/2}$. In the ideal limit, $\theta_{\omega\phi}=\theta_0$, in which the ϕ meson takes a pure $\bar{s}s$ configuration, the presence of the strict quark-line rule $g_{\phi\rho\pi}=0$ is thus assured within the theory. Therefore, the overall consistency is remarkable and it provides a strong justification for the claim that both the $|\Delta\vec{I}|=\frac{1}{2}$ rule and the quark-line rule have the *same* dynamical origin. We now solve Eqs. (11)—(18).

In the limit $\vec{p}\to\infty$, asymptotic $SU_f(3)$ symmetry permits the $SU_f(3)$ parametrization of the asymptotic axial-vector matrix elements. If we write (for $\vec{p}\to\infty$)

$$\langle\omega(\vec{p})|A_{\pi^+}|\rho^-\rangle=\langle\rho^+(\vec{p})|A_{\pi^+}|\omega\rangle\equiv S\ ,$$
$$\langle\phi(\vec{p})|A_{\pi^+}|\rho^-\rangle=\langle\rho^+(\vec{p})|A_{\pi^+}|\phi\rangle\equiv D\ , \quad (23)$$

we then obtain (from the algebra $[V_\alpha,A_\beta]=if_{\alpha\beta\gamma}A_\gamma$ and asymptotic SU(3) symmetry)

$$\langle K^{*+}(\vec{p})|A_{\pi^+}|K^{*0}\rangle=\langle K^{*0}(\vec{p})|A_{\pi^-}|K^{*+}\rangle=(\tfrac{3}{2})^{1/2}(cD+sS)\ , \quad (24)$$

where

$$c\equiv\cos\theta_{\omega\phi}\ \text{ and }\ s\equiv\sin\theta_{\omega\phi}\ . \quad (25)$$

We introduce the following abbreviations for the asymptotic weak matrix elements under consideration:

$$\langle\rho^+|H|K^{*+}\rangle\equiv\lambda\ ,\ \langle\rho^0|H|K^{*0}\rangle\equiv\alpha\ ,$$
$$\langle\omega|H|K^{*0}\rangle\equiv\beta\ ,\ \langle\phi|H|K^{*0}\rangle\equiv\gamma\ . \quad (26)$$

Then Eqs. (11)—(14) will take the form

$$-(\tfrac{3}{2})^{1/2}(\beta S+\gamma D)(cD+sS)+\lambda(S^2+D^2)=(\sqrt{2}\alpha+2\lambda)k\ , \tag{11'}$$

$$\tfrac{3}{2}\alpha(cD+sS)^2=(3\alpha+\sqrt{2}\lambda)k\ , \tag{12'}$$

$$\tfrac{3}{2}\beta(cD+sS)^2-(\tfrac{3}{2})^{1/2}\lambda S(cD+sS)+\beta S^2+\gamma DS=\beta k\ , \tag{13'}$$

$$\tfrac{3}{2}\gamma(cD+sS)^2-(\tfrac{3}{2})^{1/2}\lambda D(cD+sS)+\beta DS+\gamma D^2=\gamma k\ , \tag{14'}$$

where k is the asymptotic fraction of the ground-state contributions ($k\equiv f^{00}$).

Equations (15)—(18) will also become

$$\tfrac{3}{2}\lambda(cD+sS)^2-(\tfrac{3}{2})^{1/2}(\beta S+\gamma D)(cD+sS)=(\sqrt{2}\alpha+\lambda)k'\ , \tag{15'}$$

$$0=(\sqrt{2}\alpha+\lambda)k'\ , \tag{16'}$$

$$-(\tfrac{3}{2})^{1/2}\lambda(cD+sS)+\beta S+\gamma D=(0\cdot k')=0\ , \tag{17'}$$

$$-(\tfrac{3}{2})^{1/2}\lambda(cD+sS)+\beta S+\gamma D=(0\cdot k')=0\ , \tag{18'}$$

where k' denotes the asymptotic fraction of the ground-state contribution. Among the set of constraints, Eqs. (15′)—(18′), the independent ones are

$$(\lambda+\sqrt{2}\alpha)k'=0\ , \tag{16''}$$

$$(\tfrac{3}{2})^{1/2}\lambda(cD+sS)-(\beta S+\gamma D)=0\ . \tag{17''}$$

Since $(cD+sS)\neq 0$[27] and $\alpha\neq 0$, we know from Eq. (12′) $k\neq 0$. With Eq. (17″), Eqs. (13′) and (14′) become

$$\tfrac{3}{2}(cD+sS)^2\beta=\beta k\ , \tag{13''}$$

$$\tfrac{3}{2}(cD+sS)^2\gamma=\gamma k\ , \tag{14''}$$

respectively, which imply (since $\beta\neq 0$ and $\gamma\neq 0$) for the asymptotic fraction k

$$k=\tfrac{3}{2}(cD+sS)^2>0\ . \tag{27}$$

If we insert this value of k into Eq. (12′) we obtain

$$\sqrt{2\alpha}+\lambda=0\ . \tag{28}$$

Equation (28) is the desired $|\Delta\vec{\mathrm{I}}|=\frac{1}{2}$ rule stated in Eq. (19). If we use this information, Eq. (28), in Eq. (17″), we obtain

$$\sqrt{3}\alpha(cD+sS)+(\beta S+\gamma D)=0\ , \tag{29}$$

which is a new *nontrivial* constraint [it is *not* an SU(3) relation], imposed on the weak matrix elements involving neutral vector mesons as well as the strong couplings D and S, i.e.,

$$\sqrt{3}(\cos\theta_{\omega\phi}D+\sin\theta_{\omega\phi}S)\langle\rho^0|H|K^{*0}\rangle+S\langle\omega|H|K^{*0}\rangle+D\langle\phi|H|K^{*0}\rangle=0\ . \tag{30}$$

If we multiply λ ($\lambda=-\sqrt{2\alpha}$) on both sides of Eq. (11′) and use Eqs. (27) and (29), we then obtain, since $\alpha\neq 0$,

$$3(cD+sS)^2-(S^2+D^2)=0 \tag{31}$$

which is a constraint on the asymptotic axial-vector matrix elements (i.e., strong couplings) and the physical ω-ϕ mixing angle $\theta_{\omega\phi}$, i.e.,

$$3(\cos\theta_{\omega\phi}D+\sin\theta_{\omega\phi}S)^2=S^2+D^2\ . \tag{32}$$

One of the two possible solutions for the ratio D/S of Eq. (32) is the correct one[25,26] given by Eq. (21), which leads to the strict OZI rule in the ideal limit and prescribes the degree of the violation of the selection rule correctly. The level realization of the algebra, Eq. (9a), is achieved with a universal asymptotic fraction k of the ground-state contribution given by Eq. (27). In the ideal limit of 1^{--} mesons, i.e., $\theta_{\omega\phi}=\theta_0$,

$$D=0 \text{ and } k=\tfrac{1}{2}S^2\ . \tag{33}$$

A comparison between the level realization ansatz and the *truncation* assumption may be in order here. The difference between these two approaches can be best seen in Eq. (27). The truncation assumption is usually based on the *saturation* of algebras by low-lying states; thus the asymptotic fraction k must be equal to one. Then Eq. (27) becomes, with Eq. (31), $S^2+D^2=2(k=1)$, which yields a result inconsistent with experiment.[27] Namely, it spoils the overall consistency of Eq. (27) with both the $|\Delta\vec{\mathrm{I}}|=\frac{1}{2}$ rule, Eq. (19), and the quark-line rule, Eq. (21). In contrast to the truncation assumption, the level realization allows Eq. (27) to be consistent with all other equations, since the fraction k is now an adjustable parameter to be fixed. In Appendix A, SU(3) constraint among the weak matrix elements involving neutral vector-mesons is derived for illustration.

B. $\lambda=0$ sum rules

We again consider the level realization of asymptotic SU(3) symmetry, which yields the sum rules involving the helicity $\lambda=0$ states, i.e., $\langle B(\alpha,\vec{p},\lambda=0)|H|B(\beta,\vec{p}\,',\lambda=0)\rangle$. We insert the algebras, Eqs. (9a) and (9b), between the states $\langle B(\alpha)|$ and $|B(\beta)\rangle$ where (α,β) runs:

(i) (π^+,K^+), (π^0,K^0), (η,K^0), (η',K^0);

(ii) (ρ^+,K^{*+}), (ρ^0,K^{*0}), (ω,K^{*0}), (ϕ,K^{*0});

(iii) (ρ^+,K^+), (ρ^0,K^0), (ω,K^0), (ϕ,K^0);

(iv) (π^+,K^{*+}), (π^0,K^{*0}), (η,K^{*0}), (η',K^{*0}).

In contrast with the case considered in Sec. IV A both 0^- and 1^- of the ground-state mesons with $\lambda=0$ can now participate in the level realization. We explicitly write down the ground-state-meson contribution to the level realization of the algebra, Eq. (9a). For the case (i), we obtain ($H=H^{\rm PC}$ ($\Delta C=0,\Delta S=-1$))

$$-\langle \pi^+ | A_{\pi^+} | \rho^0 \rangle \langle \rho^0 | H | K^{*0} \rangle \langle K^{*0} | A_{\pi^-} | K^+ \rangle + \langle \pi^+ | A_{\pi^+} | \rho^0 \rangle \langle \rho^0 | A_{\pi^-} | \pi^+ \rangle \langle \pi^+ | H | K^+ \rangle$$
$$+ \text{higher-level contributions} = \sqrt{2} \langle \pi^0 | H | K^0 \rangle + 2 \langle \pi^+ | H | K^+ \rangle \,, \tag{34}$$

$$\langle \pi^0 | H | K^0 \rangle \langle K^0 | A_{\pi^-} | K^{*+} \rangle \langle K^{*+} | A_{\pi^+} | K^0 \rangle - \langle \pi^0 | A_{\pi^-} | \rho^+ \rangle \langle \rho^+ | H | K^{*+} \rangle \langle K^{*+} | A_{\pi^+} | K^0 \rangle$$
$$+ \langle \pi^0 | A_{\pi^+} | \rho^- \rangle \langle \rho^- | A_{\pi^-} | \pi^0 \rangle \langle \pi^0 | H | K^0 \rangle$$
$$+ \text{higher-level contributions} = 3 \langle \pi^0 | H | K^0 \rangle + \sqrt{2} \langle \pi^+ | H | K^+ \rangle \,, \tag{35}$$

$$\langle \eta | H | K^0 \rangle \langle K^0 | A_{\pi^-} | K^{*+} \rangle \langle K^{*+} | A_{\pi^+} | K^0 \rangle + \text{higher-level contributions} = \langle \eta | H | K^0 \rangle \,, \tag{36}$$

$$\langle \eta' | H | K^0 \rangle \langle K^0 | A_{\pi^-} | K^{*+} \rangle \langle K^{*+} | A_{\pi^+} | K^0 \rangle + \text{higher-level contributions} = \langle \eta' | H | K^0 \rangle \,. \tag{37}$$

Similarly for the algebra, Eq. (9b), we obtain for the case (i),

$$\langle \pi^+ | H | K^+ \rangle \langle K^+ | A_{\pi^+} | K^{*0} \rangle \langle K^{*0} | A_{\pi^-} | K^+ \rangle - \langle \pi^+ | A_{\pi^+} | \rho^0 \rangle \langle \rho^0 | H | K^{*0} \rangle \langle K^{*0} | A_{\pi^-} | K^+ \rangle$$
$$+ \text{higher-level contributions} = \sqrt{2} \langle \pi^0 | H | K^0 \rangle + \langle \pi^+ | H | K^+ \rangle \,, \tag{38}$$

$$-\langle \pi^0 | A_{\pi^-} | \rho^+ \rangle \langle \rho^+ | H | K^{*+} \rangle \langle K^{*+} | A_{\pi^+} | K^0 \rangle + \langle \pi^0 | A_{\pi^-} | \rho^+ \rangle \langle \rho^+ | A_{\pi^+} | \pi^0 \rangle \langle \pi^0 | H | K^0 \rangle$$
$$+ \text{higher-level contributions} = 2 \langle \pi^0 | H | K^0 \rangle + \sqrt{2} \langle \pi^+ | H | K^+ \rangle \,, \tag{39}$$

$$(\text{no ground-state contribution}) + \text{higher- level contributions} = \langle \eta | [[H, V_{\pi^+}], V_{\pi^-}] | K^0 \rangle = 0 \,, \tag{40}$$

$$(\text{no ground-state contribution}) + \text{higher-level contributions} = \langle \eta' | [[H, V_{\pi^+}], V_{\pi^-}] | K^0 \rangle = 0 \,. \tag{41}$$

Equations (40) and (41) immediately suggest that the realization pattern of the algebra, Eq. (9b), in the present case is the same as in the case of $\lambda = 1$ which is given by Eq. (20) (no ground-state contribution)$+0+\cdots=0$. Then, the RHS of Eqs. (38) and (39) immediately imply

$$\sqrt{2} \langle \pi^0(\vec{p}) | H | K^0(\vec{p}\,') \rangle + \langle \pi^+(\vec{p}) | H | K^+(\vec{p}\,') \rangle = 0, \vec{p}\,' = \vec{p} \to \infty \,. \tag{42}$$

This is nothing but the asymptotic $|\Delta \vec{I}| = \frac{1}{2}$ rule for the π-K vertices. Our remaining task is to check whether this is consistent with the whole set of constraint Eqs. (34)—(41). Again we parametrize the asymptotic axial-vector matrix elements using asymptotic SU(3) symmetry as follows:

$$\langle \pi^+(\vec{p}) | A_{\pi^+} | \rho^0(\vec{p}) \rangle = -\sqrt{2} \langle K^+(\vec{p}) | A_{\pi^+} | K^{*0}(\vec{p}) \rangle = F \neq 0, \vec{p} \to \infty \,. \tag{43}$$

Then Eqs. (34)—(37) become

$$(\tfrac{1}{2})^{1/2} F^2 \langle \rho^0 | H | K^{*0} \rangle + F^2 \langle \pi^+ | H | K^+ \rangle = (\sqrt{2} \langle \pi^0 | H | K^0 \rangle + 2 \langle \pi^+ | H | K^+ \rangle) l \,, \tag{44}$$

$$\tfrac{3}{2} F^2 \langle \pi^0 | H | K^0 \rangle + (\tfrac{1}{2})^{1/2} F^2 \langle \rho^+ | H | K^{*+} \rangle = (3 \langle \pi^0 | H | K^0 \rangle + \sqrt{2} \langle \pi^+ | H | K^+ \rangle) l \,, \tag{45}$$

$$\tfrac{1}{2} F^2 \langle \eta | H | K^0 \rangle = \langle \eta | H | K^0 \rangle l \,, \tag{46}$$

$$\tfrac{1}{2} F^2 \langle \eta' | H | K^0 \rangle = \langle \eta' | H | K^0 \rangle l \,, \tag{47}$$

where l is the asymptotic fraction of the ground-state contribution to the algebra, Eq. (9a). Equations (46) and (47) then yield

$$l = \tfrac{1}{2} F^2 \,. \tag{48}$$

Inserting this value of l into Eqs. (44) and (45), we now obtain the SU(6)-type constraints on the asymptotic weak matrix elements:

$$\langle \rho^0(\vec{p}, \lambda=0) | H | K^{*0}(\vec{p}\,', \lambda=0) \rangle$$
$$= \langle \pi^0(\vec{p}) | H | K^0(\vec{p}\,') \rangle \,, \quad \vec{p}\,' = \vec{p} \to \infty \tag{49}$$

and

$$\langle \rho^+(\vec{p}, \lambda=0) | H | K^{*+}(\vec{p}\,', \lambda=0) \rangle$$
$$= \langle \pi^+(\vec{p}) | H | K^+(\vec{p}\,') \rangle \,, \quad \vec{p}\,' = \vec{p} \to \infty \,. \tag{50}$$

If we combine Eqs. (49) and (50) with Eqs. (42), we obtain

$$\sqrt{2} \langle \rho^0(\vec{p}, \lambda=0) | H | K^{*0}(\vec{p}\,', \lambda=0) \rangle$$
$$+ \langle \rho^+(\vec{p}, \lambda=0) | H | K^{*+}(\vec{p}\,', \lambda=0) \rangle = 0 \,,$$
$$\vec{p}\,' = \vec{p} \to \infty \,, \tag{51}$$

which is exactly the statement of the $|\Delta \vec{I}| = \frac{1}{2}$ rule for the asymptotic weak vertices $\langle \rho(\lambda=0) | H | K^*(\lambda=0) \rangle$. The ground-state contributions on the LHS of Eqs. (34) and (35) are $F^2(\langle \pi^+ | H | K^+ \rangle + \sqrt{2} \langle \rho^0 | H | K^{*0} \rangle)$ and $(\frac{1}{2})^{1/2} F^2 (\langle \rho^+ | H | K^{*+} \rangle + \sqrt{2} \langle \pi^0 | H | K^0 \rangle)$, respectively, both of which vanish (as is also required by the level-realization pattern) because of Eqs. (42), (49), and (50). Therefore, the whole set of constraints derived from Eqs. (34)—(41), is remarkably consistent with each other and requires the existence of the $|\Delta\vec{I}|=\frac{1}{2}$ rule, Eqs. (42) and (51), as well as the SU(6)-type constraints, Eqs. (49) and (50). Another interesting fact is that the fractions, k and l of the ground states to the algebra Eq. (9a) for the $\lambda=1$ and $\lambda=0$ cases are the same, i.e., $k=l$. This is because from the level realization of the algebra Eq. (3), a relation, $F^2=3(cD+sS)^2$, was obtained in Ref. 26. This relation implies $(g_{\rho^+\pi^-\pi^0}/m_\rho)^2=g_{\omega\pi^-\rho^+}$ consistent with experiment. The pattern found is thus impressive.

The application of the same procedure to case (ii) repeats exactly the same information. For cases (iii) [and (iv)], the level-realization ansatz produces again SU(6)-type constraints for $H=H^{\rm PV}(\Delta C=0,\Delta S=-1)$:

$$\langle\rho^0(\vec{p},\lambda=0)|H|K^0(\vec{p}')\rangle =\langle\pi^0(\vec{p})|H|K^{*0}(\vec{p}',\lambda=0)\rangle\ ,\quad \vec{p}'=\vec{p}\to\infty\ , \tag{52}$$

$$\langle\rho^+(\vec{p},\lambda=0)|H|K^+(\vec{p}')\rangle =\langle\pi^+(\vec{p})|H|K^{*+}(\vec{p}',\lambda=0)\rangle\ ,\quad \vec{p}'=\vec{p}\to\infty\ , \tag{53}$$

as well as the $|\Delta\vec{I}|=\frac{1}{2}$ rule for the asymptotic weak vertices:

$$\sqrt{2}\langle\rho^0(\vec{p},\lambda=0)|H|K^0(\vec{p}')\rangle +\langle\rho^+(\vec{p},\lambda=0)|H|K^+(\vec{p}')\rangle=0\ , \tag{54}$$

$$\sqrt{2}\langle\pi^0(\vec{p})|H|K^{*0}(\vec{p}',\lambda=0)\rangle +\langle\pi^+(\vec{p})|H|K^{*+}(\vec{p}',\lambda=0)\rangle=0\ . \tag{55}$$

The rest of this section is concerned with presenting further SU(6)-type constraints on asymptotic weak vertices involving the helicity $\lambda=0$ states. The following algebra is used for the level realization of asymptotic SU(3) symmetry (see, however, Ref. 23):

$$[H^{\rm PC},A_3]=\tfrac{1}{2}H^{\rm PV}\ . \tag{56}$$

By repeating the same procedure of level realization for cases (i) and (iii) [or equivalently (ii) and (iv)], we obtain, with the aid of Eqs. (49) and (50), and also of Eqs. (52) and (53)

$$\langle\rho^0(\vec{p},\lambda=0)|H^{\rm PC}|K^{*0}(\vec{p}',\lambda=0)\rangle =\pm\langle\pi^0(\vec{p})|H^{\rm PV}|K^{*0}(\vec{p}',\lambda=0)\rangle\ ,\quad \vec{p}'=\vec{p}\to\infty\ , \tag{57}$$

and

$$\langle\rho^+(\vec{p},\lambda=0)|H^{\rm PC}|K^{*+}(\vec{p}',\lambda=0)\rangle =\pm\langle\pi^+(\vec{p})|H^{\rm PV}|K^{*+}(\vec{p}',\lambda=0)\rangle\ ,\quad \vec{p}'=\vec{p}\to\infty\ , \tag{58}$$

respectively. The signs appearing in Eqs. (57) and (58) correspond to the two possible solutions for the asymptotic axial-vector matrix element F, respectively,

$$F=\pm\sqrt{2}l\ , \tag{59}$$

where l is the universal asymptotic fraction of the ground-state contribution given by Eq. (48). We will present the charm counterpart of asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule and other constraints on the asymptotic two-body ground states involving charm mesons in the following section.

V. DERIVATION OF THE CHARM COUNTERPART OF THE ASYMPTOTIC $|\Delta\vec{I}|=\frac{1}{2}$ RULE

We now use the commutator, Eq. (10b), which involves the Cabibbo-angle-favored charm-lowering Hamiltonian $H^{(-,-)}$ and the axial-vector charges A_K and A_π, to derive the charm counterpart of the $|\Delta\vec{I}|=\frac{1}{2}$ rule for the asymptotic matrix elements of $H^{(-,-)}$. We here address to the problem of how to extract a reliable information on the matrix elements involving charm particles in spite of the presence of large symmetry breaking of $SU_f(4)$. The method used in Ref. 14 is less stringent in this respect. In deriving the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule in Sec. IV from Eqs. (9a) and (9b), we needed to deal *only* with the $SU_f(3)$ parametrization of the asymptotic matrix elements of the axial-vector charges A_π. However, in the present case, the realization of Eqs. (10a) and (10b) requires in general the $SU_f(4)$ parametrization[28] of the asymptotic matrix elements of the axial-vector charges A_K in addition to those of the axial charges A_π. The $SU_f(4)$ parametrization of these asymptotic matrix elements will be less accurate than in the case of $SU_f(3)$, even if we take into account the $SU_f(4)$ mixing among the $I=Y=0$ members of $SU_f(4)$ 16-plet thoroughly.[29] However, we find a way to relate the asymptotic two-body matrix elements such as $\langle\pi^+|H^{(-,-)}|F^+\rangle$ and $\langle\bar{K}^0|H^{(-,-)}|D^0\rangle$ *without* confronting with the $SU_f(4)$ parametrization of A_K and A_π.

To see this, we apply the level realization in Eq. (10b) for the asymptotic states (with $\lambda=0$), i.e., (π^+,D^+), (π^0,D^0), (K^+,F^+), and (η,η' and η_c,D^0). Writing out explicitly the ground-state-meson contribution to the level realization of the algebra, Eq. (10b), we obtain $(H\equiv H^{(-,-)})$

$$\langle\pi^+|H|F^+\rangle\langle F^+|A_{K^+}|D^{*0}\rangle\langle D^{*0}|A_{\pi^-}|D^+\rangle-\langle\pi^+|A_{K^+}|\bar{K}^{*0}\rangle\langle\bar{K}^{*0}|H|D^{*0}\rangle\langle D^{*0}|A_{\pi^-}|D^+\rangle + \text{higher-level contributions}=\langle\pi^+|H|F^+\rangle+\langle\bar{K}^0|H|D^0\rangle\ , \tag{60}$$

$$-\langle\pi^0|A_{\pi^-}|\rho^+\rangle\langle\rho^+|H|F^{*+}\rangle\langle F^{*+}|A_{K^+}|D^0\rangle+\langle\pi^0|A_{\pi^-}|\rho^+\rangle\langle\rho^+|A_{K^+}|\bar{K}^0\rangle\langle\bar{K}^0|H|D^0\rangle + \text{higher-level contributions}=\sqrt{2}(\langle\pi^+|H|F^+\rangle+\langle\bar{K}^0|H|D^0\rangle)\ , \tag{61}$$

$$\text{(zero ,i.e., no ground-state contribution)+higher-level contributions}=\langle K^+ \,|\, [[H,V_{K^+}],V_{\pi^-}] \,|\, F^+\rangle=0\ , \tag{62}$$

$$\text{(zero, i.e., no ground-state contribution)+higher-level contributions}=\langle \eta,\eta',\eta_c \,|\, [[H,V_{K^+}],V_{\pi^-}] \,|\, F^+\rangle=0\ . \tag{63}$$

On the RHS of Eqs. (60)—(63) we have already used exact $SU_I(2)$ symmetry and asymptotic $SU_f(3)$ symmetry for the matrix elements of the charge V_K. It is important to realize that the zeros which appear on the LHS of Eqs. (62) and (63) are due to conservation of G parity and the strangeness in the strong couplings involved. Therefore, the validity of Eqs. (62) and (63) is independent of the $SU_f(4)$ parametrization of the asymptotic matrix elements of A_K. Then, by the same pattern recognized in Sec. IV, we immediately conclude that the ground-state contributions on the LHS of Eqs. (60) and (61) must vanish, i.e.,

$$\langle \pi^+ | H | F^+\rangle\langle F^+ | A_{K^+} | D^{*0}\rangle - \langle \pi^+ | A_{K^+} | \bar{K}^{*0}\rangle\langle \bar{K}^{*0} | H | D^{*0}\rangle=0 \tag{64}$$

and

$$\langle \rho^+ | H | F^{*0}\rangle\langle F^{*+} | A_{K^+} | D^0\rangle - \langle \rho^+ | A_{K^+} | \bar{K}^0 | H | D^0\rangle=0\ , \tag{65}$$

and that the RHS of Eqs. (60) and (61) also vanish, i.e.,

$$\langle \pi^+(\vec{p}\,') | H | F^+)\vec{p})\rangle+\langle \bar{K}^0(\vec{p}\,') | H | D^0(\vec{p})\rangle =0,\ \ \vec{p}\,'=\vec{p}\to\infty\ . \tag{66}$$

Equation (66) constitutes the charmed counterpart of the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule, Eq. (42), and plays an important role for the Cabibbo-angle-favored D-meson decays. Equation (66) can then be read as implying the asymptotic $\Delta V=0$ rule[7,30] (conservation of the V-spin and V_3) for the *asymptotic* two-body ground-state-meson weak matrix elements, since (π^+,K^0) and $(F^+,-D^0)$ or $(\bar{D}^0,F^-)$ with infinite momenta may be regarded as the V-spin doublets. Equation (66) is *exact* to the extent of the validity of asymptotic $SU_f(3)$ symmetry under consideration. As for Eqs. (64) and (65), suffice it to say that they read in the asymptotic $SU_f(4)$ limit

$$\langle \pi^+(\vec{p}\,') | H | F^+(\vec{p})\rangle+\langle \bar{K}^{*0}(\vec{p}\,') | H | D^{*0}(\vec{p})\rangle =0\ ,\ \vec{p}\,'=\vec{p}\to\infty\ , \tag{67}$$

and

$$\langle \rho^+(\vec{p}\,') | H | F^{*+}(\vec{p})\rangle+\langle \bar{K}^0(\vec{p}\,') | H | D^0(\vec{p})\rangle =0\ ,\ \vec{p}\,'=\vec{p}\to\infty\ , \tag{68}$$

since the strong axial-vector couplings in Eqs. (64) and (65) are uniquely fixed in this limit. [Equations (67) and (68) could be less accurate than Eq. (66) because of the use of asymptotic $SU_f(4)$ rotation. In Ref. 14, however, all the above sum rules were obtained, on the same footing, using asymptotic $SU_f(4)$ symmetry.] In the same way as above, we can also derive other asymptotic $\Delta V=0$ in addition to Eq. (66), i.e.,

$$\begin{aligned}&\langle \pi^+ | H | F^{*+}\rangle+\langle \bar{K}^0 | H | D^{*0}\rangle=0\ ,\\ &\langle \rho^+ | H | F^+\rangle+\langle \bar{K}^{*0} | H | D^0\rangle=0\ ,\end{aligned} \tag{69}$$

and

$$\langle \rho^+ | H | F^{*+}\rangle+\langle \bar{K}^{*0} | H | D^{*0}\rangle=0\ ,\ \ \vec{p}\,'=\vec{p}\to\infty\ ,$$

and the counterparts of Eqs. (67) and (68) which we do not write down. The relations between the $\lambda=0$ asymptotic matrix elements of H^{PC} and H^{PV} are obtained through the realization of the algebra

$$[H^{PV},A_3]=H^{PC} \tag{70}$$

corresponding to Eqs. (57) and (58), we get

$$\langle \rho^+ | H^{PV} | F^+(\vec{p})\rangle =\pm\langle \pi^+ | H^{PC} | F^+(\vec{p})\rangle\ ,\ \ \vec{p}\to\infty\ , \tag{71}$$

$$\langle \rho^+ | H^{PC} | F^{*+}(\vec{p})\rangle =\pm\langle \pi^+ | H^{PV} | F^{*+}(\vec{p})\rangle\ ,\ \ \vec{p}\to\infty\ , \tag{72}$$

$$\langle \bar{K}^0 | H^{PC} | D^0(\vec{p})\rangle =\pm\langle \bar{K}^0 | H^{PV} | D^{*0}(\vec{p})\rangle\ ,\ \ \vec{p}\to\infty\ . \tag{73}$$

The two possible signs in Eqs. (71)—(73) are due to Eq. (59). In Ref. 14, we took a more naive point of view using asymptotic SU(4) symmetry and also the algebra

$$[H^{(-,-)},V_{D^0}]=\cot\theta_c H^{(0,-)}\ . \tag{74}$$

For the two-particle asymptotic matrix elements, we then obtained

$$\sqrt{2}\langle \pi^0 | H^{(0,-)} | K^0\rangle+\langle \pi^+ | H^{(0,-)} | K^+\rangle =\tan\theta_C(\langle \bar{K}^0 | H^{(-,-)} | D^0\rangle+\langle \pi^+ | H^{(-,-)} | F^+\rangle)\ . \tag{75}$$

We thus see that the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule obtained in Eq. (42) immediately leads to its charm counterpart Eq. (66). With Eq. (74), one therefore may relate the two-particle asymptotic matrix elements of $H^{(0,-)}$ and $H^{(-,-)}$ by using asymptotic SU(4) symmetry. We obtained[14]

$$\langle \bar{K}^0 | H | D^0(\vec{p})\rangle=-\langle \pi^+ | H | F^+(\vec{p})\rangle=-\cot\theta_C\langle \pi^+ | H | K^+(\vec{p})\rangle\ , \tag{76}$$

$$\langle \bar{K}^{*0} | H | D^{*0}(\vec{p})\rangle=-\langle \rho^+ | H | F^{*+}(\vec{p})\rangle=-\cot\theta_C\langle \rho^+ | H | K^{*+}(\vec{p})\rangle\ , \tag{77}$$

$$\langle \bar{K}^0 | H | D^{*0}(\vec{p}) \rangle = -\langle \pi^+ | H | F^{*+}(\vec{p}) \rangle = -\cot\theta_C \langle \pi^+ | H | K^{*+}(\vec{p}) \rangle \ , \tag{78}$$

$$\langle \bar{K}^0 | H | D^0(\vec{p}) \rangle = \langle \bar{K}^{*0} | H | D^{*0}(\vec{p}) \rangle = \pm \langle \bar{K}^0 | H | D^{*0}(\vec{p}) \rangle \ , \ \vec{p} \to \infty \ . \tag{79}$$

VI. CONSTRAINTS ON THE TWO-BODY WEAK MATRIX ELEMENTS INVOLVING HIGHER STATES

So far, we have studied only the ground-state-meson matrix elements, $\langle L=0|H|L=0\rangle$. As will be discussed in the subsequent paper, one needs a knowledge of nondiagonal matrix elements of H, in particular the ones involving the $L=1$ states, i.e., $\langle L=0|H|L=1\rangle$, if one tries to compute the rates of weak processes by using a *new* soft-meson technique which involves a much milder extrapolation than the usual soft-meson approximation. The constraints on $\langle L=0|H|L=1\rangle$ can be obtained by extending the realization procedure discussed in Secs. IV and V to include the $L=1$ as well as $L=0$ states in the intermediate states. Since there are four mesons $(1^{+-},0^{++},1^{++},2^{++})$ in the $L=1$ states, the derivation of the constraints become more intricate and a detailed discussion is beyond the scope of this paper. As demonstrated in Sec. IV the realization is expected to produce constraints not only on the weak matrix elements but also on the asymptotic matrix elements of axial-vector charges involving the $L=0$ and $L=1$ states. Another independent way to derive the constraints is to insert the algebras, Eqs. (9) and (10), between the appropriate $\langle L=1|$ and $|L=1\rangle$ states and study the realization at the $L=0$ level. Although we have not yet carried out a thorough study, the following features found by preliminary study may be noted: (1) As seen from the simple patterns in which the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule and $\Delta V=0$ rule emerged from the level realization of the algebras, all the *diagonal* matrix elements of H, such as $\langle L=0|H|L=0\rangle$, $\langle L=1|H|L=1\rangle, \ldots$, do satisfy these selection rules asymptotically. However, for the nondiagonal asymptotic two-body matrix elements such as $\langle L=0|H|L=1\rangle$, level realization requires that the rules are violated in some cases. This then gives the *explicit* sources of the violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rules, octet rule, and their charm counterparts. (2) Among the diagonal and nondiagonal asymptotic matrix elements of H and the corresponding ones of the axial-vector charge A, we find the following interesting relation (in a rather symbolic notation):

$$\frac{\langle L=0|H^{(0,-)}|L=1\rangle}{\langle L=0|H^{(0,-)}|L=0\rangle} \simeq \frac{\langle L=0|A_\pi|L=1\rangle}{\langle L=0|A_\pi|L=0\rangle} \ . \tag{80}$$

Equation (80) gives us an insight into the magnitude of the nondiagonal matrix elements of H, which will be needed in computing the magnitude of the violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule, etc., for the real processes. The prediction of Eq. (80) is, intuitively speaking, a very plausible simple result.

VII. DERIVATION OF ASYMPTOTIC $|\Delta\vec{I}|=\frac{1}{2}$ AND OCTET RULE IN THE CASE OF BARYONS

Exactly the same procedure can be applied to baryons and recently two of us (K.T. and S.O.) have derived $|\Delta\vec{I}|=\frac{1}{2}$ and octet rule for the *asymptotic ground-state* baryon two-particle matrix elements of $H^{(0,-)}$. Since the details are already published[31] in Physical Review Letters, the result will only be briefly summarized in Sec. II of paper III in this series. Here, we only wish to call the attention of the readers to the analog in the baryon case of the OZI-type selection rules obtained in Sec. IV for the meson case. The rules were found[31] as the constraints on strong vertices along with the asymptotic $|\Delta\vec{I}|=\frac{1}{2}$ rule for the weak vertices.

In the case of baryons the analog of the OZI-type rule, Eq. (21), is the following constraints on the asymptotic matrix elements of A_π:

$$d=(\tfrac{2}{3}k)^{1/2}\ , \ f=-\tfrac{2}{3}(2k)^{1/2}\ ,$$
$$g=-\tfrac{1}{3}(2k)^{1/2}\ , \ h=\pm\tfrac{2}{3}\sqrt{k}\ , \tag{81}$$

with

$$d \equiv \langle \Sigma^- | A_{\pi^-} | \Lambda(\vec{p}) \rangle \ ,$$
$$f \equiv \langle \Sigma^0 | A_{\pi^-} | \Sigma^+(\vec{p}) \rangle \ ,$$
$$g \equiv \langle Y^0 | A_{\pi^-} | Y^+(\vec{p}) \rangle \ ,$$

and $h \equiv \langle \Sigma^0 | A_{\pi^-} | Y^+(\vec{p}) \rangle$, where $\vec{p}\to\infty$. k denotes the fractional contribution of the ground-state baryons to the algebra, Eq. (1). From Eq. (81) and the width of $\Delta\to p\pi$ decay, k can be estimated to be around 0.6. Then Eq. (81) predicts for the value of $g_A(0), g_A(0)=(\frac{5}{3})\sqrt{k}\simeq 1.2$. This demonstrates that the sum rule $g_A(0)=\frac{5}{3}\sqrt{k}$, which produces a correct value of $g_A(0)$, is on the same footing as the OZI-type rule Eq. (21) in the present theoretical framework.

VIII. A NEW PERSPECTIVE TOWARD THE $|\Delta\vec{I}|=\frac{1}{2}$ RULE

The idea of level realization seems to provide a powerful tool to recognize the *pattern* of dynamical constraints in the world of hadrons. It explains, on the same footing, a wide variety of seemingly unrelated important physics as mentioned in Sec. I and the list may grow.

Because of our insufficient knowledge about quark confinement, it is probably a difficult task to assess whether the present approach is compatible with the enhancement of penguins at the hadronic level or not, although the algebras used, Eqs. (9a) and (9b), are compatible with the presence of penguin operators. In any case, our result seems to give a different perspective toward the understanding of the $|\Delta \vec{I}|=\frac{1}{2}$ rule and its charm counterpart from that of the traditional approach.

(1) Our point is that the origin of the observed $|\Delta \vec{I}|=\frac{1}{2}$ rule, etc., may lie in the presence of much *weaker* selection rules, *asymptotic* $|\Delta \vec{I}|=\frac{1}{2}$ and $\Delta V=0$ rule (i.e., the $|\Delta \vec{I}|=\frac{3}{2}$ part, for example, vanishes asymptotically at the zero-four-momentum-transfer-squared limit), for presumably the most important *two-particle ground-state* hadron matrix elements.

In this connection, our observation, the smallness of the isoscalar parts of the nucleon anomalous magnetic moments and also of the isoscalar neutral-current axial-vector couplings can also be explained[11] in exactly the same way, deserves an attention.

(2) The presence of $|\Delta \vec{I}|=\frac{3}{2}$ part in the original Hamiltonian shows up explicitly as the violation of asymptotic $|\Delta \vec{I}|=\frac{1}{2}$ rule in the presumably less important weak two-particle vertices involving higher excited states. This will provide us with an explicit (not necessarily very small) violation of the $|\Delta \vec{I}|=\frac{1}{2}$ rule and also with a much larger violation of the $\Delta V=0$ rule in observed strange- and charm-meson nonleptonic decays. In the succeeding paper (II), we discuss the implications upon the physical processes of the asymptotic selection rules obtained in this paper by using PCAC and the new soft-meson technique which will be developed.

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APPENDIX A: ASYMPTOTIC SU(3) SYMMETRY CONSTRAINTS

Consider the algebra $[H\equiv H^{\rm PC}(\Delta C=0,\Delta S=-1)$ and H^+ denotes the adjoint of $H]$,

$$[[H^+,V_{\bar{K}^0}],V_{\bar{K}^0}]=-2H\ . \tag{A1}$$

Insert Eq. (A1) between the asymptotic states $\langle \rho^0(\vec{p},\lambda=1)|$ and $|K^{*0}(\vec{p}\,',\lambda'=1)\rangle$ with $\vec{p}\,'=\vec{p}\rightarrow\infty$. We then obtain

$$-2\langle\rho^0|H^+|\bar{K}^{*0}\rangle-\{\langle K^{*0}|H^+|\rho^0\rangle-\sqrt{3}\cos\theta_{\omega\phi}\langle K^{*0}|H^+|\phi\rangle-\sqrt{3}\sin\theta_{\omega\phi}\langle K^{*0}|H^+|\omega\rangle\}=-2\langle\rho^0|H|K^{*0}\rangle\ . \tag{A2}$$

With the aid of the charge-conjugation property

$$\langle\rho^0|H|K^{*0}\rangle=\langle\rho^0|H^+|\bar{K}^{*0}\rangle\ , \tag{A3}$$

Eq. (A2) becomes

$$\langle\rho^0|H|K^{*0}\rangle-\sqrt{3}\cos\theta_{\omega\phi}\langle\phi|H|K^{*0}\rangle-\sqrt{3}\sin\theta_{\omega\phi}\langle\omega|H|K^{*0}\rangle=0\ . \tag{A4}$$

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