Higher-twist contributions to high-p_T inclusive meson production in two-photon collisions

J. A. Bagger*
Department of Physics, Princeton University, Princeton, New Jersey 08544

J. F. Gunion
Department of Physics, University of California, Davis, California 95616
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The most important higher-twist subprocess contributing to inclusive single-meson production at high p_T is photon + quark→meson + quark. We consider two-photon collisions and compare this higher-twist contribution to that arising from the leading-twist subprocess, photon + photon →quark + antiquark. We find that the higher-twist subprocess, while not important for direct π production, is significant for ρ production, especially when enhanced by suitable trigger requirements.

As experiments examining high-p_T particle production in two-photon collisions are improved, it becomes important to reassess the various contributions which arise in quantum chromodynamics. Predictions for the higher-twist contributions, originally obtained in Ref. 1 (see also the closely related works of Ref. 2), may now be refined using the exclusive-process QCD formalism developed in Ref. 3. In this paper we discuss photon-photon production of high-p_T jets and π and ρ mesons. We conclude that the higher-twist subprocess cross sections are substantially smaller than estimated in Ref. 1. Nonetheless, the relative magnitude of these subprocesses may be enhanced by two means: by tagging both the e^+ and the e^-, so as to constrain the photon-photon center-of-mass energy to lie substantially above the minimum required for production of a particle at given p_T; and by using a three-jet trigger[1] (see Appendix A), which requires a beam or target jet in addition to the two high-p_T jets. Both requirements discriminate against the $\gamma\gamma\to q\bar{q}$ subprocess relative to the higher-twist reaction.

The formalism for this calculation has been thoroughly developed in Ref. 3. An application similar to the one given here appears in Ref. 4, from which our notation is taken. Conventions regarding the moving coupling constant and techniques for minimizing higher-order corrections are also discussed in that reference. Here we present our final formulas.

The contribution from the minimum-twist subprocess $\gamma\gamma\to q\bar{q}$ is shown in Fig. 1(a). The corresponding inclusive cross section for production of a meson M is given by

$$\left[E\frac{d\sigma}{d^3p}\right]^{\gamma\gamma\to M+X}=\frac{3}{\pi}\sum_{Q=q,\bar{q}}\int_0^1\frac{dz}{z^2}\delta(\hat{s}+\hat{t}+\hat{u})\hat{s}\times D_{M/Q}(z,-\hat{t})\left.\frac{d\sigma}{d\hat{t}}\right|_{\gamma\gamma\to Q\bar{Q}}, \quad (1)$$

where the cross section for producing a quark of given color is

$$\left.\frac{d\sigma}{d\hat{t}}\right|_{\gamma\gamma\to Q\bar{Q}}=\frac{2\pi\alpha^2}{\hat{s}^2}\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right]e_Q{}^4, \quad (2)$$

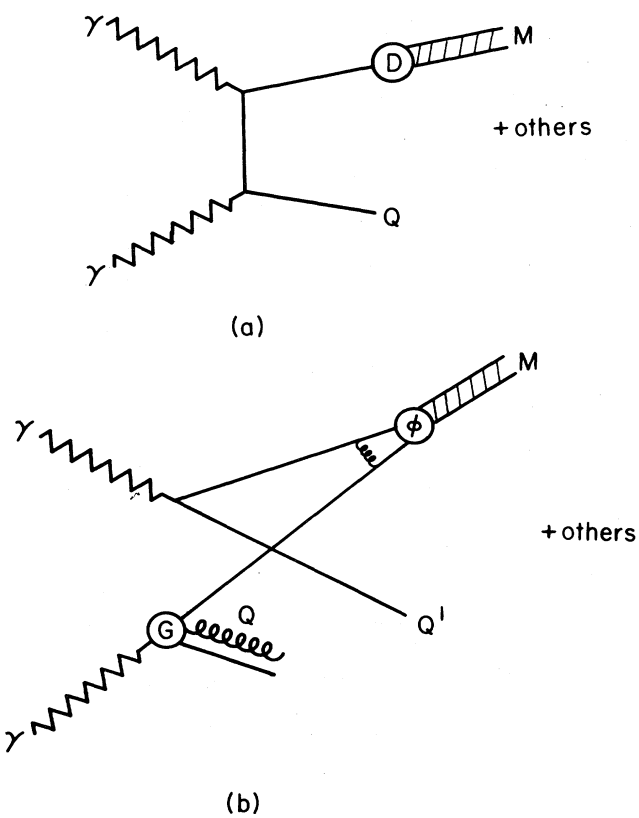

FIG. 1. (a) The minimum-twist contribution to $\gamma\gamma\to MX$. (b) The higher-twist contribution to $\gamma\gamma\to MX$.

and

$$\hat{s}=s\ ,\quad \hat{t}=\frac{t}{z}\ ,\quad \hat{u}=\frac{u}{z}\ . \tag{3}$$

Here s, t, and u refer to the overall $\gamma\gamma\to MX$ reaction. For π^+ production we assume $D_{\pi^+/u}=D_{\pi^+/\bar{d}}$. Equations (1) and (2) give

$$\left[E\frac{d\sigma}{d^3p}\right]^{\gamma\gamma\to\pi^+ +X}=\tfrac{34}{27}\alpha^2\frac{1}{z}D_{\pi^+/u}(z,-\hat{t})\frac{\hat{t}}{\hat{s}^2\hat{u}}+(t\leftrightarrow u)\ , \tag{4}$$

where

$$z=\frac{|t|}{s}+\frac{|u|}{s}\equiv x_t+x_u$$
$$=\frac{2p_T}{\sqrt{s}}\ \text{at}\ 90^\circ\ .$$

In our numerical calculations we used the Feynman-Field parametrization[5] for $D_{\pi^+/u}$. As discussed in Ref. 4, the experimental data can also be used for $D_{\pi^+/u}$, with little effect upon our results. The ρ-production cross sections presented later use the Feynman-Field parametrization for $D_{\rho^+/u}$, as given in Refs. 4 and 5. Note that D/z behaves as $1/z^2$ as $z\to 0$. For the kinematic range considered in our numerical calculations, D/z increases even more rapidly. Noteworthy features of the final cross section, Eq. (4), are as follows.

(1) At fixed p_T, the cross section decreases with s asymptotically as $1/s$ (but less rapidly in the kinematic region of our calculations). If the e^+ and e^- are not tagged, the cross section (4) must be convoluted with distribution functions G_{γ/e^-} and G_{γ/e^+}. In this case the inclusive spectrum approaches a constant as $s\to\infty$ at fixed p_T (Feynman scaling). This occurs because the dominant contribution to the cross section comes from events for which the subprocess energy $\hat{s}$ is of the order of $4p_T^2$, the minimum consistent with energy-momentum conservation.

(2) At fixed s, the D function causes the cross section to decrease rapidly as p_T increases towards the phase-space boundary ($z\to 1$). The rest of the cross section varies slowly at fixed s. As s increases, the phase-space boundary moves to higher p_T, and the p_T distribution broadens.

The higher-twist subprocess $\gamma q\to Mq$ contributes to $\gamma\gamma\to MX$ through the diagram of Fig. 1(b). The cross section is expressed as

$$\left[E\frac{d\sigma}{d^3p}\right]^{\gamma\gamma\to M+X}=\frac{3}{\pi}\sum_{Q=q,\bar{q}}\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})\hat{s}\,G_{Q/\gamma}(x,-\hat{t})\times\left.\frac{d\sigma}{d\hat{t}}\right|_{\gamma Q\to MQ'}+(t\leftrightarrow u)\ . \tag{5}$$

Here $G_{Q/\gamma}$ is the per color distribution function for a quark in a photon. The subprocess cross section for π^+ production[4]

$$\left.\frac{d\sigma}{d\hat{t}}\right|_{\gamma Q\to\pi^+Q'}=\frac{8\pi^2\alpha C_F}{9}[\Delta(\hat{s},\hat{u},e_Q,e_{Q'})]^2\times\frac{1}{\hat{s}^2}\frac{1}{(-\hat{t})}\left[\frac{1}{\hat{s}^2}+\frac{1}{\hat{u}^2}\right] \tag{6}$$

includes the full gauge-invariant set of amplitudes. In (5) and (6), the subprocess invariants are

$$\hat{s}=xs\ ,\quad \hat{t}=t\ ,\quad \hat{u}=xu\ , \tag{7}$$

and

$$\Delta(\hat{s},\hat{u},e_Q,e_{Q'})=\hat{u}e_Q\alpha_s\left[\frac{\hat{s}}{2}\right]I_\pi\left[\frac{\hat{s}}{2}\right]+\hat{s}e_{Q'}\alpha_s\left[-\frac{\hat{u}}{2}\right]I_\pi\left[-\frac{\hat{u}}{2}\right]\ . \tag{8}$$

The I_π factors reflect the exclusive form factor of the pion and are discussed thoroughly in Ref. 4, as is the motivation behind the arguments of α_s and I_π. Note that the relation between I_π and the pion form factor completely fixes the normalization of the higher-twist subprocess.

The full cross section for π^+ production is given by

$$\left[E\frac{d\sigma}{d^3p}\right]^{\gamma\gamma\to\pi^+ +X}=\left[\frac{x}{1-x_u}\right]\sum_{q=u,\bar{d}}G_{q/\gamma}(x,-\hat{t})\frac{8\pi\alpha C_F}{3}\frac{\Delta^2(\hat{s},\hat{u},e_q,e_{q'})}{\hat{s}^2(-\hat{t})}\left[\frac{1}{\hat{s}^2}+\frac{1}{\hat{u}^2}\right]\Bigg|_{x=x_t/(1-x_u)}+(t\leftrightarrow u)\ . \tag{9}$$

Corresponding results hold for ρ-meson production. The subprocess cross section for longitudinal-ρ production is very similar to that for π production, but its overall normalization is a factor of 2.5 higher. The transverse-ρ subprocess cross section is numerically larger and has a quite different form—see Ref. 4 for details.

The form for $G_{q/\gamma}$ is taken from Ref. 6. One should recall that a typical $G_{q/\gamma}(x)$ vanishes very slowly (logarithmically) as $x\to 1$. It is flat over the range $0.2<x<0.8$, and behaves as $1/x$ as $x\to 0$. These facts, together with the form of Eq. (9), result in the following features of the higher-twist cross section.

(1) At fixed p_T, the cross section falls very slowly with s.

(2) At fixed s, the cross section decreases[7] as $1/p_T^5$, multiplied by a slowly varying logarithmic function which vanishes at the phase-space boundary. Thus, the p_T spectrum is fairly independent of s except near the kinematic limit.

Our results for π^+ production are plotted in Figs.

2(a)—2(c). The π^- cross sections are, of course, identical. In Fig. 2 we see that at low energies the higher-twist and minimum-twist (HT and MT) cross sections fall with p_T at about the same rate. The overall normalization of the higher-twist contribution decreases slowly with energy. The p_T behavior, however, remains almost the same. In contrast, as s increases, the minimum-twist cross section decreases significantly at the lower p_T values, and the falloff in p_T [controlled primarily by $z \propto p_T/\sqrt{s}$, see (4)] becomes less rapid.

In the case of inclusive π^+ production, the minimum-twist contribution dominates the higher-twist piece by a factor of 20 at the lowest p_T value and highest s value considered, $p_T=2.0$ GeV/c and $\sqrt{s}=25$ GeV. In less favorable kinematic regions, the factor is even greater. Even with a three-jet trigger it seems unlikely that the higher-twist process can be isolated.

In the case of ρ^+ production [Figs. 3(a)—3(c)], the relative magnitude of the higher-twist contribution is much larger. This is partly due to the extra spin degrees of freedom and partly due to the more favorable normalization of the ρ subprocess cross sections (see Ref. 4). We find

$$\left[\frac{\mathrm{HT}}{\mathrm{MT}}\right]_{p_T=2.5\ \mathrm{GeV}/c} \cong \begin{cases} \frac{1}{4}, & \sqrt{s}=7\ \mathrm{GeV} \\ 2, & \sqrt{s}=25\ \mathrm{GeV}\ . \end{cases} \tag{10}$$

Equation (10) illustrates the fact that the higher-twist subprocess may be enhanced by tagging the e^+ and e^- so as to fix $\sqrt{s}$ to be much larger than kinematically required for a given p_T.

Current experiments do not perform such tagging. To compare with their results, the two-photon cross sections presented here must be convoluted with the distribution functions $G_{\gamma/e^+}(z^+)$ and $G_{\gamma/e^-}(z^-)$ for photons in the incoming e^+ and e^- beams. Under these conditions, the largest contributions arise when z^+ and z^- take on values such that the subprocess energies are only somewhat larger than the minimum required to produce the observed high-p_T particle. Thus, for both the higher-twist and minimum-twist reactions, $\langle s_{\gamma\gamma}\rangle \sim a(4p_T^2)$, $\langle t_{\gamma\gamma}\rangle \sim b(2p_T^2)$, and $\langle u_{\gamma\gamma}\rangle \sim c(2p_T^2)$, where a, b, and c are somewhat larger than 1, and are determined by the distribution functions and subprocess cross sections being convoluted;

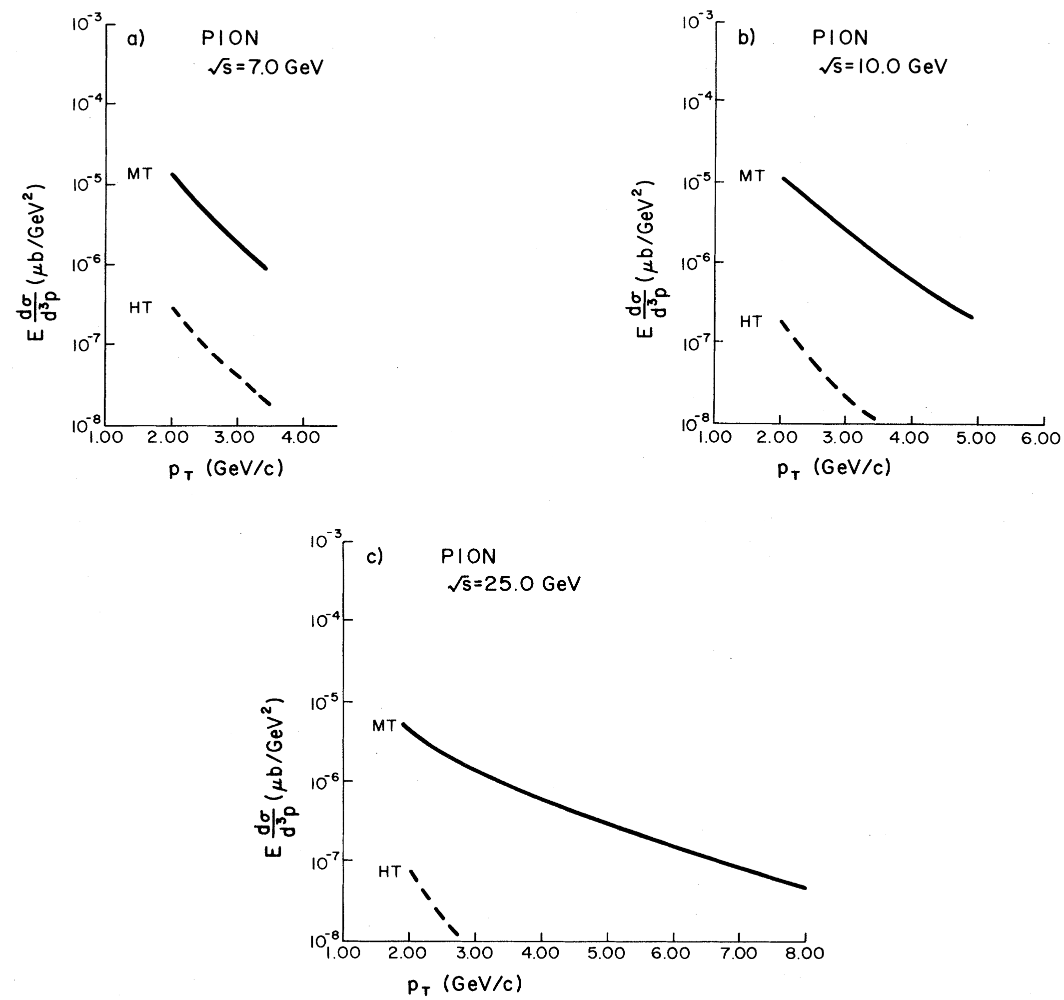

FIG. 2. Direct π^+ production at zero rapidity (90°) as a function of p_T at (a) $\sqrt{s}=7$ GeV, (b) $\sqrt{s}=10$ GeV, and (c) $\sqrt{s}=25$ GeV. Dashed curves are higher-twist contributions. Solid curves are minimum-twist contributions.

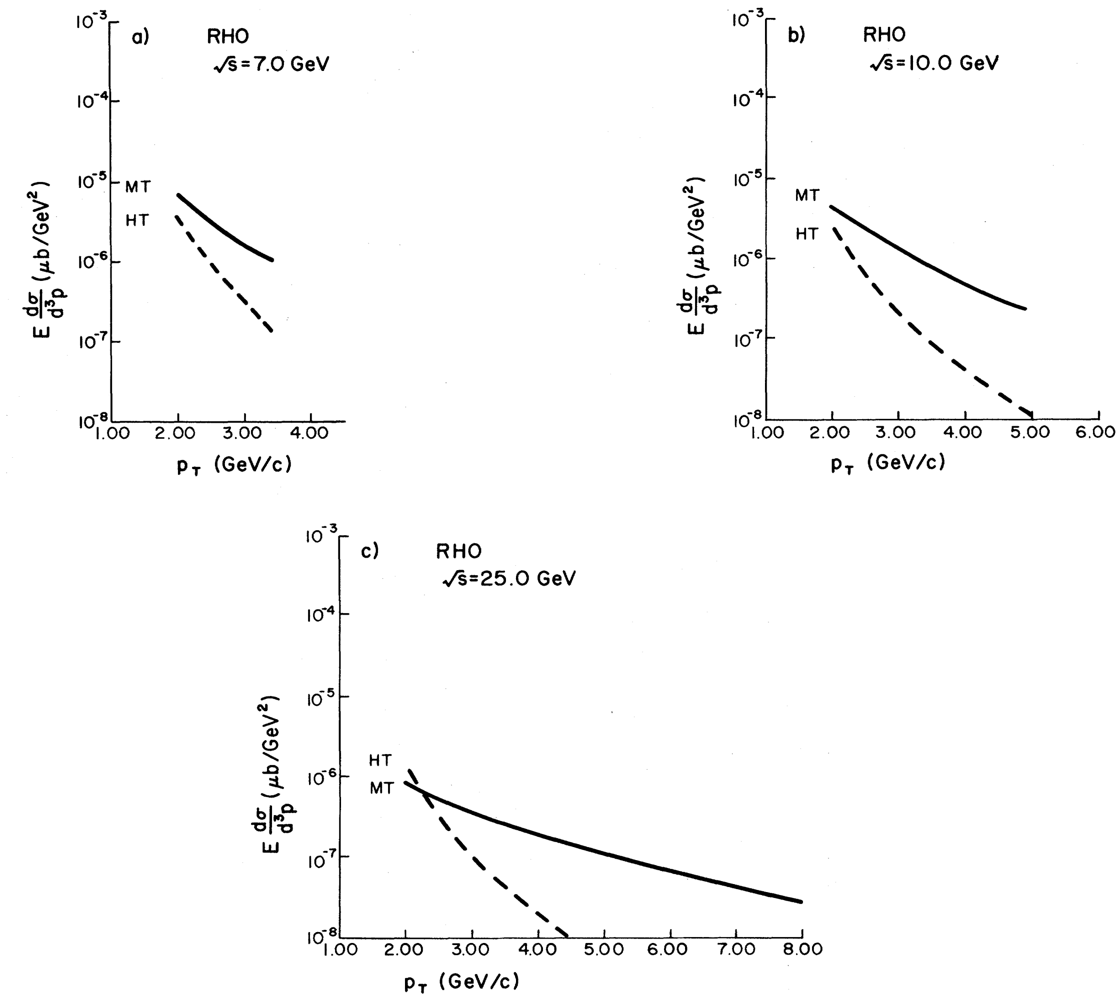

FIG. 3. Direct ρ^+ production at zero rapidity (90°) as a function of p_T at (a) $\sqrt{s}=7$ GeV, (b) $\sqrt{s}=10$ GeV, and (c) $\sqrt{s}=25$ GeV. Dashed curves are higher-twist contributions. Solid curves are minimum-twist contributions.

see Ref. 1 for more details. For instance, at $p_T=2.5$ GeV/c, the average $\sqrt{s_{\gamma\gamma}}$ value is closer to 10 than to 25 GeV. Using these values as an example, we find

$$\left[\frac{\text{HT}}{\text{MT}}\right]_{\pi^+\ \text{direct}} \sim 0.03\ , \qquad \left[\frac{\text{HT}}{\text{MT}}\right]_{\rho^+\ \text{direct}} \sim 0.3 \tag{11}$$

to be more representative of untagged events.

These estimates, based upon the direct calculations of this paper, can be confirmed by modifying the results of Ref. 1 as follows. In Ref. 1 the higher-twist subprocess is normalized through a dimensional coupling

$$\frac{1}{3}\frac{g^2}{4\pi} \sim 2\ \text{GeV}^2\ .$$

Comparing Eq. (3.20) of Ref. 1 to Eqs. (6) and (8), we see that

$$\frac{1}{3}\frac{g^2}{4\pi} \sim \frac{8\pi C_F}{9}\langle(\alpha_s I_\pi)^2\rangle\ . \tag{12}$$

A typical value for $(\alpha_s I_\pi)^2$ is of the order 0.004 GeV^2. This results in a corrected value of

$$\frac{1}{3}\frac{g^2}{4\pi} \sim 0.015\ \text{GeV}^2\ .$$

Therefore, the higher-twist estimates given in Ref. 1 are too large by a factor of ~ 130.[8] At $\sqrt{s_{e^+e^-}}=30$ GeV, Figs. 7 and 9 of Ref. 1 show that the ratio of higher-twist to minimum-twist direct π^+ production is approximately 5 at the most favorable p_T value, $p_T=2$ GeV/c. The result must now be corrected by a factor of $\frac{1}{130}$, giving

$$\left[\frac{\text{HT}}{\text{MT}}\right]_{\pi^+\ \text{direct}} \underset{\text{no tagging}}{\sim} 0.04$$

$$(\sqrt{s_{e^+e^-}}=30\ \text{GeV}\ ,\ p_T=2\ \text{GeV}/c)\ . \tag{13}$$

By p_T of 5 GeV/c, this ratio is below 1%. For single direct ρ^+ production the relative higher-twist contribution is about 10 times larger.

The results of Ref. 1, with the corrected value of $\frac{1}{3}g^2/4\pi \sim 0.015$ GeV^2, can also be used to estimate the higher-twist contribution to inclusive single-jet production (see Figs. 6 and 8 of Ref. 1). It is necessary to incorporate the fact that higher-twist ρ production is larger relative to higher-twist π production by more than the spin-statistical factor of 3 assumed in Ref. 1. We find a ratio of ~ 10. Assuming this holds for all vector mesons relative to their pseudoscalar counter parts, we conclude

$$\left[\frac{\mathrm{HT}}{\mathrm{MT}}\right]_{\mathrm{jet}} \sim 0.1 \quad (\sqrt{s_{e^+e^-}}=30\ \mathrm{GeV}\ ,\ p_T=2\ \mathrm{GeV}/c)\ . \qquad (14)$$

By p_T of 5 GeV/c (at $\sqrt{s_{e^+e^-}}=30$ GeV), this ratio decreases to approximately 2%. A three-jet trigger would significantly increase this percentage.

Current two-photon data for inclusive single-particle and jet production (with no tagging) exhibit more events at low $p_T \gtrsim 2$ GeV/c than are expected from minimum-twist predictions.[9] The above estimates indicate that higher-twist subprocesses considered here are unable to account for this excess.

In conclusion, it will be difficult to study the higher-twist subprocesses in photon-photon collisions unless one can isolate ρ- (or vector-) meson production in the presence of a three-jet trigger bias. Even for a three-jet trigger, there will be some background from the $\gamma q \to gq$ subprocess[1] (see Appendix A). The relative size of the higher-twist piece is larger in photon-hadron collisions. However, because the distribution function of a quark in a hadron is more complex than that of a quark in a photon, direct extraction of the higher-twist subprocess may be difficult. If trigger biases sufficient to isolate the higher-twist subprocess are possible, then photon-photon collisions could prove to be the most direct probe of exclusive reactions in QCD.

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APPENDIX A: THE THREE-JET TRIGGER

Since the three-jet trigger may play a substantial role in bringing higher-twist subprocesses to an observable level in $\gamma\gamma$ collisions, we devote a few paragraphs to remind the reader of its nature and use in the high-p_T environment.

The basic point is simple. Each QCD high-p_T subprocess produces a characteristic event structure. For instance, when the meson produced by the $\gamma\gamma \to q\bar{q}$ collision subprocess of Fig. 1(a) has a large momentum transverse to the $\gamma\gamma$ center-of-mass collision axis (essentially the same as the e^+e^- collision axis since the colliding photons are predominantly collinear with the e^+ and e^-), the other quark jet produced by the subprocess must also carry a large transverse momentum. Thus, for this subprocess the primordial final-state particles are confined to jets with large transverse momenta relative to the collision axis. No jets are produced along either beam axis. There will be hadrons, produced as part of both of the large-p_T jets, which are very slow and, because of fluctuations in their transverse momenta about their parent-jet axis, may move slowly along the $\gamma\gamma$ center-of-mass axis. But they will not form a distinguishable jet. Analysis of such an event in the $\gamma\gamma$ center of mass using two-jet techniques of the type employed for e^+e^- collisions ($e^+e^- \to q\bar{q}$) should yield an entirely satisfactory description.

In contrast, the higher-twist subprocess of Fig. 1(b) produces a three-jet topology. The target photon breaks into a collinear quark which participates in the high-p_T subprocess and a (collinear) residue (antiquark + gluons + $\cdots$). The quark is scattered to high p_T by the other (active) photon, producing a high-p_T meson and a balancing quark jet [Q' of Fig. 1(b)].

Thus, the primordial final state consists of (i) a high-p_T meson, (ii) a balancing high-p_T jet, and (iii) a residue jet along the target-photon direction. Analysis of such an event using a two-jet assumption would yield a very poor fit—one would discover the need to hypothesize three jets (one composed of only one or very few particles), just as required for the $e^+e^- \to q\bar{q}g$ annihilation topology.

Another process is that in which both incoming photons break into collinear quarks + residue jets, followed by high-p_T scattering of the two collinear quarks. This produces a topology in which there are four jets—two balancing high-p_T jets and two residue jets in opposite directions along the $\gamma\gamma$ c.m. axis. Such an event can only be adequately described by a four-jet hypothesis.

There are very few subprocesses for which the final-state topology is of the three- (and only three) jet type given by the higher-twist subprocess of Fig. 1(b). The percentage of events arising from this subprocess is clearly enhanced by requiring a three-jet topology. The only competing subprocess is that in which the meson of Fig. 1(b) is replaced by a gluon. Because the gluon jet is not efficient in producing single particles carrying a large fraction of the total jet momentum, it is possible for the direct-production mechanism of Fig. 1(b) to compete provided one demands a large p_T for the single meson. The three-jet events should fall into two categories: (i) those in which the high-p_T transverse jets are each composed of a significant number of particles sharing the total transverse momentum of the jet, and (ii) those in which one transverse jet consists of one high-momentum particle with no or very few accompanying hadrons. Events of the type (ii) should receive a substantial contribution from the higher-twist subprocess of Fig. 1(b).

*Present address: Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305.

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[7]The $1/p_T{}^5$ behavior follows from Eq. (9) in the region where $x \sim x_t \sim x_u \sim p_T/\sqrt{s}$ is small. There $\hat{s} \sim p_T\sqrt{s}$, $\hat{t} \sim p_T\sqrt{s}$, $\hat{u} \sim p_T{}^2$, and $G_{q/\gamma} \sim 1/(p_T/\sqrt{s}\,)$.

[8]This factor is very similar to that obtained earlier by G. R. Farrar and G. Fox, Nucl. Phys. B167, 205 (1980) in their work on the $qM \rightarrow qM$ higher-twist subprocess.

[9]See, for example, R. J. Wedemeyer, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Universität Bonn, Bonn, 1981), p. 410; D. Burke, in *Proceedings of the 21st International Conference on High Energy Physics, Paris,* 1982, edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. 43 (1982)].