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Dirac- and Majorana-neutrino-mass effects in neutrino-electron elastic scattering

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A covariant formulation is given for the mass-dependent differential cross sections for neutrino (antineutrino)-electron elastic scattering with either Dirac or Majorana neutrinos. It is explained how these cross sections along with a formulation for neutrino oscillations may be used to describe the helicity-transformation effect for neutrinos passing through matter.

I. INTRODUCTION

In recent years much effort has been devoted to developing a better understanding of the properties of massive neutrinos[1] and their relation to the gauge theory of electroweak forces. In this paper, I investigate phenomena which are associated with the scattering in matter of massive Dirac or Majorana neutrinos from electrons.

An important consequence of the massiveness of neutrinos is the prediction of oscillations which can occur both in vacuum and in matter. Among the oscillations which have been considered are those which change the lepton type (flavor changes),[2] particle-antiparticle oscillations,[3] helicity changes, and doublet-singlet neutrino changes.[4] Other consequences are also associated with massive neutrinos. These include mass-dependent effects in the scattering cross sections and changes in the direction of the neutrino's spin-polarization vector as the result of scattering or as the result of interacting with a strong magnetic field.[5]

In this paper, I give invariant cross sections for the elastic scattering of both massive Dirac and Majorana neutrinos from electrons. From these expressions, one may determine the changes in the spin polarization of the neutrino as the result of scattering from electrons in matter. This effect is shown to be enhanced in the presence of neutrino oscillations. An estimate is made for the helicity transformations which can occur for neutrinos passing through matter. The transformation effect depends on the ratio of the neutrino mass to the target mass, and it is suppressed for scattering from nucleons. It is suggested that scattering-induced helicity transformations may appear in certain astrophysical environments.

II. NEUTRINO-MASS-DEPENDENT CROSS SECTIONS

In this section I present the details of a derivation of the cross sections for the elastic scattering of a massive neutrino from an electron. In the derivation, I use the units $m_e=\hbar=c=1$ and the conventions which may be found in Ref. 6. I consider the process in which a neutrino of mass m, polarization four-vector s_a, and four-momentum a is scattered from an electron to a final state of spin polarization s_c and four-momentum c. Initially, the electron is unpolarized and has four-momentum b. Its final-state four-momentum is d.

The neutrino and the electron are described, respectively, by the current four-vectors

$$j_\nu(c,a)^\mu=\bar{\nu}(c)\Gamma(\sigma,\lambda)^\mu\nu(a)\ , \tag{2.1a}$$

$$j_e(d,b)^\mu=\bar{u}(d)\Gamma(V,A)^\mu u(b)\ , \tag{2.1b}$$

where

$$\Gamma(V,A)^\mu=\gamma^\mu(V+A\gamma^5)/2\ . \tag{2.2}$$

In (2.1a) for the Dirac case $\sigma=1$ and λ has the value 1 for a right-handed projection and the value -1 for a left-handed projection of the neutrino's helicity. For the elastic scattering of a Dirac neutrino produced by a $V-A$ process, these values of λ are also associated with the antineutrino and the neutrino, respectively. For a Majorana neutrino[7] where $\nu(p)=C\bar{\nu}^\tau(-p)$ (C represents charge conjugation), $\sigma=0$, $\lambda=1$, and (2.1a) is multiplied by 2. Although the Dirac and the Majorana neutrinos behave similarly in the low-neutrino-mass limit for elastic scattering, they behave differently in matter. A right-handed Dirac neutrino will not behave as an antineutrino. As pointed out by Kayser and Shrock in Ref. 1, lepton-number conservation will allow a left-handed Dirac neutrino to produce an e^-, μ^-, or τ^- in a charged-current weak interaction in matter; however, a right-handed Dirac neutrino will not produce an antilepton of the associated flavor as would be the case for the antineutrino. Furthermore a Dirac neutrino (antineutrino) produced in a $V-A$ process appears with a left-handed (right-handed) projection operator so that the interaction of a helicity reversed Dirac neutrino is highly suppressed. On the other hand, a Majorana neutrino, where $\nu=\bar{\nu}$, can produce in matter either its associated lepton or antilepton depending upon its spin orientation and the Lorentz properties of the charged current. In the remainder of this section, I will assume that neutrinos are produced from $V-A$ processes and described by the standard model so that they are predominantly left-handed.

In the standard model of electroweak interactions,[8] modified so as to include massive neutrinos, the electron neutrino interacts with both the neutral Z and with the charged W vector bosons. The effects for both the neutral and the charged vector bosons may be accounted for with different values for the constants V and A in (2.2). The interaction amplitude for the process under consideration is

$$\mathscr{L}(s_a,s_c)=\frac{g}{\pi\sqrt{2}}j_\nu(c,a)\cdot Z(d,b)\ , \quad (2.3)$$

where

$$Z(d,b)^\mu=\frac{4\pi}{(q^2-M^2)}(g^{\mu\nu}-q^\mu q^\nu/M^2)j_e(d,b)_\nu\ , \quad (2.4)$$

with $q=a-c$. The invariant differential cross section with the amplitude (2.3) for the process $\nu+e\to\nu+e$ becomes

$$\frac{d\sigma}{dt}(s_a,s_c,\lambda,m)=\frac{2}{4\pi f(s,a,b)}\mathscr{M}(s_a,s_c,\lambda,m)\ , \quad (2.5)$$

where

$$f(s,a,b)=4((a\cdot b)^2-m^2)\ ,$$

$$4s=(a+b)^2\ ,\ \ 4t=(a-c)^2\ ,$$

and

$$2(s+t+u)=m^2+1\ .$$

In (2.5), the polarization function is

$$\mathscr{M}(s_a,s_c,\lambda,m)=8G^2\mathrm{Tr}[\rho_c\Gamma(\sigma,\lambda)^\mu\rho_a\overline{\Gamma}^\nu(\sigma,\lambda)] \times\mathrm{Tr}[\rho_d\Gamma(V,A)_\mu\rho_b\overline{\Gamma}(V,A)_\nu]\ , \quad (2.6a)$$

with

$$\overline{\Gamma}=\gamma^0\Gamma^\dagger\gamma^0\ ,$$

where ρ_i ($i=a$, b, c, or d) denotes the polarization density matrices. Upon evaluation,[9] one finds in the large-M limit for the Dirac case

$$\begin{aligned}\mathscr{M}(s_a,s_c,\lambda,m)_D=\ &2G^2\{(a\cdot b)^2+(a\cdot d)^2+m^2[(s_c\cdot d)(s_a\cdot b)+(s_c\cdot b)(s_a\cdot d)]\\&-\lambda m[(s_c\cdot d)(a\cdot b)+(s_c\cdot b)(a\cdot d)]-\lambda m[(a\cdot b)(s_a\cdot b)+(a\cdot d)(s_a\cdot d)]\}(|V|^2+|A|^2)\\&+2G^2[-(a\cdot c)-m^2(s_a\cdot s_c)+\lambda m(s_c\cdot a)+\lambda m(s_a\cdot c)](|V|^2-|A|^2)\\&-2G^2\{-\lambda[(a\cdot b)^2-(a\cdot d)^2]-\lambda m^2[(s_c\cdot d)(s_a\cdot b)-(s_c\cdot b)(s_a\cdot d)]\\&+m[(s_c\cdot d)(a\cdot b)-(s_c\cdot b)(a\cdot d)]+m[(a\cdot b)(s_a\cdot b)-(a\cdot d)(s_a\cdot d)]\}\\&\times(VA^*+V^*A)\ . \end{aligned} \quad (2.6b)$$

For the case of the Majorana neutrino, the polarization function is found from four times (2.6a) when $\lambda=1$ and $\sigma=0$. Upon evaluation of the traces, one finds

$$\begin{aligned}\mathscr{M}(s_a,s_c)_M=4G^2\{&(|V|^2+|A|^2)[(a\cdot b)^2+(a\cdot d)^2+m^2(d\cdot b)](1+s_a\cdot s_c)\\&-(|V|^2-|A|^2)[2m^2+a\cdot c+m^2(s_a\cdot s_c)]\\&-2VAma\cdot(b+d)[s_a\cdot c+s_c\cdot a]\\&-(|V|^2+|A|^2)[s_c\cdot a(s_a\cdot da\cdot d+s_a\cdot ba\cdot b)+s_a\cdot c(s_c\cdot da\cdot b+s_c\cdot ba\cdot d)\\&-(m^2+a\cdot c)(s_a\cdot ds_c\cdot b+s_a\cdot bs_c\cdot d-s_a\cdot s_cd\cdot b)-s_c\cdot as_a\cdot cd\cdot b]\}\ . \end{aligned} \quad (2.6c)$$

Although information for all scattering configurations with different values for s_a and s_c can be found from (2.6), a considerable simplification occurs when the neutrino mass is small relative to its energy ($m/\omega_a<1$). With this approximation, one finds for the scattering of helical Dirac neutrinos

$$\begin{aligned}\mathscr{M}(\lambda_a,\lambda_c,\lambda,m)\cong\ &2G^2(1+\lambda_a\lambda_c-\lambda\lambda_a-\lambda\lambda_c)\{[(a\cdot b)^2+(a\cdot d)^2](|V|^2+|A|^2)-(a\cdot c)(|V|^2-|A|^2)\\&+\lambda[(a\cdot b)^2-(a\cdot d)^2](VA^*+V^*A)\}\\&+m^2 2G^2\{(\lambda-\lambda_c)\lambda_a[f_a(b)a\cdot b+f_a(d)a\cdot d]\\&+(\lambda-\lambda_a)\lambda_c[f_c(d)a\cdot b+f_c(b)a\cdot d]\}(|V|^2+|A|^2)\\&-m^2 2G^2[(\lambda-\lambda_c)\lambda_a f_a(c)+(\lambda-\lambda_a)\lambda_c f_c(a)](|V|^2-|A|^2)\\&-m^2 2G^2\{(\lambda\lambda_c-1)\lambda_a[f_a(b)a\cdot b-f_a(d)a\cdot d]\\&+(\lambda\lambda_a-1)\lambda_c[f_c(d)a\cdot b-f_c(b)a\cdot d]\}(VA^*+V^*A)\ . \end{aligned} \quad (2.7)$$

If one follows the same approximation procedure, a similar expression for the Majorana case can be easily obtained from (2.6c). In obtaining the approximation (2.7), I have used the representation

$$s_p=\lambda_p\left[\frac{|\vec{p}|}{m},\frac{\omega_p}{m}\vec{e}_p,0,0\right] \tag{2.8}$$

for the polarization four-vector of a particle of mass m, energy ω_p, and momentum $|\vec{p}|\vec{e}_p$. I have also defined the scalar products

$$\begin{aligned} s_p\cdot q &\cong \frac{\lambda_p}{m}[p\cdot q-m^2 f_p(q)] , \\ s_a\cdot s_c &\cong \frac{\lambda_a\lambda_c}{m^2}[a\cdot c-m^2 f_c(a)-m^2 f_a(c)] , \end{aligned} \tag{2.9}$$

where

$$f_p(q)=\frac{\omega_q}{2\omega_p}+\frac{|\vec{q}|}{2|\vec{p}|}\vec{e}_p\cdot\vec{e}_q .$$

The scattering of a helical Dirac neutrino without a change in helicity is described with the values $\lambda=-\lambda_a-\lambda_c=1$. The corresponding scattering for an antineutrino is described when these parameters have the opposite values. The case of a helicity transformation is described when $\lambda_c=-\lambda_a$.

In the standard model, the parameters V and A have the values

$$\begin{aligned} V&=\tfrac{1}{2}+2\sin^2\theta_W , \quad \sin^2\theta_W\cong 0.23 , \\ A&=\tfrac{1}{2} , \end{aligned} \tag{2.10a}$$

for the scattering of a neutrino and a charged lepton from the same flavor family where both neutral and charged vector bosons contribute. For the scattering of a neutrino and a charged lepton from a different flavor family, when only the neutral vector boson contributes, the parameters have the values

$$\begin{aligned} V&=-\tfrac{1}{2}+2\sin^2\theta_W , \\ A&=-\tfrac{1}{2} . \end{aligned} \tag{2.10b}$$

As a check on the formulas, it is easy to see that the cross section for the scattering of a massless neutrino can be recovered in the limit $m\to 0$. If one introduces the variable

$$y=b\cdot(a-c)/a\cdot b ,$$

then

$$\frac{d\sigma}{dy}(V,A)=-(\omega_a/2)\frac{d\sigma}{dt}(V,A) . \tag{2.11a}$$

Upon integrating y from 0 to 1, one finds the total cross section for elastic electron-neutrino scattering:

$$\sigma(V,A)=\frac{\omega_a G^2}{2\pi}\left[|V+A|^2+\frac{|V-A|^2}{3}-\frac{(|V|^2-|A|^2)}{2\omega_a}\right] . \tag{2.11b}$$

The result for antineutrino scattering is found from the above expression when $A\to -A$. This result in the mass-zero limit is the same for both the Dirac and the Majorana neutrinos.

The differential cross sections in the center-of-mass system which are a special case of (2.6) when the spin of the scattered neutrino is undetected have appeared in the cited work of Kayser and Shrock. Their equations (3.6) and (3.7) can be recovered from (2.6) when $s_c=0$ and when the differential cross section is multiplied by a factor of 2 to account for the summation over the spin of the final state neutrino. In the center-of-mass system, the differential cross section is given by

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_a,0,m)=\frac{1}{16\pi^2 S}\mathscr{M}(s_a,0,m)_{D,M} , \tag{2.12a}$$

where $S=4s=(a+b)^2$. The polarization functions for the Dirac and Majorana cases are, respectively,

$$\mathscr{M}(s_a,0,1,m)_D=2G^2\{[(a\cdot b)^2-a\cdot bms_a\cdot b](V+A)^2+[(a\cdot d)^2-a\cdot dms_a\cdot d](V-A)^2+[ms_a\cdot c-a\cdot c](V^2-A^2)\} , \tag{2.12b}$$

$$\mathscr{M}(s_a,0,m)_M=4G^2\{[(a\cdot b)^2+(a\cdot d)^2+m^2(d\cdot b)](|V|^2+|A|^2)-(2m^2+a\cdot c)(|V|^2-|A|^2)-2VAma\cdot(b+d)s_a\cdot c)\} \cdot \tag{2.12c}$$

In the center-of-mass system, one has

$$\begin{aligned} &\omega_a=\omega_c , \quad \omega_b=\omega_d , \quad \vec{a}=|\vec{a}|\vec{e}_a , \quad \vec{c}=|\vec{a}|\vec{e}_c , \quad |\vec{a}|=|\vec{b}|=|\vec{c}|=|\vec{d}| , \\ &\vec{e}_c=\cos\theta\,\vec{e}_a+\sin\theta\sin\phi\,\vec{e}_2+\sin\theta\cos\phi\,\vec{e}_1 , \\ &a\cdot b=\omega_a\omega_b+|\vec{a}|^2 , \quad a\cdot c=\omega_a{}^2-|\vec{a}|^2\cos\theta , \quad a\cdot d=\omega_a\omega_b+|\vec{a}|^2\cos\theta , \quad \theta=\theta_{\text{c.m.}} , \\ &s_a=\left[\frac{|\vec{a}|}{m}\xi_{||},\frac{\omega_a}{m}\xi_{||}\vec{e}_a,\xi_{\perp}\vec{e}_1\right] . \end{aligned} \tag{2.13}$$

The polarization three-vector in the rest frame of the incident neutrino is

$$\vec{\xi}_a = \xi_{\parallel}\vec{e}_a + \xi_{\perp}\vec{e}_1 .$$

III. OSCILLATION EFFECTS

As a massive neutrino passes through matter, its helicity may be reversed as the result of interacting with electrons. As one can see from the previous discussion, this effect has the energy dependence m^2/ω_a and is expected to be small. If on the other hand, the neutrino oscillates to a neutrino type with a larger mass, then the helicity-reversal effect can become enhanced. After scattering in the larger-mass state, the neutrino can oscillate back to the original or to another lepton type with reversed helicity. For the Majorana neutrino this combined effect could be confused with a Dirac $\nu \to \bar{\nu}$ transformation in $V-A$ processes.

In this section, I give a formulation to estimate the significance of these combined effects. Although it now appears that $\nu_e \leftrightarrow \nu_\mu$ oscillations have a small probability,[10] oscillations of the type $\nu_e \leftrightarrow \nu_\tau$ may still be significant, especially if the mass of ν_τ is such that $m \sim 1$. At present, this is allowed as the result of the current terrestrial experimental bounds for the neutrino masses.

To begin the discussion, one can use $|\nu_\sigma)$ ($\sigma = 1$, 2, or 3) to represent eigenstates of the neutrino energy operator with mass eigenvalues m_σ, and $|\nu_l)$ ($l = e$, μ, or τ) to represent the observed physical neutrinos. To describe oscillations, one considers the superposition

$$|\nu_l) = U_{\sigma l}|\nu_\sigma) , \tag{3.1}$$

where there is a summation on repeated indices. The time development of this state is generated by the Hamiltonian H so that

$$|\nu_l(\tau)) = e^{-iH\tau}|\nu_l(0)) . \tag{3.2}$$

The probability to observe a neutrino of lepton type l' at time τ, if a neutrino of lepton type l is present at time zero, is

$$\begin{aligned} W_{ll'}(\tau) &= |a_{l'l}(\tau)|^2 \\ &= U_{\sigma' l'} U_{\sigma l'} U_{\sigma' l} U_{\sigma l} \cos[(E_{\sigma'} - E_\sigma)\tau] , \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} a_{l'l}(\tau) &= (l' | l(\tau)) , \\ (\sigma | l) &= U_{\sigma l} . \end{aligned} \tag{3.4}$$

If the neutrino energy is large relative to its mass, one finds in vacuum for $l = \mu$, or τ

$$\begin{aligned} W_{el}(x) &= 1 - W_{ee}(x) \\ &= \tfrac{1}{2}\sin^2(2\theta_v)[1 - \cos(2\pi x / L_v)] , \end{aligned} \tag{3.5}$$

where θ_v is the vacuum mixing angle and where L_v is the vacuum oscillation length which can be found in Bilenky and Pontecorvo[1] for Dirac and Majorana neutrinos.

As the result of coherent-scattering effects, the probabilities (3.3) become modified for oscillations in matter. The appropriate expressions for oscillations in matter can be found in Ref. 11. For either $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \leftrightarrow \nu_\tau$ oscillations in matter, the transition probabilities become

$$\begin{aligned} W_{el}(x) &= 1 - |(\nu_e | \nu_e(x))|^2 \\ &= \tfrac{1}{2}\sin^2(2\theta_v)(L_m/L_v)^2[1 - \cos(2\pi x / L_m)] . \end{aligned} \tag{3.6}$$

Here the oscillation length in matter is

$$L_m = L_v \left[1 + \left[\frac{L_v}{L_0} \right]^2 - 2\cos 2\theta_v \left[\frac{L_v}{L_0} \right] \right]^{-1/2} \tag{3.7}$$

and $L_0 = 2\pi / G n_e$, where n_e is the electron density.

One can now use the above results along with the cross sections derived in Sec. II to obtain expressions for the passage of neutrinos through matter when they interact with electrons. This provides a description of the helicity-transformation effect which is coupled with lepton-type oscillations. If one assumes that the flavor of the neutrino is unchanged as the result of the interaction, then the differential cross section at time τ becomes

$$\begin{aligned} &\frac{d\sigma}{dt}(s_a, s_c, \lambda, \tau_f, \tau, \tau_0)_{l_f, l'} \\ &\quad = \sum_{l = e, \nu, \tau} A(\tau_f, \tau, \tau_0)_l \frac{d\sigma}{dt}(s_a, s_c, \lambda, m)_l , \end{aligned} \tag{3.8}$$

where

$$A(\tau_f, \tau, \tau_0) = |a_{ll_f}(\tau_f - \tau)|^2 |a_{l'l}(\tau - \tau_0)|^2 .$$

In the process described by this differential cross section, a neutrino of lepton type l' initially in vacuum at time τ_0 can oscillate and interact in matter with an electron at time τ as a neutrino of lepton type l. After interacting, the neutrino can continue to oscillate and be detected in vacuum at time τ_f as a neutrino of lepton type l_f. If the neutrino enters matter at time τ_1 and emerges at time τ_2, then one must use the vacuum probability functions (3.5) for τ_0 to τ_1 and for τ_2 to τ_f. However, for $\tau_1 \le \tau \le \tau_2$ when the neutrino is in matter, one must use the matter probability functions (3.6). In general the differential cross section must be multiplied by the neutrino momentum distribution function and integrated over this distribution. Furthermore, during the passage through matter, it is to be multiplied by the electron density function and integrated from the position at τ_1 to the position at τ_2.

If only the final-state electron is detected, then one finds upon summing over the final neutrino types and integrating over t as done in deriving (2.11) the expressions for the total cross section for an interaction at time τ to be

$$\sigma(\tau_f, \tau, \tau_0) = \sum_{l = e, \mu, \tau} W_{le}(\tau_f, \tau, \tau_0)\sigma(\tau)_l . \tag{3.9}$$

In the limit where one neglects terms which depend upon m/ω_a, one finds the expression used in Ref. 12 to study oscillations in ν_e-e scattering. In the same limit, one obtains from (3.8) the differential cross section used in Ref. 13 to study oscillation effects in $\bar{\nu}_e$-e scattering.

As a final contribution in this paper, I give an estimate for the helicity-transformation effect as neutrinos pass through matter. I start by considering the passage of a left-handed electron-type neutrino through matter. I consider only the contribution which comes from the elastic scattering from electrons since a helicity transformation is small for the scattering from a target of large mass; furthermore, absorption is negligible. I use $\alpha(x)$ to represent the absorption coefficient for the left-handed to right-handed transition $\nu_L \rightarrow \nu_R$, and I use $\beta(x)$ to represent the inverse-transition absorption coefficient. The densities $N_a(x)$ and $N_b(x)$ of left-handed and right-handed neutrinos found at a distance x from the origin are found from the differential equation

$$\begin{bmatrix} dN_a/dx \\ dN_b/dx \end{bmatrix} = \begin{bmatrix} -\alpha & \beta \\ \alpha & -\beta \end{bmatrix} \begin{bmatrix} N_a \\ N_b \end{bmatrix} . \quad (3.10)$$

With the conditions $N_b(0)=0$ and $N_a(0)=N_a(x)+N_b(x)$ for matter of uniform electron density $n_e=\rho_e N_0$ ($N_0 = 6.022\times 10^{23}$), the solutions are

$$\frac{N_a(x)}{N_a(0)} = \frac{1}{1+\beta/\alpha}(\beta/\alpha + e^{-(\alpha+\beta)x}) ,$$

$$\frac{N_b(x)}{N_a(0)} = \frac{1}{1+\beta/\alpha}(1 - e^{-(\alpha+\beta)x}) . \quad (3.11)$$

For the Dirac neutrino $\alpha > \beta$, and for the Majorana neutrino $\alpha \sim \beta$.

For the interesting cases of terrestrial and stellar processes, the quantity $(\alpha+\beta)x$ is small and the fraction of helicity-transformed neutrinos becomes at a distance L

$$N_b(L)/N_a(0) \sim Q = \alpha L . \quad (3.12)$$

The absorption coefficient $\alpha(x)$ can be found in either the Dirac or the Majorana cases from

$$\alpha(x)_{D,M} = \sum \langle W_{el}(x) \rangle \rho_e N_0 \sigma(L \rightarrow R, m)_{D,M} , \quad (3.13)$$

where $W_{el}(x)$ are the matter oscillation probabilities which are found from (3.6). The total cross section for a helicity transformation can be found in a manner similar to that used to derive (2.11). An estimate of its value can be obtained if one considers scattering in the forward direction. In this way, one can use

$$\sigma(L \rightarrow R) \sim \int \frac{d\sigma}{dy}(s_a, -s_a)\delta(y)dy$$

$$\sim -\frac{\omega_a}{2}\frac{d\sigma}{dt}(s_a, -s_a, \theta=0) . \quad (3.14)$$

For the Dirac case one finds from (2.7)

$$\sigma(L \rightarrow R)_D \cong \frac{G^2}{2\pi} \frac{m^2 |A|^2}{\omega_a(1-(m/\omega_a)^2)} . \quad (3.15a)$$

In a similar manner, one finds for the Majorana case

$$\sigma(L \leftrightarrow R)_M \cong \frac{G^2}{\pi} \frac{m^2(|A|^2 - |V|^2)}{\omega_a(1-(m/\omega_a)^2)} . \quad (3.15b)$$

As discussed by Wolfenstein, when $L_v < L_0$ in (3.7), $\theta_m \sim \theta_v$, and there is little difference between the oscillation probabilities in matter and in vacuum. When $L_v \sim L_0$ these oscillation probabilities can differ considerably. Numerical values for these probabilities can be found from Table II of Ref. 11. In the present numerical estimate, $L_v \le L_0$ for terrestrial and condensing stellar matter, and it is reasonable to approximate the oscillation probability with the value 0.5. The electron-density parameter ρ_e has the following approximate values:

$\rho_e \sim 1-2$, sun, Earth ,

$10^{10} < \rho_e < 10^{13}$, neutron star ,

$10^{16} < \rho_e$, black hole .

With the representative values $m \sim 1$, $\omega_a \sim 10$, and $\langle W_{el} \rangle \sim 0.5$, one finds from (3.12) and (3.15) for the Dirac case

$$N_b(L)/N_a(0) \sim 0.17\times 10^{-17} \rho_e [L \text{ (km)}] . \quad (3.16)$$

The result is approximately twice this value for the Majorana case.

Although one can conclude from this numerical estimate that the helicity transformation effect is unlikely to be observed for neutrinos passing through the Earth or the sun, the effect may be present in very dense stars if $m \sim 1-10^{-1}$. Values in this range are within the experimental bounds for ν_μ or ν_τ, but they are larger than the cosmological bounds,[14] $m_e + m_\mu + m_\tau + m_x \sim 40$ eV. If one now considers a condensing star where electron neutrinos are produced from the reaction $n + e \rightarrow p + \nu$ and observes from (3.6) that the probabilities with small oscillation lengths in matter for $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \leftrightarrow \nu_\tau$ oscillations are approximately $\frac{1}{3} - \frac{1}{2}$, then the helicity transformation effect could be significant in producing helicity-transformed neutrinos in the universe. This effect would have to be considered along with the expected precession of the neutrino's polarization vector which can occur if the neutrino has a magnetic moment and passes through a dense magnetic field.[15] As a final remark, it is worthwhile to note, until such time that the cosmological bounds on the neutrino masses are better established, that it may be of interest to use the cross sections (2.5) in looking for (m/ω_a) dependence in precision ν-e scattering. In this way one might be able to distinguish between Dirac and Majorana neutrinos.

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