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Mean life of the positive muon

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Positive pions were stopped in a water Cherenkov counter and the time distribution of the positrons from $\pi^+\to\mu^+\to e^+$ was measured. The μ^+ mean life obtained, $2.196\,95\pm0.000\,06$ μs, is slightly less than previous results.

I. INTRODUCTION

It is of considerable interest to determine the mean life τ of the positive muon to good accuracy. As shown and discussed by Sirlin,[1] the Weinberg-Salam description of the electroweak interaction gives, with the neglect of terms in $m_e{}^3/m_\mu{}^3$,

$$\frac{1}{\tau}=\frac{G_\mu{}^2 m_\mu{}^5}{192\pi^3}\left[1-\frac{8m_e{}^2}{m_\mu{}^2}\right]\left[1+\frac{3m_\mu{}^2}{5m_W{}^2}-\frac{\alpha}{2\pi}(\pi^2-\tfrac{25}{4})\right]. \quad (1)$$

The quantity G_μ is the dimensional form of the phenomenological weak coupling constant. The masses of the electron, muon, and W vector boson are given by m_e, m_μ, and m_W, and α is the fine-structure constant. The muon mass is known to a precision of 1.7×10^{-6} and α to 8.0×10^{-7} (Ref. 2), while $3m_\mu{}^2/5m_W{}^2$ is 1.0×10^{-6} for $m_W\simeq80$ GeV, so that the uncertainty in G_μ is dominated by the uncertainty in τ. Furthermore, as used by Bardin *et al.*,[3] the difference between the mean lives of μ^+ and μ^- in hydrogen gives the rate of capture of μ^- by protons and thus information on muon-nucleon coupling. We have therefore made a precision measurement of the mean life τ of the positive muon.

II. EXPERIMENT

The experiment was done at the Tri-University Meson Facility (TRIUMF) in Vancouver, partly with beam line M-13 and partly with beam line M-11.[4] Both beams consisted primarily of 150—170-MeV/c π^+ produced by 500-MeV protons. The protons arrived at the production target in 2—5-ns bursts every 43 ns so that time of flight could be used for particle identification, with fair $\pi/\mu/e$ separation in M-13, and better separation in the longer line M-11. Protons were removed by degrader at the first focus in both lines. Table I gives the characteristics of the beams.

Our approach was much like that of Balandin *et al.*[5] in that we used a water-filled container in which π^+ and μ^+ stopped, and in which the decay e^+ were detected through their Cherenkov radiation. The water-target and beam-counter arrangement is shown in Fig. 1. Coincidences between scintillation counters $S1$ and $S2$ signaled the arrival of beam particles, primarily π^+, with some μ^+ and e^+ contamination. Nuclear reactions removed about 5% of the π^+ as they slowed in the water counter. The remaining 95% stopped in the water, about 8 cm upstream from the geometric center of the counter, and decayed into μ^+. The μ^+ in turn came to rest after traveling 0.14 cm and then decayed into e^+. The angular distribution of these

TABLE I. Characteristics of M-11 and M-13 beam lines.

	M-13	M-11
Momenta (MeV/c)	150—170	150—170
Momentum resolution (%)	2	3
Composition (%) ($\pi/\mu/e$)	93/4/3—96/4/0.25	72/19/10—80/13/6
Intensities (10^3/s)	5—30	5—30
Proton intensities	200 nA—3 μA	200 nA—30 μA
Production targets	28 mm H_2O, 10 mm C	2 mm C, 10 mm C, 28 mm H_2O
Takeoff angle	135°	2.5°
Channel length (m)	10	14

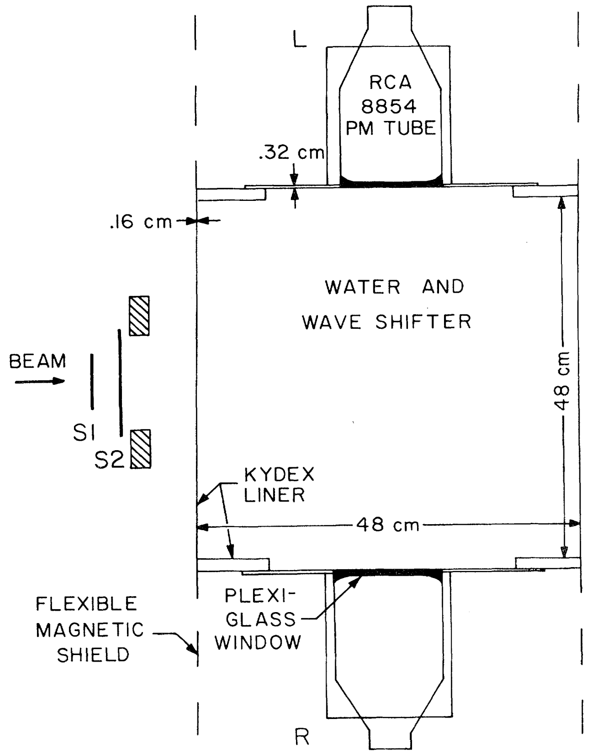

FIG. 1. Target and detector arrangement, top view. Entering beam particles were detected by scintillation counters *S*1 and *S*2. In the water, muon decay positrons caused Cherenkov radiation. This radiation was wavelength shifted by dissolved Na-*G* amino acid and then detected by the phototubes *R* and *L*.

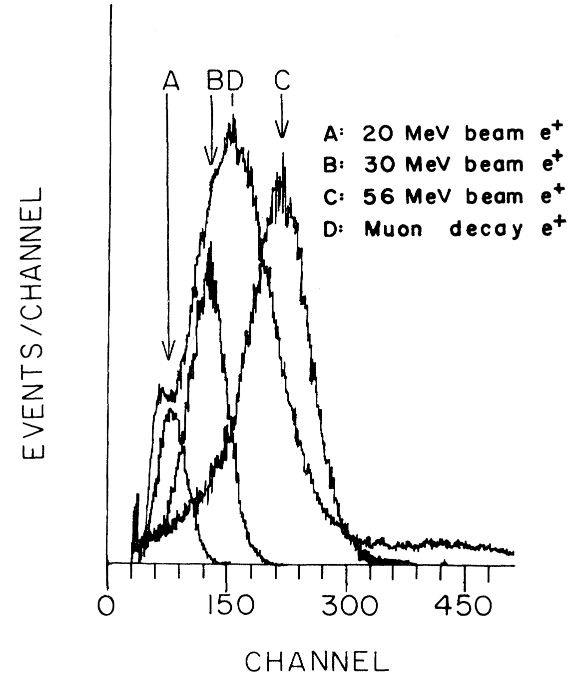

FIG. 2. Energy calibration of the water Cherenkov counter. Curves A, B, and C were obtained with monoenergetic e^+ beams. Curve D was obtained with decay e^+ from stopped muons, and includes a high-energy contribution from direct beam e^+.

e^+ was necessarily isotropic. However, the e^+ that came from beam μ^+ could in principle be emitted with a time-dependent directional preference; this effect is examined below.

The water Cherenkov counter was a 48-cm-long, 48-cm-diameter stainless-steel cylinder, lined with Kydex, an efficient diffuse reflector.[6] Na-*G* amino acid[7] was added to the deionized water as a wavelength shifter to absorb ultraviolet Cherenkov light and re-emit it near the wavelength of maximum response of the photomultiplier tubes. Studies with cosmic-ray muons showed that the presence of the wavelength shifter increased the amplitude of the signal by a factor of approximately 4. Also, the isotropic re-emission of the shifted photons helped to reduce the effects of any anisotropies in the response of the detector. The light was detected by two 12.7-cm-diameter RCA 8854 photomultiplier tubes connected in fast coincidence to reduce noise-induced events. The analog sum of the two signals was used to divide the events into the following energy categories: low (~ 10 to ~ 20 MeV), medium (~ 20 to ~ 50 MeV), and high ($\gtrsim 50$ MeV). Figure 2 shows the response of the Cherenkov counter for three beam e^+ energies and for the muon decay spectrum. Light from beam π^+ and μ^+ appeared below or at the lower boundary of the low region, while light from beam e^+ appeared in the high region. The medium-energy data were used for the actual determination of τ.

As shown in Table I, runs were taken with beams that contained up to 19% μ^+ component. Such μ^+ are about 90% polarized, and retain 60% of their polarization after they stop in water.[8] The e^+ momentum is correlated with the μ^+ spin direction. If the detector response is not sufficiently isotropic, and if the μ^+ precess or depolarize, systematic errors can arise because events can be shifted into and out of the medium-energy region with time. The effect of the e^+ path direction on the energy calibration of the Cherenkov counter was therefore studied with e^+ beams at the National Bureau of Standards linac. The Cherenkov detector was set at various positions and angles with respect to the e^+ beam, and it was found that the resulting change in pulse height was less than 4%. It follows that, if muonium is not formed, a magnetic field of less than 1.6 G leads to precession sufficiently slow to cause an error in τ of less than 5×10^{-6}. If, as has been observed in thoroughly degassed water,[8] muonium is formed by 20% of the μ^+, a magnetic field of less than 0.08 G is required. Since the water contained considerable dissolved air, muonium formation was not expected. Nevertheless, we held the field below 0.05 G throughout the stopping volume by wrapping the detector in a magnetic shield[9] and placing the entire assembly in a three-axis coil system.[10]

Depolarization of muons can produce time-dependent

TABLE II. Histogram definitions.

Histogram	Character	Criteria
1	Low clean (pion)	1 and only 1 positron in 20 μs
2	Medium clean (pion)	with pre- and post-pile-up gate
3	High clean (pion)	cleaning for π^+ start
4	Low pre only	Up to a multiplicity of 3 positrons
5	Medium pre only	with 20-μs postbeam cleaning
6	High pre only	
7	Low clean (muon)	1 and only 1 positron in 20 μs
8	Medium clean (muon)	with pre- and post-pile-up gate
9	High clean (muon)	cleaning for μ^+ start
10	Low post only	Up to 3 positrons and at least
11	Medium post only	one postbeam signal
12	High post only	
13	Beam pion	1 and only 1 postbeam event in the
14	Beam muon	20-μs period after a START

effects similar to those of precession. However, the depolarization time of muons in water exceeds 100 μs,[8] while even 10 μs would make these effects negligible. To verify that the Na-*G* amino-acid wavelength shifter did not cause any unexpected shortening of depolarization times, the spin-relaxation times of protons in water and in the solution were compared;[11] no difference was found. Further, a value of τ was determined directly from beam muons selected by time of flight during the M-11 runs, in which there was fairly good time of flight separation of μ^+ from π^+. Within the limits of the μ^+ statistics, there was no significant difference between this value of τ and the one obtained from stopping π^+.

The basic data were time intervals between electronic logic signals. The signals were used to gate a 100-MHz crystal-controlled oscillator and the number of oscillator pulses within the gate was recorded. The time intervals associated with a common $S1 \cdot S2$ START signal comprised one interval set. After a START, the associated STOP signals were accepted for 20 μs. The data from any one interval set consisted of a maximum of four intervals, three with STOP signals from the Cherenkov counter, and one consisting of a second $S1 \cdot S2$ beam signal. If more than three Cherenkov signals or more than one additional beam signal occurred, the interval set was canceled. Most interval sets consisted of the single interval between the arrival of a π^+ and the detection of the decay e^+ from the resulting μ^+. The other classes of interval sets provided information about the structure of the beam and the nature of the backgrounds.

Details of the instrumentation, with emphasis on the clock and control logic, will be described in a paper that is being prepared, and only a brief summary is given here. Two separate CAMAC timing channels, channel A and channel B, were used. In a typical interval set, a START signal was routed to one of two channels, for example, channel A, and started four scalers[12] which counted the 100-MHz crystal-oscillator pulses. Subsequent Cherenkov signals then stopped first scaler 2, then scaler 3, and then scaler 4, and a subsequent beam particle stopped scaler 1. At the end of the interval set, that is, 20 μs after the START signal, it took from 20 to 50 μs to store the data, and the next interval set was routed to channel B. Dead-time losses were thus reduced by the use of the two parallel timing channels. Also, consistency between the sets of data processed by the two channels provided an additional check on component behavior. The use of a microprogrammable branch driver between the CAMAC unit and the computer[13] reduced processing time by approximately a factor of 4.

The clock system was subjected to extensive testing with random signals from two scintillators that viewed separate radioactive sources. In addition, tests were made with START signals whose arrival time was varied with respect to the oscillator phase. The system showed no nonlinearity or distortions that could affect τ at a level of 10^{-5}. The crystal-controlled oscillator frequency and stability were checked regularly against the U.S.A. network-television color-carrier frequencies, which are based on the NBS atomic-clock standard.[14] The oscillator was adjusted to 100 MHz at the beginning of the experiment, and remained stable to better than one part in 10^6 throughout.

The data from each interval set were routed into histograms according to STOP multiplicity and the amplitudes (low, medium, high) of the Cherenkov signals. In all, the 14 histograms listed in Table II were constructed for each of the two timing channels. A pile-up protection circuit used signals from scintillation counter *S*2 to identify interval sets that did not contain a second beam particle in the time from 20 μs before to 20 μs after the START signal. If further these interval sets contained only one Cherenkov signal, they were routed into the histograms designated as "clean." Time-of-flight tags separated these histograms fairly well into those that were π^+ initiated (histograms 1, 2, or 3, depending on Cherenkov-signal amplitude) and those that were μ^+ initiated (histograms 7, 8, or 9, de-

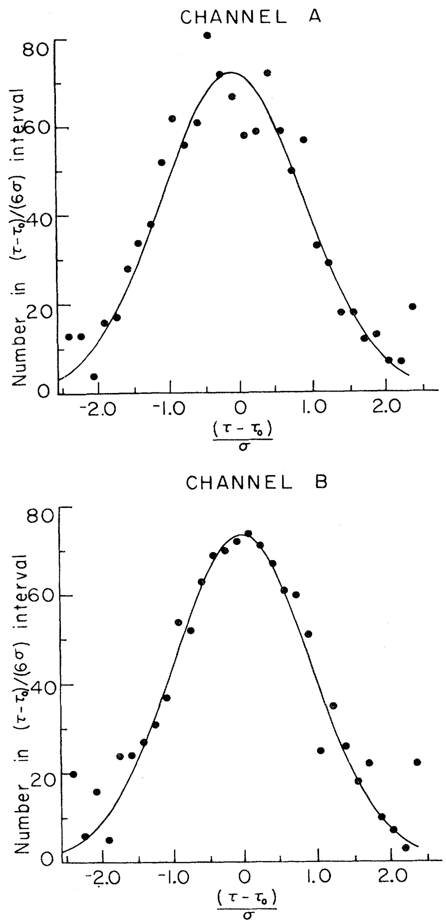

FIG. 3. Distributions of τ values for the individual updates, for timing channel A and timing channel B. The data from each update were fitted to expression (2). The distributions are plotted as a function of $(\tau-\tau_0)/\sigma$, where σ is the standard deviation for the fit and $\tau_0=2.196\,95$ μs. The last points on the left and on the right give the total number of updates with larger negative and positive deviations.

pending on Cherenkov-signal amplitude). As described in Sec. III, histograms 2 and 8, the clean medium-energy distributions, were used for the actual determination of τ. Histograms 4, 5, 6, 10, 11, and 12 contained the multiple-signal events. Histograms 13 and 14 registered the arrival pattern of second-beam particles.

The histogram data were written to tape, on the average every 15 min, along with the contents of 30 monitoring scalers that reflected the running conditions. These updates assured that losses caused by component failure were low, and allowed a variety of data groupings to be analyzed separately.

III. ANALYSIS

To determine τ, a standard least-squares fit of the function

$$N e^{-t/\tau}+B \qquad (2)$$

was made to the decay histograms with N, τ, and B being the adjustable parameters. This fit was made for time intervals beginning between 0.30 and 0.45 μs and ending between 17.40 and 17.55 μs after the START. Because 0.30 μs equals 11.5 pion mean lives, there was no need to include in this expression the description of the initial growth of the muon population. The fits obtained showed that expression (2) sufficed, but it is useful to discuss some of the reasons for the effective absence of more complicated terms.

Suppose that a π^+ or μ^+ arrived and was detected by $S1\cdot S2$, that its e^+ was detected at t, but that there was an additional π^+ or μ^+ whose arrival was not detected. If the e^+ from the undetected π^+ or μ^+ fell within ± 50 ns of t, only a single Cherenkov pulse might have been seen. Events of this kind can contribute a term that decays like $e^{-2t/\tau}$. However, in this experiment, the additional π^+ or μ^+ must either have arrived at least 20 μs before the $t=0$ defined by the detected π^+ or μ^+, or have entered without triggering $S2$. The first possibility contributed very little because of the $e^{-20\mu s/\tau}$ factor. The second possibility contributed very little because $S2$ was highly efficient: An examination of events in the 1.7-μs interval that preceded the $S1\cdot S2$ START showed that $S2$ missed $\lesssim 10^{-3}$ of the incoming π^+ and μ^+, and that therefore the amplitude of the $e^{-2t/\tau}$ term was expected to be $\lesssim 10^{-7}$ of the $e^{-t/\tau}$ amplitude. Nevertheless, fits were made with the addition of a term $D e^{-2t/\tau}$ to expression (2). The coefficient D was indeed consistent with zero, and its introduction did not lower the normalized χ^2.

The 43-ns microstructure period of the beam was too short to affect the value of τ. Unless there were substantial beam intensity variations with time scales of the order of τ, there are no remaining *a priori* reasons for expecting a significant background term other than the constant B in expression (2). The signal-to-noise ratio was excellent for the medium-energy clean spectra; the weighted average of N/B was 15 480 for the data used in the analysis. Because B was so small, a 6% deviation from constancy over the interval used for analysis would have had to be present to give a change in τ of 1 standard deviation. That the deviation was far less could be inferred from the multiple-event histograms as follows. Separate determinations of τ were made from these histograms, which had signal-to-noise ratios $\lesssim 100$. These determinations were consistent with that obtained from the medium-energy clean data. With the much poorer signal-to-noise ratios, the multiple histogram analysis was much more sensitive to the background shape. If the shapes of the backgrounds in all of the histograms were even roughly similar, this agreement implied that any nonflat component in the background was too small to affect the result.

For the final data analysis, all of the individual updates were first examined for evidence of any malfunctions and irregularities, and for any inconsistencies between data processed by timing channel A and those processed by timing channel B. The rates for the various classes of events and beam counter signals were required to have ratios that were consistent within statistical fluctuations unless there was a well-understood change in running conditions. Some data were discarded because of clear evidence

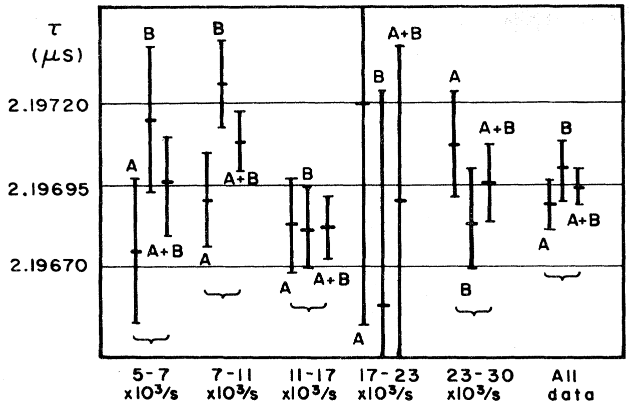

FIG. 4. Values of τ and their standard deviations determined from data obtained with beam intensities in five different ranges, and from all of the data. For each, there are shown separately the results from timing channel A, from timing channel B, and from the combination A + B.

of tape errors, computer failures, or incorrect settings of the beam-defining logic. In addition, some data were discarded because the cyclotron was operating in a low-duty-cycle, high-instantaneous-current mode, or because there were oscillations in the beam-monitoring histograms (13 and 14). Such beam oscillations were detected only when the cyclotron was operating in a nonstandard low-intensity mode. Also, there was eliminated from the data which determined τ a 10-h run during which timing channel B developed an unusually large odd-even fluctuation; that is, the odd and even numbered time intervals differed appreciably in length.

A total of 2.32×10^9 medium-energy clean interval sets was used in the final analysis. For most of the runs, these sets came from four histograms: the π^+ START data (histogram 2) and the μ^+ START data (histogram 8), with each divided into events processed by timing channel A and those processed by timing channel B.

The values of τ obtained from the separate updates follow the expected statistical behavior, as shown by the Gaussian distributions in Fig. 3. Updates were also sorted according to five ranges of beam intensity, and a value of τ was obtained from each of these five sets of data. Figure 4 shows the results, along with that obtained from all of the data. The consistency of these values indicates the absence of rate effects.

Other tests of the data were made. Values of τ were determined from fitting intervals other than the 0.3-to-17.5-μs range and showed no inconsistencies. Binning by various factors, that is, combining various numbers of adjacent channels before fitting, caused no significant change in τ. However, binning by 2 did lower the normalized χ^2 from 1.07 to 1.02. It follows that there was present an odd-even effect, with a mean fractional difference in the effective lengths of the odd and even time intervals of 2×10^{-4}.

Our final result is

$$\tau = 2.196\,95 \pm 0.000\,06\ \mu\text{s} \tag{3}$$

with the error shown equaling 1 standard deviation, and with a normalized χ^2 of 1.02 for the data binned by 2. This result and the three other most recent measurements are shown in Table III. From Eq. (1), we find that with our value of τ the dimensional form of the electroweak coupling constant is given by

$$G_\mu = (1.166\,365 \pm 0.000\,016)\times 10^{-5}\ \text{GeV}^{-2}\,. \tag{4}$$

TABLE III. Recent measurement of the μ^+ mean life τ.

	τ (μs)	One standard deviation (μs)
J. Duclos *et al.* (1973) (Ref. 15)	2.197 3	0.000 3
M. P. Balandin *et al.* (1974) (Ref. 5)	2.197 11	0.000 08
G. Bardin *et al.* (1981) (Ref. 3)	2.197 18	0.000 12
This measurement	2.196 95	0.000 06

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[6]Kydex, Rohm and Haas Co. The use of this material in Cherenkov counters was studied by S. Willis (Ph.D thesis, Yale University, 1979). The Kydex used in our counter was provided by R. Burman of LASL.

[7]Di-sodium 2-amino-6, 8-naphthalene-disulforic acid, 8 mg/l. The use of this material as a wavelength shifter is described by A. Bezaguet *et al.*, Nucl. Instrum. Methods 158, 303 (1979), and references therein.

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[9]G. J. Sellers, in *Proceedings of the IEEE International Symposium on Electromagnetic Compatibility, Seattle, 1977* (IEEE, New York, 1977), pp. 129–132.

[10]G. Marshall, Ph.D. thesis, University of British Columbia, 1980. C. Oram provided essential advice and assistance with the mounting and operation of the coil system.

[11]Pulsed NMR experiments were done through the cooperation of M. S. Conradi and P. Kuhns of the William and Mary Department of Physics.

[12]Joerger Enterprises, Inc., Time Digitizer Model TD, substantially modified. We added an external fast reset, and replaced the front end to have zero pickoff rather than amplitude pickoff to avoid amplitude sensitivity.

[13]Digital Equipment Corporation PDP-11, provided by TRIUMF. We wish to thank T. Miles for his assistance with the computing system.

[14]*Time and Frequency Users' Manual,* edited by G. Kamas (National Bureau of Standards Technical Note No. 695, 1977), Chap. 7.

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