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Supersymmetry transformation rules for Kaluza-Klein supergravity on the seven-sphere

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We show that the truncation to the massless supermultiplet of the $N=8$ gauged SO(8) supergravity theory obtained by spontaneous compactification of eleven-dimensional supergravity on the round seven-sphere yields consistent linearized supersymmetry transformation rules in four dimensions. They coincide with the linearized transformation rules of the de Wit-Nicolai theory.

Spontaneous compactification of the $N=1$ supergravity theory in eleven dimensions[1] admits as a ground-state solution the product of anti-de Sitter (AdS) spacetime with the seven-sphere equipped with its standard SO(8)-invariant metric.[2,3] This yields, via the Kaluza-Klein mechanism, a four-dimensional supergravity theory which has an SO(8) gauge symmetry and $N=8$ supersymmetry. The field content of this theory comprises an $N=8$ massless supermultiplet, together with an infinite tower of massive supermultiplets. (It has recently been shown that the next-to-lightest massive supermultiplet actually includes further massless states, namely, a $\underline{294}$ of scalars.[4]) It is natural to enquire whether the four-dimensional theory can be consistently truncated to just the massless supermultiplet, and if so whether the resulting theory is that of de Wit and Nicolai.[5] A previous objection to this conjecture was the apparent discrepancy between the relation of the SO(8) gauge coupling constant and cosmological constant calculated on the one hand in this Kaluza-Klein theory, and on the other hand in the de Wit-Nicolai theory. This conflict was resolved with the realization that the presence of the A_{MNP} field (an antisymmetric tensor gauge field) in the eleven-dimensional theory modifies the usual formula for the coupling constant derived from a pure gravity theory in higher dimensions.[6]

A remaining objection to the equivalence of these theories is that if one substitutes the linearized *Ansätze* for the fields of the massless supermultiplet into the $d=11$ supersymmetry transformation rules, one appears to obtain inconsistent equations.[7] In this Rapid Communication we describe how this is remedied by realizing that the expansion of the $d=11$ supersymmetry parameter in spinor harmonics on S^7 contains additional terms, linear in scalar and pseudoscalar fields, which had previously been overlooked.

The $d=11$ supergravity Lagrangian contains a graviton field $e_M{}^A$, a gravitino Ψ_M, and an antisymmetric tensor gauge field A_{MNP}. This Lagrangian is invariant under the $N=1$ supersymmetry transformations

$$\delta e_M{}^A = -i\bar{\epsilon}\hat{\Gamma}^A\Psi_M \ , \tag{1}$$

$$\delta A_{MNP} = \tfrac{3}{2}\bar{\epsilon}\hat{\Gamma}_{[MN}\Psi_{P]} \ , \tag{2}$$

$$\delta\Psi_M = D_M(\hat{\omega})\epsilon - \frac{i}{144}(\hat{\Gamma}_M{}^{NPQR} + 8\delta_M{}^N\hat{\Gamma}^{PQR})\hat{F}_{NPQR}\epsilon \ , \tag{3}$$

where $A,B,\ldots$ are tangent-space indices, $M,N,\ldots$ are coordinate indices, $\hat{\Gamma}_A$ are the $d=11$ Dirac matrices, and $\hat{\omega}$ and $\hat{F}$ denote the supercovariant formulations of the connection form ω and field strength F (with components $F_{MNPQ}=4\partial_{[M}A_{NPQ]}$). Following the conventions of Ref. 3, spontaneous compactification occurs by taking the Freund-Rubin ground-state *Ansatz*[8] for $F^{(0)}_{mnpq}$ in which all components vanish except

$$F^{(0)}_{\mu\nu\rho\sigma} = 3m\,\epsilon_{\mu\nu\rho\sigma} \ , \tag{4}$$

where we split $d=11$ indices $M,N,\ldots$ as $\mu,\nu,\ldots$ running over spacetime with coordinates x^μ, and $m,n,\ldots$ in the extra dimensions with coordinates y^m. Tangent-space indices are similarly split as $\alpha,\beta,\ldots$ in spacetime and $a,b,\ldots$ in the extra dimensions. The $N=8$ gauged SO(8) theory is obtained by taking the $\mathrm{AdS}\times S^7$ solution of the resulting $d=11$ bosonic field equations

$$R^{(0)}_{\mu\nu} = -12m^2 g^{(0)}_{\mu\nu}, \quad R^{(0)}_{mn} = 6m^2 g^{(0)}_{mn} \ . \tag{5}$$

The fields of the four-dimensional theory correspond to fluctuations around the ground state. We therefore write

$$g_{MN}(x,y) = g^{(0)}_{MN}(x,y) + h_{MN}(x,y) \ , \tag{6}$$

$$F_{MNPQ}(x,y) = F^{(0)}_{MNPQ}(x,y) + f_{MNPQ}(x,y) \ . \tag{7}$$

Making the field redefinitions

$$h'_{MN}=h_{MN}+\tfrac{1}{2}g^{(0)}_{MN}h_q{}^q \ , \tag{8}$$

$$\Psi'_M=\Psi_M-\tfrac{1}{2}\hat{\Gamma}_M\hat{\Gamma}^q\Psi_q \ , \tag{9}$$

we can write the *Ansätze* for the bosons of the massless supermultiplet as[3,9]

$$h'_{\mu\nu}(x,y)=h_{\mu\nu}(x) \ , \tag{10}$$

$$h'_{\mu n}(x,y)=\tfrac{1}{2}B^{IJ}_\mu(x)\eta^{IJ}_n \ , \tag{11}$$

$$h'_{mn}(x,y)=S^{IJKL}(x)\eta^{IJKL}_{mn} \ , \tag{12}$$

$$f_{\mu\nu\rho\sigma}(x,y)=\frac{3m}{2}\epsilon_{\mu\nu\rho\sigma}(h'_\tau{}^\tau-\tfrac{2}{9}h'_t{}^t) \ , \tag{13}$$

$$f_{\mu\nu pq}(x,y)=\frac{1}{24m}\epsilon_{\mu\nu\rho\sigma}\nabla^\sigma\nabla_q h'_t{}^t \ , \tag{14}$$

$$f_{\mu\nu pq}(x,y)=-\frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\nabla_{[p}h'_{q]}{}^\sigma \ , \tag{15}$$

$$f_{\mu npq}(x,y)=-\tfrac{1}{2}\partial_\mu P^{IJKL}(x)\eta^{IJKL}_{npq} \ , \tag{16}$$

$$f_{mnpq}(x,y)=2mP^{IJKL}(x)\eta^{IJKL}_{mnpq} \ , \tag{17}$$

where the vector fields B^{IJ}_μ, the scalar fields S^{IJKL}, and pseudoscalar fieldP^{IJKL} are antisymmetric in spinor SO(8) indices $I,J,\ldots$ and S^{IJKL} is self-dual and P^{IJKL} is anti-self-dual in $IJKL$. The y-dependent tensors η are defined in terms of the eight Killing spinors $\eta^I(y)$ which satisfy

$$\bar{D}_a{}^{(0)}\eta^I\equiv D_a{}^{(0)}\eta^I-\frac{m}{2}\Gamma_a\eta^I \ , \tag{18}$$

where we have split $d=11$ Dirac matrices $\hat{\Gamma}_A$ into $d=4$ and $d=7$ matrices as $\hat{\Gamma}_A=(\gamma_\alpha\otimes 1,\gamma_5\otimes\Gamma_a)$. The definitions are

$$\begin{aligned}&\eta^{IJ}_m=\bar{\eta}^I\Gamma_m\eta^J, \quad \eta^{IJKL}_{mn}=\eta^{[IJ}_m\eta^{KL]}_n \ ,\\ &\eta^{IJKL}_{mnp}=\eta^{[IJ}_{[mn}\eta^{KL]}_{p]}, \quad \eta^{IJKL}_{mnpq}=\eta^{[IJ}_{[mn}\eta^{KL]}_{pq]} \ ,\end{aligned} \tag{19}$$

where we also define $\eta^{IJ}_{mn}=\bar{\eta}^I\Gamma_{mn}\eta^J$. In (11), (16), and (17) we have changed the normalization conventions from those of Ref. 3, in order to simplify comparisons with the theory of de Wit and Nicolai. We have also corrected a sign error in Eq. (15).

For the fermions, the massless *Ansätze* are[3,9]

$$\Psi'_\mu(x,y)=\psi^I_\mu(x)\eta^I \ , \tag{20}$$

$$\Psi'_m(x,y)=\chi^{IJK}(x)\eta^{IJK}_m \ , \tag{21}$$

where the spin-$\frac{1}{2}$ fields χ^{IJK} are antisymmetric in IJK and

$$\eta^{IJK}_m=\eta^{[I}\eta^{JK]}_m \ . \tag{22}$$

The expansion of the supersymmetry parameter, which we shall modify later, will for now be taken to be

$$\epsilon(x,y)=\epsilon^I(x)\eta^I \ . \tag{23}$$

The idea now is to substitute these *Ansätze* into (1), (2), and (3), in order to verify that to first order in fields, four-dimensional supersymmetry transformation rules are obtainable; i.e., that the y dependence of the left- and right-hand sides of each of these equations matches. Starting with (1), we first convert it into a transformation law for the linearized metric,

$$\delta h_{MN}=-2i\bar{\epsilon}\hat{\Gamma}^{(0)}_{(M}\Psi_{N)} \ , \tag{24}$$

where $\hat{\Gamma}^{(0)}_M$ is obtained from $\hat{\Gamma}_A$ using the background vielbein. The calculations here are straightforward, and indeed yield equations in which the y dependence matches. There are three cases to consider, corresponding to $MN=\mu\nu$, μn, and mn, and so dropping the y dependence these give, after some algebra,

$$\begin{aligned}&\delta h_{\mu\nu}=-2i\bar{\epsilon}^I\gamma_{(\mu}\psi^I_{\nu)} \ ,\\ &\delta B^{IJ}_\mu=-2i\bar{\epsilon}^{[I}\gamma_5\psi^{J]}_\mu-2i\bar{\epsilon}^K\gamma_\mu\chi^{KIJ} \ ,\\ &\delta S^{IJKL}=-2i\bar{\epsilon}^{[I}\gamma_5\chi^{JKL]_+} \ ,\end{aligned} \tag{25}$$

where in the final equation $[IJKL]_+$ denotes the self-dual projection of $[IJKL]$. This projection follows from the duality property of η^{IJKL}_{mn} defined in (19).

We now convert (2) into a transformation law for F_{MNPQ}, and so after linearization we obtain

$$\delta f_{MNPQ}=6D^{(0)}_{[M}(\bar{\epsilon}\hat{\Gamma}^{(0)}_{NP}\Psi_{Q]}) \ . \tag{26}$$

On substituting the *Ansätze* into (26), one again finds after some straightforward but tedious algebra that the y dependence matches. There are five cases to consider, corresponding to the various possible combinations of spacetime and internal indices. All except the cases of 3 or 4 internal indices reproduce the previously obtained transformation laws (25). The remaining cases produce the last bosonic transformation law,

$$\delta P^{IJKL}=-2\bar{\epsilon}^{[I}\chi^{JKL]_-} \ . \tag{27}$$

We now turn to the fermion transformation law (3). This is the equation which at first sight appears to give rise to a mismatch of the y dependence, and hence an inconsistent result, and so here we shall describe the calculation in greater detail, although still omitting the quite tedious but straightforward intermediate steps. Linearizing (3), we obtain

$$\begin{aligned}\delta\Psi_M=\;&\bar{D}^{(0)}_M\epsilon-\tfrac{1}{4}(2e_A^{(0)N}D^{(0)}_{[M}v_{N]B}-e_A{}^{(0)P}e_B{}^{(0)Q}e_M^{(0)D}D_P{}^{(0)}v_{QD})\hat{\Gamma}^{AB}\epsilon\\ &-\frac{i}{144}(\hat{\Gamma}_M^{(0)NPQR}+8\delta^N_M\hat{\Gamma}^{(0)PQR})f_{NPQR}\epsilon-\frac{i}{144}(\hat{\Gamma}_M^{(1)NPQR}+8\delta^N_M\hat{\Gamma}^{(1)PQR})F^{(0)}_{NPQR}\epsilon \ ,\end{aligned} \tag{28}$$

where v^A_M is the fluctuation around the background vielbein $e_M^{(0)A}$ corresponding to the metric fluctuation h_{MN} defined in (6). It is convenient to impose the gauge condition $v_m{}^\alpha=0$. The matrices $\hat{\Gamma}_M^{(1)NPQR}$ and $\hat{\Gamma}^{(1)PQR}$ are the terms linear in $v_M{}^A$ which result from converting from tangent space to world indices on the Dirac matrices $\hat{\Gamma}_A{}^{BCDE}$ and $\hat{\Gamma}^{CDE}$.

We now substitute the *Ansätze* into (28), and consider first the case $M=m$. On general grounds we expect the variation $\delta\Psi_m$ to involve terms containing B^{IJ}_μ, $\nabla_{[\mu}B^{IJ}_{\nu]}$, $\partial_\mu S$, $\partial_\mu P$, S, and P. Detailed calculation shows that the term in B_μ vanishes, while after a Fierz transformation the y dependence of the term in $\nabla_{[\mu}B_{\nu]}$ is seen to match with the left-hand side of (28). For the scalar and pseudoscalar terms, we require the Fierz transformation

$$-144\left(\delta^{I[J}\eta_m^{KLM]}+\tfrac{1}{9}\Gamma_m\delta^{I[J}\eta\!\!\!/^{KLM]}\right)=\Gamma_m{}^{npq}\eta^I\eta_{npq}^{JKLM}+6\Gamma^{np}\eta^I\eta_{mnp}^{JKLM}+18\Gamma^n\eta^I\eta_{mn}^{JKLM}$$
$$-2\Gamma_m\eta^I\eta_{pq}^{JKLM}g^{pq}+\Gamma_m{}^p\eta^I\eta_{np}^{[JK}\eta_q^{LM]}g^{nq}-2\eta^I\eta_{mn}^{[JK}\eta_p^{LM]}g^{np}\ . \qquad (29)$$

It is crucial to note that the first two terms on the right-hand side of (29) are anti-self-dual in $JKLM$, since they are associated with the 35_C of three-forms of the pseudoscalar *Ansatz,* while the remaining terms are self-dual in $JKLM$, since they are associated with the 35_V of Killing tensors of the scalar *Ansatz.*[3] The terms in $\delta\Psi_m$ involving $\partial_\mu S$ and $\partial_\mu P$ are just precisely the self-dual and anti-self-dual parts of (29), respectively, and hence the y dependence of these terms matches with $\delta\Psi_m$; they occur in the combination $\partial_\mu(S+i\gamma_s P)$.

Repeating the procedure for the S and P terms in $\delta\Psi_m$, one runs into an apparent inconsistency. The relevant terms in the transformation rule are

$$\delta\chi^{IJK}|_{s,p}\left(\eta_m^{IJK}+\tfrac{1}{9}\Gamma_m\eta\!\!\!/^{IJK}\right)=\frac{m}{72}S^{JKLM}\epsilon^I\left(-5\Gamma_m{}^p\eta^I\eta_{np}^{[JK}\eta_q^{LM]}g^{nq}-2\Gamma_m\eta^I\eta_{pq}^{JKLM}g^{pq}-18\Gamma^n\eta^I\eta_{mn}^{JKLM}\right)$$
$$+\frac{im}{72}P^{JKLM}\gamma_5\epsilon^I\left(-8\Gamma_m{}^{npq}\eta^I\eta_{npq}^{JKLM}-12\Gamma^{pq}\eta^I\eta_{mpq}^{JKLM}\right)\ . \qquad (30)$$

Using (29), one can now show that the y dependence on each side of (30) does not match. To remedy this, we note that the *Ansatz* (23) for $\epsilon(x,y)$ implies that $\bar{D}_m^{(0)}\epsilon=0$, but that the *Ansatz* can be modified by the addition of S- and P-dependent terms in such a manner that $\bar{D}_m^{(0)}\epsilon$ now produces precisely the required extra terms in (30) in order that the y dependence coincides with that of the Fierz equation (29). The modified *Ansatz* is

$$\epsilon(x,y)=\left[1+\tfrac{1}{72}S^{JKLM}\left(-2\eta_{mn}^{JKLM}g^{mn}+3\eta_{mn}^{[JK}\eta_p^{LM]}g^{np}\Gamma^m\right)+\frac{i}{72}\gamma_5P^{JKLM}\left(-3\eta_{mnp}^{JKLM}\Gamma^{mnp}\right)\right]\epsilon^I(x)\eta^I\ . \qquad (31)$$

Note that this does not upset any of the previously derived linearized transformation rules, since there ϵ always appears in terms already linear in fields. Summarizing, the transformation rule for $\delta\Psi_m$ gives, in terms of $F_{\mu\nu}^{IJ}=2\nabla_{[\mu}B_{\nu]}^{IJ}$,

$$\delta\chi^{IJK}=\tfrac{3}{8}F_{\mu\nu}^{[IJ}\gamma^{\mu\nu}\epsilon^{K]}-2\gamma_5\partial_\mu(S^{IJKL}+i\gamma_5P^{IJKL})\gamma^\mu\epsilon^L$$
$$-4m(S^{IJKL}+i\gamma_5P^{IJKL})\epsilon^L\ . \qquad (32)$$

Finally, substituting the *Ansätze* for the massless fields, and the modified *Ansatz* (31) for $\epsilon(x,y)$, into the transformation rule for $\delta\Psi_\mu$, one finds that the y dependence matches on each side of the equation, and

$$\delta\psi_\mu^I=\bar{D}_\mu\epsilon^I-2mB_\mu^{IJ}\epsilon^J$$
$$-\left[\frac{(1-\gamma_5)}{2}F_{\mu\nu}^{+IJ}-\frac{(1+\gamma_5)}{2}F_{\mu\nu}^{-IJ}\right]\gamma^\nu\epsilon^J\ , \qquad (33)$$

where $F_{\mu\nu}^{\pm}=\tfrac{1}{2}(F_{\mu\nu}\pm{}^*F_{\mu\nu})$, and

$${}^*F_{\mu\nu}=\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\ .$$

The derivative $\bar{D}_\mu$ is the anti-de Sitter covariant derivative $D_\mu+m\gamma_\mu\gamma_5$ including terms up to first order in fluctuations. In Ref. 6, it was shown that the SO(8) gauge coupling constant is given by $g^2=16\pi Gm^2$. Since in our conventions $4\pi G=1$, we therefore have $g=2m$, and hence we see that by introducing an SO(8) covariant derivative $\mathcal{D}_\mu$, which, when acting on ϵ^I gives

$$\mathcal{D}_\mu\epsilon^I=D_\mu\epsilon^I-gB_\mu^{IJ}\epsilon^J\ , \qquad (34)$$

(33) may be rewritten as

$$\delta\psi_\mu=\bar{\mathcal{D}}_\mu\epsilon^I-\left[\frac{(1-\gamma_5)}{2}F_{\mu\nu}^{+IJ}-\frac{(1+\gamma_5)}{2}F_{\mu\nu}^{-IJ}\right]\gamma^\nu\epsilon^J\ . \qquad (35)$$

Here we have defined $\bar{\mathcal{D}}_\mu$ in the same manner as $\bar{D}_\mu$, i.e., $\bar{\mathcal{D}}_\mu=\mathcal{D}_\mu+m\gamma_\mu\gamma_5$.

It is important to check that the redefinition of $\epsilon(x,y)$ in (31) does not change any of the results obtained previously in this kind of Kaluza-Klein theory. In particular, it should be emphasized that the criterion for supersymmetry[2,3] of the ground state, $\langle\delta\Psi_M\rangle=0$, remains unaltered. The reason for this is that, in the ground state, (31) reduces to the previous *Ansatz* (23) for ϵ, since $\langle S\rangle=\langle P\rangle=0$.

The important conclusions of this paper are firstly that the truncation of the theory to include just the massless supermultiplet is indeed consistent with the transformation rules at the linearized level. Secondly, as one would expect, the transformation rules coincide with those of the de Wit-Nicolai theory at the linearized level. This may be seen after some trivial rescalings of the fields. In particular, note the SO(8) covariant formulation of the derivative in (35), and the appearance throughout of the combination of fields $(S+i\gamma_5P)$. The results in this paper will be described in greater detail in a forthcoming publication.[10]

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