Renormalization-group analysis of dynamical symmetry breaking in QCD

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We perform a renormalization-group analysis of the dynamical symmetry breaking in QCD based on the Nambu—Jona-Lasinio approach. We show how the mass scale that the fermions acquire in dynamical symmetry breaking can be calculated in terms of the invariant cutoff. We also determine the high-energy behavior of the quark two-point function.

I. INTRODUCTION

It is a common belief that some quantum field theories can spontaneously generate a mass even though the underlying field-theory Lagrangian to begin with may have no mass scale associated with it. In the conventional quantum chromodynamics of massless fermions the underlying Lagrangian possesses a chiral invariance that protects the fermions from having a primordial mass. This chiral invariance, however, is believed not to be respected by the physical ground state and the physical fermions then acquire a mass scale that is associated with this "dynamical symmetry breaking." The precise nature of this mechanism has to date only been conjectured but not fully demonstrated in the theory.[1]

In this paper we begin a renormalization-group analysis of the dynamical symmetry breaking in QCD. We shall show how the mass scale that the fermions acquire in dynamical symmetry breaking can actually be *calculated* in terms of the renormalization-group-invariant cutoff of QCD, Λ. By the same renormalization-group analysis we can also determine the high-energy behavior of the two-point Green's function for the fermions, thus settling an issue which previous studies based on Schwinger-Dyson equations[2] could not decide upon.

Our renormalization-group analysis is based on the approach long ago pioneered by Nambu and Jona-Lasinio,[3] with however the crucial difference that their suggestion was in the context of an unrenormalizable theory, while QCD is fully renormalizable. Their key observation is that the usual (naive) perturbation theory perturbs around the massless gluon and massless quark free fields. To any finite order in the usual perturbation theory, the quark field remains massless because of the manifest chiral invariance of the original Lagrangian.

Much as a ferromagnet ground state is one where all spins are aligned, the true physical ground state for QCD may be one where quark-antiquark pairs are "condensed" together, in which case the usual perturbation series is an expansion around the wrong Hilbert space. Following Nambu and Jona-Lasinio, we must expand around the new (broken) vacuum. That is, we write the total Lagrangian as

$$L=(L_0-M\bar{\psi}\psi)+(L_{\rm int}+\delta M\bar{\psi}\psi) \tag{1.1}$$

with $\delta M=M$, and take the perturbation expansion around the new massive quark vacuum. The parameter M is not arbitrary, but is to be determined self-consistently from the requirement that the radiative corrections due to the new interaction Lagrangian in (1.1) vanish.

To one-loop accuracy, the full renormalized two-point function reads ($\lambda\equiv g_r{}^2/16\pi^2, T_aT_a\equiv C_F\mathbb{1}$),

$$\Gamma_r^{(2)}(p)=\gamma\cdot p-iM+i\left[\delta M+3\lambda C_f M\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]\right] \tag{1.2}$$

and the Nambu—Jona-Lasinio requirement is that for $p^2\ll M^2$

$$\Gamma_r^{(2)}(p)=\tilde{Z}^{-1}(\gamma\cdot p-iM)\,, \tag{1.3}$$

where $\tilde{Z}$ is a finite wave-function renormalization. Comparing (1.2) and (1.3), we have

$$-\delta M=3\lambda C_f M\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]. \tag{1.4}$$

Finally, for consistency with the original Lagrangian, we set $\delta M=M$. A (trivial) solution is $M=0$, the original perturbation theory, while a nontrivial, nonperturbative result is obtained provided

$$1+3\lambda C_f\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]=0\,. \tag{1.5}$$

In the original Nambu—Jona-Lasinio paper, the analog of Eq. (1.5) is the gap equation, expressed in terms of bare couplings and masses, and a quadratic cutoff. Because their theory was not renormalizable, no effort was made to rewrite their condition as a function of renormalized quantities. Here, with the theory being a renormalizable one, it is important that we check on the renormalization-group properties of the one-loop condition (1.3).

There is yet another reason for wanting to perform the renormalization-group analysis. It is clear that if one chose the renormalization scale μ to be such that λ is small, then $\log(M^2/\mu^2)$ will obviously be large and a summation over leading logarithms and next-to-leading logarithms will be necessary to give a meaningful result.

To leading-logarithm approximation (we leave the next-to-leading-logarithm approximation to Sec. III), Eq. (1.5) becomes

$$\left[1+\frac{b}{2}\lambda C_f\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]\right]^{6C_f/b}=0\,, \tag{1.6}$$

where

$$\mu\frac{\partial}{\partial\mu}\lambda=-b\lambda^2\,. \tag{1.7}$$

The nontrivial solution of (1.6) is

$$M=\mu e^{1/6}e^{-1/b\lambda}=\Lambda_c e^{1/6}\,, \tag{1.8}$$

where Λ_c here is the one-loop renormalization-group-invariant cutoff. When higher-order graphs are included in the analysis, renormalization-group methods may be used to sum over the contributions in the leading-logarithm approximation.

A side remark is necessary here. Condition (1.2) assumes that the $1/\epsilon$ counterterms have been (liberally) invoked to make all radiative correction graphs finite, including those due to the new self-mass interactions [see Eq. (1.1)]. It is quite reassuring that the *same* self-consistency condition, Eq. (1.6), is present in the M-dependent $1/\epsilon$ counterterms needed to renormalize the theory, and these extra counterterms *drop out* when the condition (1.5) is satisfied. The same will remain true to higher-loop renormalization accuracy.

The new technique, based on the Nambu—Jona-Lasinio approach, is thus fully compatible with the renormalization group and provides a tool for direct *calculations* of dynamical symmetry breaking. It can, for example, be used to settle the question of what the high-energy ($p^2 \gg M^2$) behavior of the quark propagator should be. It is no longer a question of conjecture or intuitive arguments in a study of the Schwinger-Dyson equations, but only a matter of performing the renormalization-group summations.

II. SELF-CONSISTENCY CONDITION

Consider the QCD Lagrangian L,

$$L=-\bar{\psi}\gamma\cdot D\psi-\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+\mathscr{L}_{\text{gf}}+\mathscr{L}_{\text{ghost}}+\mathscr{L}_{\text{counterterm}} \tag{2.1}$$

$$=L_c+L_{\text{int}}\,, \tag{2.2}$$

where besides the gauge-fixing and ghost terms we have included the usual mass-independent $1/\epsilon$ counterterms. Lagrangian L does not manifestly contain any mass parameter and is to be the starting point of our renormalization-group analysis.

The renormalization procedure that we shall use is the modified 't Hooft minimal-subtraction[4] scheme ($\overline{\text{MS}}$), with furthermore the convention that

$$\text{Tr}(\gamma_\mu\gamma_\nu)=4\delta_{\mu\nu} \tag{2.3}$$

even for $n=4-\epsilon$ dimensions.

As already discussed in the Introduction, Lagrangian L is invariant under local SU(N) transformations as well as under global chiral transformations of the fermion fields. The $\overline{\text{MS}}$ scheme is fully compatible with both the local gauge symmetry and the global chiral invariance. Therefore the naive perturbation series will order by order only give massless fermion propagators. For the new perturbation series around the broken vacuum, a question will arise concerning the new mass-dependent counterterms that are to be introduced into L [Eq. (2.1)] to make the resulting Green's functions finite. As in all renormalization procedures, we will for now simply introduce them *as needed* and afterward discuss the self-consistency of these additional counterterms.

A more tricky question at this stage concerns the nature of the bookkeeping needed to do the new perturbation theory. A careless application of the perturbation expansion of Eq. (1.1) can easily lead one back to the old naive vacuum. The correct bookkeeping in order to satisfy our requirement is to regard $\delta M\bar{\psi}\psi$ in (1.1) as a new interaction that is of order λ. The precise strength of this new interaction is determined by the requirement that (1.3) should hold, to a given accuracy in λ. Having so determined δM, we finally impose the self-consistency condition that

$$\delta M=M \tag{2.4}$$

in order that our Lagrangian (2.4) be the original massless QCD Lagrangian that we started out with.

To illustrate our procedure, let us calculate the two-point quark Green's function to first order in λ. For convenience of presentation, we choose to do this calculation in the Landau ($\alpha=0$) gauge. The result is ($p^2 \ll M^2$)

$$\Gamma_r^{(2)}(p)=\gamma\cdot p-iM+i\left[\delta M+3\lambda C_f M\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]\right]. \tag{2.5}$$

In (2.5), the δM term is the contribution from the new δM interaction, considered to be of order λ, while the second term is the contribution from the radiative correction due to virtual emission and absorption of gluons. Requirement (1.3) thus fixes δM to be given by

$$\delta M=-3\lambda C_f M\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]. \tag{2.6}$$

The physical meaning of (2.4) is apparent. Ordinarily, the physical mass is the sum of its intrinsic mass and the dynamical radiative self-mass. Here the quark has no intrinsic mass term to begin with so the total mass M is simply equal to the dynamical radiative self-mass.

In order to appreciate better the rearrangement of the old perturbation theory that has been effected by our new bookkeeping, we enumerate in Fig. 1 all the graphs that contribute to the renormalized two-point quark Green's function, up to two-loop accuracy.[5] In Fig. 1, the new

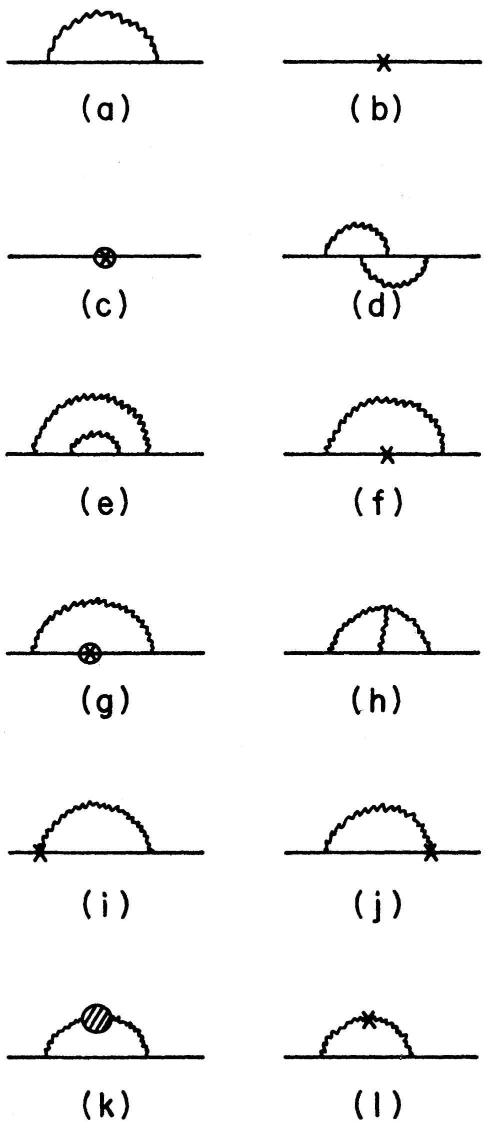

FIG. 1. Diagrams contributing to the quark self-mass, to two-loop accuracy.

mass insertions due to δM term are represented by a cross in a circle. At the one-loop level it has been used to cancel the finite part of Fig. 1(a). As remarked earlier this serves to determine δM to order λ. At the two-loop level, for the graph Fig. 1(g), only the order-λ piece of δM is to be used there so as to be consistent with the new bookkeeping. When all the finite terms that are of order λ^2 are collected together, the higher-order piece of δM can again be determined from Eq. (1.3) now required to be true to order λ^2. The net result for δM so determined up to order λ^2 is given by $[L=\ln(M^2/\mu^2)]$

$$\begin{aligned}\delta M/M=&-\lambda C_f(3L-1)-\lambda^2(3L-1)^2(\tfrac{1}{2}C_f{}^2-\tfrac{1}{12}bC_f)\\&-\lambda^2(3L-1)(\tfrac{1}{2}C_f{}^2+\tfrac{97}{18}C_fC_2-\tfrac{5}{9}C_ff)\end{aligned} \tag{2.7}$$

and the renormalized two-point Green's function for the quarks is in the form as given by Eq. (1.3), with Z being the finite wave-function renormalization, which in the Landau gauge is given by

$$\tilde{Z}=1-\lambda^2(\tfrac{3}{2}C_f{}^2-\tfrac{25}{4}C_fC_2+C_ff)\ln\frac{M^2}{\mu^2}\;. \tag{2.8}$$

In arriving at the final form (1.3) we have been fairly liberal with all the $1/\epsilon$ counterterms needed to cancel the infinities of the theory. Besides the usual (M-independent) counterterms, there will be new M-dependent counterterms. They are, to order λ^2,

$$\begin{aligned}L_{\text{counterterm}}\\=&\frac{6\lambda C_f}{\epsilon}M\bar{\psi}\psi\left[1+\lambda C_f\left[3\ln\frac{M^2}{\mu^2}-1\right]+\cdots\right]\\&+\frac{\lambda^2}{\epsilon}(\tfrac{269}{12}C_fC_2-\tfrac{8}{3}C_ff)M\bar{\psi}\psi[1+\cdots]\\&+\frac{\lambda^2}{\epsilon^2}(-18C_f{}^2-22C_fC_2+4C_ff)M\bar{\psi}\psi[1+\cdots]\;.\end{aligned} \tag{2.9}$$

As advertised earlier, we already see at the λ^2 level the beginning of the self-consistency condition (1.4) acting to suppress these additional counterterms. If this continues to be true in higher orders then we can be sure that we are really talking about the same fundamental Lagrangian as the usual massless QCD Lagrangian. To reach this conclusion we must do a complete renormalization-group analysis of the two-point Green's function.

III. RENORMALIZATION-GROUP ANALYSIS

For this analysis, it is good to first recall the renormalization-group equations for the old theory (viz., QCD of quarks with mass m). In terms of the usual variable $t=(\ln\mu)$, the equation for m_r reads to two-loop renormalization-group accuracy

$$\frac{d}{dt}m_r=-h_1\lambda m_r-h_2\lambda^2 m_r\;, \tag{3.1}$$

where [f = number of flavors, $C_2=N$ for SU(N) group]

$$h_1=6C_f\;, \tag{3.2}$$

$$h_2=3C_f{}^2+\tfrac{97}{3}C_fC_2-\tfrac{10}{3}C_ff\;, \tag{3.3}$$

and the corresponding equation for λ reads

$$\frac{d}{dt}\lambda=-b\lambda^2-c\lambda^3\;. \tag{3.4}$$

The solution to (3.1) is given by

$$m_r=m_0\left[\frac{\lambda}{\lambda_c}\right]^{h_1/b}\left[\frac{b+c\lambda}{b+c\lambda_c}\right]^{h_2/c-h_1/h}\;, \tag{3.5}$$

where m_0 is a renormalization-group invariant and λ_0, also a renormalization-group invariant, is defined as

$$\frac{1}{\lambda_c}=\frac{1}{\lambda}-\frac{c}{b}\ln\left[1+\frac{b}{c\lambda}\right]-bt+a\ . \tag{3.6}$$

Here a is an integration constant that will be specified below.

In our new theory, let us define for the two-point renormalized Green's function ($p^2 \ll M^2$)

$$\Gamma_r^{(2)}=\tilde{Z}^{-1}(\gamma\cdot p-iM+i\Sigma)\ , \tag{3.7}$$

where $\tilde{Z}$, the finite wave-function renormalization constant, and Σ are both functions of M, λ, and μ. From our previous section, we have the series expansion for Σ as

$$\Sigma=M[1+\lambda C_f(3L-1)+\lambda^2(3L-1)^2(\tfrac{1}{2}C_f{}^2-\tfrac{1}{12}bC_f)+\lambda^2(3L-1)(\tfrac{1}{2}C_f{}^2)+\tfrac{97}{18}C_fC_2-\tfrac{5}{9}C_ff)+\cdots]\ . \tag{3.8}$$

It is now easy to check that Σ satisfies the following renormalization-group equation:

$$\left[\mu\frac{\partial}{\partial\mu}-(b\lambda^2+c\lambda^3)\frac{\partial}{\partial\lambda}\right]\Sigma=-(h_1\lambda+h_2\lambda^2)\Sigma\ . \tag{3.9}$$

Equation (3.9) clearly establishes the renormalization property of Σ as being identical to that of m_r. We may thus use our series expansion (3.8) to determine the arbitrary constant a in (3.6). It is, specifically,

$$a=-b/6+\ln M\ . \tag{3.10}$$

Among the infinity of solutions to (3.9), the only one that matches the series expansion in (3.8) is

$$\Sigma=M\left[\frac{\lambda}{\lambda_0}\right]^{h_1/b}\left[\frac{b+c\lambda}{b+c\lambda_0}\right]^{h_2/c-h_1/b}\ . \tag{3.11}$$

Recall that the self-consistency condition (1.3) translates into the requirement that

$$\Sigma=0 \tag{3.12}$$

and Eq. (3.11) lets us satisfy the self-consistency condition to all orders in λ, although only to two-loop renormalization accuracy, by simply setting

$$\frac{1}{\lambda_c}=0\ . \tag{3.13}$$

If we recall the two-loop definition of Λ, the QCD cutoff,

$$\frac{1}{\lambda}+\frac{b}{2}\ln\frac{\Lambda^2}{\mu^2}-\frac{c}{b}\ln\left[1+\frac{b}{c\lambda}\right]=0 \tag{3.14}$$

we obtain finally the result of Eq. (1.7) now true to two-loop renormalization accuracy

$$M=\Lambda_c e^{1/6}\ . \tag{3.15}$$

Note that despite the superficial similarity of Eqs. (1.7) and (3.15), the Λ_c here is the two-loop QCD cutoff. Even though the two-point Green's function is a gauge-dependent quantity, the above result for M is independent of gauge parameter.

Finally we remark on the nature of the extra mass-dependent counterterms needed for our theory. In the light of our new understanding of Σ, we can really rewrite the total Lagrangian as

$$-\bar{\psi}\gamma\cdot\partial\psi-\Sigma\bar{\psi}\psi+\frac{6\lambda C_f}{\epsilon}\Sigma\bar{\psi}\psi+\lambda^2\left[-\frac{18C_f{}^2+3bC_f}{\epsilon^2}+\frac{269C_fC_2-32C_ff}{12\epsilon}\right]\Sigma\bar{\psi}\psi+\text{gauge interactions, etc.}\ , \tag{3.16}$$

and notice that formally the Lagrangian is identical to an ordinary massive QCD theory, including the mass-dependent counterterms. Where we differ from the old theory is the implementation of the condition that $m_r=0$. When this condition is satisfied, our Lagrangian becomes that of the massless QCD Lagrangian, the theory that we are ultimately interested in.

IV. HIGH-ENERGY BEHAVIOR

In this section we shall derive the high-energy ($p^2 \gg M^2$) behavior of the two-point quark Green's function. The same set of graphs in Fig. 1 evaluated now in the high-energy region yield the series expansion (up to an overall p-dependent finite wave-function renormalization)

$$\begin{aligned}\Gamma_r^{(2)}&=\tilde{\tilde{Z}}^{-1}\Gamma_R\\&=\tilde{\tilde{Z}}^{-1}\left\{\gamma\cdot p-iM\left[1+\lambda C_f\left[+3\ln\frac{M^2}{p^2}+3\right]+\lambda^2\left[\ln\frac{p^2}{\mu^2}\right]^2(\tfrac{9}{2}C_f{}^2+\tfrac{3}{4}bC_f)+\lambda^2\ln\frac{p^2}{\mu^2}(-\tfrac{27}{2}C_f{}^2-\tfrac{185}{6}C_fC_2+\tfrac{13}{3}C_ff)\right.\right.\\&\qquad+\lambda^2\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]^2(\tfrac{9}{2}C_f{}^2-\tfrac{3}{4}bC_f)+\lambda^2\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right](\tfrac{3}{2}C_f{}^2+\tfrac{97}{6}C_fC_2-\tfrac{5}{2}C_ff)\\&\qquad\left.\left.+\lambda^2\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]\left[-9C_f{}^2\ln\frac{p^2}{\mu^2}+12C_f{}^2\right]+\lambda^2O(1)\right]\right\}\ .\end{aligned} \tag{4.1}$$

Here Γ_R satisfies the simple renormalization-group equation

$$\left[\mu\frac{\partial}{\partial\mu}-(b\lambda^2+c\lambda^3)\frac{\partial}{\partial\lambda}\right]\Gamma_R=0\ . \tag{4.2}$$

Note that because we have separated out the p-dependent wave-function renormalization, the reduced Green's function has the form

$$\Gamma_R=\gamma\cdot p-i\widetilde{M} \tag{4.3}$$

and it is really $\widetilde{M}$ that we have to study.

The high-energy behavior of $\widetilde{M}$ involves a summation over all higher-order terms in λ, subject to two-loop renormalization-group accuracy. This can be done by use of the renormalization-group equation for $\widetilde{M}$. Let $e^t\widetilde{M}(\bar{\lambda},\overline{M},\mu,p)$ be the function obtained from the original $\widetilde{M}(\lambda,M,\mu,p)$ by everywhere substituting $\bar{\lambda}$ for λ and $\overline{M}$ ($\equiv e^{-t}M$) for M. From our series expansion it can easily be checked that $[t=\frac{1}{2}\ln(p^2/\mu^2)]$

$$\frac{d}{dt}\widetilde{M}=-(6\bar{\lambda}C_f+h_2\bar{\lambda}^2)\widetilde{M}\ , \tag{4.4}$$

where $\bar{\lambda}$ satisfies

$$\frac{d}{dt}\bar{\lambda}=-b\bar{\lambda}^2-c\bar{\lambda}^3\ . \tag{4.5}$$

The integration constant for Eq. (4.5) may again be determined by the implicit relation

$$\frac{1}{\bar{\lambda}}=\frac{1}{\lambda}+\frac{b}{2}[\ln(p^2/\mu^2)-\tfrac{4}{3}]+\frac{c}{b}\ln\left[1+\frac{b}{c\bar{\lambda}}\right]. \tag{4.6}$$

Equation (4.4) can be solved in the by now familiar way, and we arrive very quickly at the result that for large t

$$\widetilde{M}\to M\left[\frac{c}{b^2}\frac{\ln(b^2t/c)}{t}\right]^{b_1/b}\left[1-\frac{h_2}{c}\frac{1}{\ln(b^2t/c)}+\cdots\right]. \tag{4.7}$$

Thus far in our discussion we have suppressed all mention of the effect of the anomalous-dimension term in the renormalization-group equation for the full two-point function.[6] As is well known, in the Landau gauge, the anomalous dimension is of second order in λ and its effect in the high-energy region is simply to multiply Γ_R by a constant, C, where

$$C=\exp(-A\lambda/b) \tag{4.8}$$

and

$$A=-3C_f{}^2+\tfrac{25}{2}C_fC_2-2C_ff\ . \tag{4.9}$$

The complete result for the renormalization two-point quark function therefore reads

$$\Gamma_r^{(2)}\to\widetilde{Z}^{-1}C\left\{\gamma\cdot p-iM\left[\frac{c}{b^2}\frac{\ln(h^2t/c)}{t}\right]^{h_1/b}+\cdots\right\}. \tag{4.10}$$

V. CONCLUSION

What we have shown in this paper is that the Nambu—Jona-Lasinio condition is in fact a renormalization-group invariant. Because of that, we have the freedom to choose μ as we like. For our purposes, to make sense of our new perturbation theory, we have chosen μ large, in the domain where λ is small, and where the logarithms are correspondingly large. We have used the renormalization-group analysis to sum over these leading and next-to-leading logarithms. The remaining terms are indeed small.

One may think that discussions of low-energy chiral breaking necessarily involves an expansion in $\lambda(M)$ which is large. However, renormalization-group invariance of (1.6) assures us the freedom to work in the small-λ domain. In this domain, the $\log(M^2/\mu^2)$ will obviously be large and a summative over leading logarithms and next-to-leading logarithms will be necessary to give a meaningful result. In this way, the renormalization-group equations have been used to give meaning to the dynamically generated quark mass M, within the context of a loop expansion.

This mass M is the constituent mass of the quark as opposed to being the current mass. Because we have not included the effects of quantum flavor dynamics, we cannot at this level differentiate between the up-quark and the down-quark constituent masses. Our treatment so far also cannot distinguish between generations. Such questions properly belong in the context of grand unification and will be addressed in a future publication.

In our new perturbation theory of QCD, what we have not yet been able to accomplish at this stage is the calculation of the expectation value of $\bar{\psi}\psi$ in our (broken) vacuum.[7] The technical difficulty concerns the renormalization bookkeeping for vacuum graphs which are more highly divergent than the n-point functions ($n\geq 2$). This problem is presently under investigation.

In principle our renormalization-group analysis of the Nambu—Jona-Lasinio self-consistency condition has applications for the general tumbling phenomena in gauge theories.[8] It can take the guesswork out of the tumbling and provide a calculable scheme for the actual tumbling scenario that takes place.

Our calculation scheme can also make contact with the Schwinger-Dyson approach to dynamical symmetry breaking.[2] In this latter approach, the high-energy behavior of the quark two-point function is postulated in order to solve a truncated Schwinger-Dyson equation. As has been noted previously, this involves a certain amount of guesswork, since there are two possible high-energy behaviors that are allowed and *a priori* it would be difficult to choose between the two. By our new calculation scheme, we have been able to directly compute the high-energy behavior and find that it is the so-called "irregular" solution that the broken QCD "chooses."

This result is in disagreement with that obtained by using the standard operator-product expansion (OPE) with coefficient functions calculated in the unbroken theory.[9] As Gupta and Quinn have shown,[10] however, in the example of scalar field theories with unstable vacua, the OPE for a shifted theory *differs* from the OPE for an unshifted theory. In the Nambu—Jona-Lasinio mechanism, we shift to the massive vacuum. It amounts to putting in an explicit mass term and then taking it out through a new self-mass interaction term. Because of asymptotic freedom, the strength of this new self-mass interaction term becomes weak at high energies. In the calculation of the new coefficient functions then, it is as if the theory explicitly broke chiral invariance and the high-energy behavior is correspondingly that of the irregular solution.

Intuitively speaking, our result may be understood as follows. At low energies, we have observed the chiral-symmetry breakdown through the nonperturbative $M\neq 0$ solution. In zero-temperature QCD, the β function is negative definite, and there is no phase transition in going from low to high energies. Therefore it is not surprising that we obtain at high energies the behavior characteristic of a broken QCD theory.

So far we have not mentioned the role which the Nambu-Goldstone boson will play in the broken QCD.[11] Preliminary investigations have shown that the same condition (3.13) is responsible for producing a singularity in the associated pseudovector vertex between the quark and an external pseudovector current. In principle, therefore, we can even hope to calculate the dynamics of mass generation of broken gauge theories as well.

These and many other interesting and related questions deserve renewed attention from our new renormalization-group point of view.

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