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Modified quantum chromodynamics and ultraviolet divergences

Hae Won Lee and Pong Youl Pac
Department of Physics, Seoul National University, Seoul 151, Korea
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We present a field-theoretical gauge model which presumably describes the long-distance behavior of quantum chromodynamics (QCD). This theory, modified QCD, possesses local gauge invariance and exhibits a k^{-4} gluon propagator at the tree level, which leads to quark confinement at this level. Quantization is done using the path-integral formalism in the covariant gauge. Developing perturbation theory, we encounter two difficulties: ultraviolet (UV) divergences and infrared (IR) divergences. IR divergences are seriously bad, whereas UV divergences have simple structures and appear only in the one-loop level. We concentrate our attention on the latter and find the complete counterterms using the background-field method extended to these cases. This model turns out to be renormalizable at the expense of introducing a new free parameter.

I. INTRODUCTION

Since the idea of quark confinement was introduced, many theorists have made efforts to prove this conjecture within the framework of quantum chromodynamics (QCD). Until now, several mechanisms that may be responsible for confinement have been proposed.[1] On the other hand, some tried to solve the Schwinger-Dyson equations for this purpose.[2] Recently, West proposed a proof of confinement which relies on general field-theoretical arguments.[3] However, all these approaches do not give us satisfactory explanations to this problem.

In contrast with these efforts, there have been field-theoretical approaches[4,5] which are based on the k^{-4} bare propagators. In Ref. 4, a dipole—vector-gluon model was proposed. In Ref. 5, Kiskis considered a scalar field theory involving higher derivatives. At this stage, it might be helpful to recall that one of the criteria for confinement is a k^{-4} behavior of the gluon propagator at small k^2, as shown in connection with the Wilson loop by West.[6] In this paper, we generalize this idea and present a field-theoretical model, by modifying QCD, where colored quarks interact with gluons as in QCD except that the bare gluon propagator is of the form k^{-4}. This model also possesses local $SU(3)_c$ gauge invariance.

Since our Lagrangian contains terms with second-order spacetime derivatives, the usual canonical quantization procedure cannot be applied. For these kinds of theories, Kiskis presented a quantization procedure[5] which resembles canonical quantization in many respects. This formalism is suitable to see the particle structure of the theory. However, in deriving various Green's functions, there is no difference between this method and the path-integral method. Here, we follow the latter. The corresponding Feynman rules can be obtained following the Faddeev-Popov trick. We adopt the covariant gauge and our rules involve ghost fields.

Feynman rules in this method are the same as in QCD except for the gluon propagator and the gluon vertices. By careful studies of power-counting rules, we could find that in this theory primitive UV divergences appear only in the one-loop level. Compared with this simple behavior of UV divergences, our model suffers serious IR divergences. These two kinds of divergences are the main difficulties encountered in perturbation theory. In this work, we concentrate on the ultraviolet divergences and will find the complete divergent terms of the effective action. We believe that this study is important to understand the quantum-theoretical structure of our model.

To find UV-divergent terms, we use the background-field method.[7] The gauge fixing is done in the so-called background gauge. In this formalism, the gauge invariance is maintained even in the quantum level, and each term appearing in the one-loop effective action is given by the logarithm of the Fredholm determinant of the corresponding operator. From this we could find that the divergent piece of each term has a similar structure to that in QCD, and the complete UV-divergent term is proportional to the ordinary Yang-Mills action.

This paper is organized as follows. In Sec. II the presentation of modified QCD is given. In Sec. III the path-integral quantization of this model is given and the Feynman rules are obtained. In Sec IV we evaluate the one-loop divergences in the background gauge. The final section contains brief discussions on renormalizability.

II. MODIFIED QCD

Generalizing the idea suggested in Refs. 4 and 5—quark confinement through the k^{-4}-type bare propagators—to gauge theories and modifying QCD, we propose a model Lagrangian for the long-range phenomena in QCD as

$$\mathscr{L}_{\rm MQCD}=a^2{\rm tr}[D_\mu,F^{\mu\nu}][D^\lambda,F_{\lambda\nu}]+\bar{\psi}(i\not{D}-m)\psi\ , \quad (1)$$

where

$$D_\mu=\partial_\mu-igA_\mu^aT^a,\ \ [D_\mu,D_\nu]=-igF_{\mu\nu}\ ,$$

and A_μ^a and ψ denote the gluon and quark fields, respectively. Here, the quark field denoted by ψ carries both flavors and colors as in QCD and the T^a's denote the generators of the color gauge group in the appropriate representations. This model contains three parameters: dimensionless coupling constant g, color-singlet mass matrix m, and constant a carrying the dimension of length. However, our model can be regarded as a theory with two parameters by absorbing a into A_μ^a and g.[8] In this case $(a=1)$, the A_μ^a's are dimensionless and g is of mass dimension.[9] Hereafter, we shall follow this convention.

Now, we list some interesting features of this model, modified QCD. First, it is obvious from Eq. (1) that this model possesses the same local gauge invariance as in QCD and the gluon propagator is of the form k^{-4} as we required. We also note that the first term in Eq. (1), when continued analytically into imaginary time, is negative definite. Because of this, one can define the Euclidean version of quantum field theory determined by the Lagrangian (1).[10] Finally, we investigate the dynamical properties of this model. From Eq. (1) we can find that the quarks interact with the gluons in the same manner as in QCD, whereas the dynamics in the gluon sector have been modified.[11] Consequently, the equation of motion for the quark field is the same as in QCD, whereas the field equation for the gluons is modified to be

$$(-D^2 g_{\mu\nu}+2igF_{\mu\nu})D_\lambda F^{\lambda\nu}=gJ_\mu \ , \tag{2}$$

with

$$J_\mu^a=\bar{\psi}\gamma_\mu T^a\psi \ .$$

III. QUANTIZATION

Quantization of our model is not simple due to the presence of higher-order spacetime derivatives and local gauge invariance in Eq. (1). Because of the former we cannot follow the conventional procedure of canonical quantization. In this work, we avoid this difficulty by adopting the path-integral formalism for quantization.[12] The difficulty associated with the latter can also be overcome using the Faddeev-Popov trick. Here, we choose the covariant gauge, i.e.,

$$\partial_\mu A_a^\mu = C_a(x), \quad a=1,\ldots,\dim(G) \ , \tag{3}$$

where the C_a's denote arbitrary constants.

In this gauge, the path-integral method combined with the Faddeev-Popov trick says that

$$e^{iW[J_\mu,K,\bar{K}]}=N\int DA_\mu^a D\psi D\bar{\psi}\det(\partial\cdot D)\prod_{x,a}\delta(\partial\cdot A^a-C^a)\exp\left\{i\int d^4x[\mathscr{L}_{\mathrm{MQCD}}(A,\psi,\bar{\psi})+J_\mu^a A_a^\mu+\bar{K}\psi+\bar{\psi}K]\right\} , \tag{4}$$

where W denotes the generating functional for the connected Green's functions and $K,\bar{K}$ denote antiferromagnetic c-number fields. Since the physical amplitudes do not depend on the choice of gauge, we can insert in Eq. (4) the factor

$$\int DC_a \exp\left[-i\int d^4x \frac{1}{2\xi}C^a\partial^2 C^a\right] , \tag{5}$$

where ξ denotes an arbitrary constant. After integrating for the C^a variables, we obtain

$$e^{iW[J_\mu,K,\bar{K}]}=N\int DA_\mu^a D\psi D\bar{\psi}\det(\partial\cdot D)\exp\left[i\int d^4x\left[\mathscr{L}_{\mathrm{MQCD}}(A,\psi,\bar{\psi})-\frac{1}{2\xi}(\partial\cdot A^a)\partial^2(\partial\cdot A^a)+J_\mu^a A_a^\mu+\bar{K}\psi+\bar{\psi}K\right]\right] . \tag{6}$$

Feynman rules for this model can be derived from this expression. Then it is obvious that our Feynman rules are the same as in QCD except for the gluon propagator and the gluon self-interacting vertices. In this model, the gluon propagator is

$$D_{\mu\nu}^{ab}(k)=i\delta^{ab}\frac{1}{(k^2)^2}\left[g_{\mu\nu}-\left[1-\frac{1}{\xi}\right]\frac{k_\mu k_\nu}{k^2}\right] . \tag{7}$$

This is just what we expected. On the other hand, the rules for the gluon vertices are different from those in QCD in that (a) gluons can also interact with each other via the five- and the six-point vertices and (b) the l-point gluon vertex contains the factor with $(6\text{-}l)$th power of momentum $(l=3,4,5,6)$. Here we omit the detailed rules for the gluon vertices. In Sec. IV we investigate the structure of the UV divergences and evaluate the complete infinite term in the background gauge.

IV. ULTRAVIOLET DIVERGENCES

A. General theory

Any Green's functions in MQCD can be evaluated perturbatively using the Feynman rules derived in the previous section. In such perturbative calculations, we encounter two types of divergences as in QCD: ultraviolet and infrared divergences. However, the detailed structures of these divergences are quite different from those of QCD, especially in the former. The infrared divergences of this model are much stronger than those of QCD because our gluon propagator described in Sec. III is more singular than that of QCD. On the other hand, the ultraviolet divergences have a simple structure. In this work, we concentrate on the latter problem.

First, let us find power-counting rules which tell us the superficial degree of divergences for a given Feynman graph. Let Γ be a one-particle irreducible diagram. Denoting the numbers of external fermion, gluon, and ghost lines by E_F, E_G, E_{gh}, and the numbers of the corresponding internal lines by I_F, I_G, I_{gh}, respectively, we obtain

$$E_F+2I_F=\sum_{i=1}^{V} f_i, \quad E_G+2I_G=\sum_{i=1}^{V} g_i \ ,$$
$$E_{\mathrm{gh}}+2I_{\mathrm{gh}}=\sum_{i=1}^{V} n_i^{\mathrm{gh}} \ , \tag{8}$$

where V denotes the total number of vertices in the graph

Γ and f_i, g_i, n_i^{gh} denote the numbers of fermion, gluon, and ghost lines, respectively, in the ith vertex. Then the superficial degree of divergences $D(\Gamma)$ is

$$D(\Gamma)=\left[\sum d_i\right]+3I_F+2I_{\mathrm{gh}}-4V+4\ , \tag{9}$$

where d_i denotes the order of derivative coupling in the ith vertex. Then making use of Eq. (8), we can reduce Eq. (9) to

$$D(\Gamma)=4-\tfrac{3}{2}E_F-E_{\mathrm{gh}}+\sum\delta i\ , \tag{10}$$

with

$$\delta_i=\tfrac{3}{2}f_i+d_i+n_i^{\mathrm{gh}}-4\ .$$

Note that our expressions for $D(\Gamma)$ and δ_i are different from those of QCD.

With this formula we can find all superficially divergent graphs, i.e., graphs with $D(\Gamma)\geq 0$. First, let us evaluate δ for various vertices in MQCD. From the Feynman rules discussed in the previous section, it is easy to see that $\delta=2-i$ for any i-point vertex ($i=3,4,5,6$). Since $\delta\leq -1$, diagrams with $D(\Gamma)\geq 0$ can appear only in the one-loop or the two-loop levels. It is also evident that for every graph having external fermion or ghost lines $D(\Gamma)<0$.[13] Hence, whenever $D(\Gamma)\geq 0$, we have $E_F=E_{\mathrm{gh}}=0$. By a similar reason, we also have $E_G\leq 4$. Considering all the possible diagrams, we could make the following conclusions. The one-loop corrections to the gluon n-point function ($n=2,3,4$) are superficially divergent and $D(\Gamma)=4-n$. Besides these, there are a few two-loop diagrams for which $D(\Gamma)=0$—the two-loop corrections to the gluon self-energy. However, since the gauge invariance reduces the superficial degree of divergence for the gluon self-energy by 2, these two-loop diagrams are convergent in practice. By the same reason, the one-loop corrections to gluon self-energy diverge logarithmically in practice. On the other hand, the apparent linear divergence of the gluon three-point function is also reduced to a logarithmic one due to Lorentz invariance. In conclusion, all possible divergences are appearing in the one-loop corrections to the gluon n-point functions ($n=2,3,4$) and are logarithmic. In the higher-order calculations, UV divergences are confined within such one-loop subgraphs. In other words, there is no primitive UV divergence beyond the one-loop level.

Once we succeed in resolving the one-loop divergence problem, we need not care about any other ultraviolet divergences. This problem may be stated by two questions—(a) What kinds of form do these divergences have? (b) How should we treat these divergences? As for the first question, we give the answer in Sec. IV B. According to the analysis given above, these one-loop divergences seem to be appearing in the effective action $\Gamma[A,\psi,\bar{\psi}]$, which is the Legendre transform of $W[J,K,\bar{K}]$, as a fourth-degree polynomial of A_μ^a. The detailed form of this infinite term may depend on the regularization prescription. However, the gauge invariance imposed on the regularization procedure excludes many possibilities. In Sec. IV B we calculate this infinite term in the background gauge. In this gauge, the gauge invariance is maintained even in the quantum theory and thus we expect that these terms are proportional to the ordinary Yang-Mills action. The second question as well as renormalizability will be investigated in the final section.

B. Calculation of one-loop divergences

For the utmost utilization of gauge invariance, hereafter we work on the so-called background gauge[17] defined by the condition

$$D_\mu^{\mathrm{cl}}(A-A_{\mathrm{cl}})_a^\mu=C_a(x)\ , \tag{11}$$

where $D_\mu^{\mathrm{cl}}=\partial_\mu-igA_\mu^{\mathrm{cl}}$ and A_μ^{cl} denotes a background gauge field. The merit of this gauge is that, when we evaluate the generating functional of the proper vertices $\Gamma(A,\psi,\bar{\psi})$ and $A_\mu^{\mathrm{cl}}=A_\mu$, Γ is gauge invariant. Following the steps as in Sec. IV A, we obtain

$$e^{iW[J_\mu,K,\bar{K}]}=N\int DA_\mu^a D\psi D\bar{\psi}\,\det{}^{1/2}(D_{\mathrm{cl}}{}^2)\det(D_{\mathrm{cl}}\cdot D)\exp\left[i\int d^4x\left[\mathscr{L}_{\mathrm{MQCD}}(A,\psi,\bar{\psi})-\frac{1}{2\xi}(D_{\mathrm{cl}}\cdot Q)D_{\mathrm{cl}}{}^2(D_{\mathrm{cl}}\cdot Q)+J\cdot A+\bar{K}\psi+\bar{\psi}K\right]\right]\ , \tag{12}$$

where $Q=A-A_{\mathrm{cl}}$ and we have inserted the identity

$$1=\int DC^a(x)\exp\left[-\frac{i}{2\xi}\int d^4x\, CD_{\mathrm{cl}}{}^2C\right]\det{}^{1/2}(D_{\mathrm{cl}}{}^2) \tag{13}$$

instead of Eq. (5). On the other hand, according to the background-field method,[7] $\Gamma[A,\psi,\bar{\psi}]$ is equal to $\tilde{\Gamma}(0,\psi,\bar{\psi})$, where $\tilde{\Gamma}(Q_\mu^a,\psi,\bar{\psi})$ denotes the Legendre transform of the generating functional $\tilde{W}[\tilde{J},K,\bar{K}]$ defined by

$$e^{i\tilde{W}[\tilde{J},K,\bar{K}]}=N\int DQ_\mu^a D\psi D\bar{\psi}\,\det{}^{1/2}(D_{\mathrm{cl}}{}^2)\det[D_{\mathrm{cl}}\cdot(D_{\mathrm{cl}}-igQ)]$$

$$\times\exp\left[i\int d^4x\left[\mathscr{L}_{\mathrm{MQCD}}(A_{\mathrm{cl}}+Q,\psi,\bar{\psi})-\frac{1}{2\xi}(D_{\mathrm{cl}}\cdot Q)D_{\mathrm{cl}}{}^2(D_{\mathrm{cl}}\cdot Q)+\tilde{J}\cdot Q+\bar{K}\psi+\bar{\psi}K\right]\right]\ . \tag{14}$$

The arguments on the UV divergences given in Sec. IV A are also valid in this gauge. Since the UV divergences are absent in the graphs having external quark lines, all the UV divergences appear in $\Gamma(A,\psi=\bar{\psi}=0)$. The one-loop expressions for $\Gamma(A)\equiv\Gamma(A,\psi=\bar{\psi}=0)$ is easily found from Eq. (14), viz.,

$$\Gamma^{(1)}(A)=-\tfrac{3}{2}i\ln\det D^2-i\ln\det(i\not{D}-m)+\frac{i}{2}\ln\det M\ , \tag{15}$$

where

$$M_{\mu\nu}=(D^2-2igF)^2{}_{\mu\nu}+2X\cdot Dg_{\mu\nu}-4X_\nu D_\mu \tag{16}$$

with

$$X_\lambda=-ig[D_\mu,F^\mu_\lambda]\ . \tag{17}$$

In Eq. (16), we have chosen $\xi=1$.

We now evaluate the divergent terms contained in the formal expression (15). First note that, evaluating $\ln\det M$, the second and third terms in Eq. (16) do not contribute to the ultraviolet divergences. The reason is that the divergent terms should be of dimension 4, and all such terms which are gauge invariant and contain X_μ must be of the form $\mathrm{tr}[D_\mu,X^\mu]$, whereas $[D_\mu,X^\mu]=0$ identically. Therefore for our purpose it is sufficient to consider

$$\ln\det(D^2-2igF)^2=2\ln\det(D^2-2igF) \tag{18}$$

instead of $\ln\det M$.[14] On the other hand, the one-loop effective action in QCD has the expression

$$-\tfrac{1}{2}i\ln\det D^2-i\ln\det(i\not{D}-m)+\frac{i}{2}\ln\det(D^2-2igF)\ . \tag{19}$$

Note that the first term of this formula differs from ours by the factor 3. The divergent piece of each term in Eq. (19) is well known to us.[15] Exploiting these formulas, we find our desired expression for the one-loop divergences, viz.,

$$\Gamma_{\mathrm{div}}(A)=\frac{g^2}{32\pi^2}\left[\frac{23}{2}-\frac{N_f}{3}\right]\ln\Lambda^2\int d^4x\,F^a_{\mu\nu}F^{\mu\nu}_a\ , \tag{20}$$

where Λ denotes the ultraviolet cutoff and N_f the number of flavors.

V. DISCUSSIONS

The divergences evaluated in the preceding section can be eliminated by adding a counterterm to the bare Lagrangian. According to these calculations, this counterterm should be

$$-\frac{g^2}{32\pi^2}\left[\frac{23}{2}-\frac{N_f}{3}\right]\ln\left[\frac{\Lambda^2}{\mu^2}\right]\int d^4x\,F^a_{\mu\nu}F^{\mu\nu}_a\ ,$$

where a new parameter μ^2 is introduced. The introduction of this new parameter is inevitable to reflect the finite ambiguities inherent in the counterterm. According to the analyses given in Sec. IV A, no other counterterm is needed and with this counterterm every Green's function is free from the ultraviolet divergences even in the higher-order calculations. As a result, we may conclude that our model is renormalizable at the expense of introducing a new free parameter. A decade ago, Coleman and Weinberg experienced similar situations[16] when considering various massless Higgs models, where they had to introduce counterterms with the form of mass terms which are absent in the Lagrangian. In contrast with our case, they need not introduce a new free parameter because they could maintain masslessness by a renormalization condition. In our case, however, we could not find a similar renormalization condition due to the serious infrared divergences of our model.

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[8]This means the replacements $aA_\mu\to A_\mu$, $a^{-1}g\to g$.

[9]Because of this, the ultraviolet divergences in the higher-order calculations become weaker. The detailed analysis will be given in Sec. IV.

[10]According to the Euclidicity postulate, this may be necessary for a quantum field theory to be well defined in the Lorentz space.

[11]There exists the other possible Lagrangian which satisfies those four properties stated above. It has the form $\mathrm{tr}[D_\alpha,F_{\beta\gamma}][D^\alpha,F^{\beta\gamma}]$. However, the inclusion of such a term does not alter our main conclusions concerning the ultraviolet divergences, and thus we will omit this possibility.

[12]Otherwise, one may follow the quantization procedure given in Ref. 5. However, there exists no difference between these two methods when we calculate various Green's functions.

[13]In reality, $D(\Gamma)=0$ for the one-loop correction to the ghost self-energy. However, since one of the two vertices in this graph carries an external-momentum factor, this correction is UV finite.

[14]Otherwise, we could evaluate the ultraviolet divergences directly from the expression $-\ln\det M$. To be able to use Schwinger's proper-time method for this calculation, we should find the small-τ expansion of the coincident limit, $\langle x\,|\,e^{iM\tau}\,|\,x\rangle$. We have developed a powerful method with which we can find this expansion for such a k^4-type operator M. Our new general method will be reported elsewhere.

[15]See, for example, G. 't Hooft, Nucl. Phys. B62, 444 (1973); C. Lee, H. W. Lee, and P. Y. Pac, *ibid.* B201, 429 (1982).

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