New one-loop counterterms for quantum gravity from Becchi-Rouet-Stora invariance

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If Faddeev-Popov ghosts are added to the asymptotic states of quantum gravity in order to preserve the Becchi-Rouet-Stora invariance, then new Lagrangian counterterms appear in the one-loop approximation.

Quantum gravity is known to be unrenormalizable in perturbation theory. This is a consequence of the wrong dimensionality of Newton's coupling constant. Furthermore, in order to make the physical S matrix unitary, it is necessary to introduce two vector-valued ghost fields obeying Fermi statistics,[1] which are called Faddeev-Popov ghosts. Conventionally, the Faddeev-Popov ghosts are simply discarded as fictitious, i.e., they do not appear in the initial and final states because they go round only internal lines. However, such a standpoint is incompatible with asymptotic completeness.[2] Every particle appearing in the intermediate states of the unitarity relations must have its own asymptotic field and state appearing in both initial and final states. As long as we introduce the Faddeev-Popov ghosts, we must take into account their asymptotic fields because they cannot decay spontaneously. It is perhaps worthwhile to mention here that the ghost fields are unobservables and hence some subsidiary condition (see Ref. 2) on the asymptotic physical states must be assumed. The requirement to introduce the Faddeev-Popov ghosts in the asymptotic states is equivalent, in a background-field method, to the introduction of the classical fields of ghosts as Grassmann-type background fields. Consequently, new possible candidates for, say, one-loop counterterms could arise after multiplicative renormalization.[3]

In this paper, we shall work within the covariant background-field formalism of 't Hooft and Veltman.[4] The only difference from it will be the introduction, by a background-field decomposition, of the classical (or external) fields of Faddeev-Popov ghosts as odd Grassmann variables. As a result we shall find new one-loop counterterms for quantum gravity and their effect will be to spoil, in the case of pure gravity, the renormalizability of the S-matrix elements already in the one-loop approximation.

The starting point is the Lagrangian of gravity interacting with a real scalar field (in units with $16\pi G=c=\hbar=1$)

$$\mathscr{L}_E=(-g)^{1/2}(R-\tfrac{1}{2}\nabla_\mu\phi g^{\mu\nu}\nabla_\nu\phi)\ , \tag{1}$$

where R is the scalar of curvature constructed from $g_{\mu\nu}$ and the metric is assumed of Lorentzian signature. Following the procedure of the background-field method, we write

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}\ ,$$
$$\phi=\bar{\varphi}+\varphi\ ,$$

where the c numbers (background fields) $\bar{g}_{\mu\nu}$ and $\bar{\varphi}$ are understood as solutions of the classical equations of motion. The one-loop Lagrangian for (1) is obtained by expanding $\mathscr{L}_E$ about the classical background fields and taking only the quadratic term in the quantum fluctuations, i.e.,

$$\begin{aligned}\mathscr{L}_2=(-\bar{g})^{1/2}\{&-\tfrac{1}{2}\bar{\nabla}_\mu\varphi\bar{\nabla}_\nu\bar{\varphi}(\bar{g}^{\mu\nu}h^\alpha{}_\alpha-2h^{\mu\nu})-\tfrac{1}{2}\bar{\nabla}_\mu\bar{\varphi}\bar{\nabla}_\nu\bar{\varphi}(h^\mu{}_\alpha h^{\alpha\nu}-\tfrac{1}{2}h^\alpha{}_\alpha h^{\mu\nu})-\tfrac{1}{2}\bar{\nabla}_\mu\varphi\bar{g}^{\mu\nu}\bar{\nabla}_\nu\varphi\\&-[\tfrac{1}{8}(h^\alpha{}_\alpha)^2-\tfrac{1}{4}h^\alpha{}_\beta h^\beta{}_\alpha](\bar{R}+\tfrac{1}{2}\bar{\nabla}_\mu\bar{\varphi}\bar{g}^{\mu\nu}\bar{\nabla}_\nu\bar{\varphi})-h^\nu{}_\beta h^\beta{}_\alpha R^\alpha{}_\nu+\tfrac{1}{2}h^\alpha{}_\alpha h^\nu{}_\beta\bar{R}^\beta{}_\nu-\tfrac{1}{4}\bar{\nabla}_\mu h^\beta{}_\alpha\bar{g}^{\mu\nu}\bar{\nabla}_\nu h^\alpha{}_\beta\\&+\tfrac{1}{4}\bar{\nabla}_\mu h^\alpha{}_\alpha\bar{g}^{\mu\nu}\bar{\nabla}_\nu h^\beta{}_\beta-\tfrac{1}{2}\bar{\nabla}_\beta h^\alpha{}_\alpha\bar{\nabla}_\mu h^{\beta\mu}+\tfrac{1}{2}\bar{\nabla}_\alpha h^\nu{}_\beta\bar{\nabla}_\nu h^{\alpha\beta}\}\ ,\end{aligned} \tag{2}$$

where the covariant derivative denoted by $\bar{\nabla}_\mu$ contains the Christoffel symbol made up of the classical field $\bar{g}_{\mu\nu}$. Let us observe that the greek indices in Eq. (2), as in the following, are raised and lowered by means of the background metric $\bar{g}_{\mu\nu}$. The Lagrangian (2) is invariant under the gauge transformations induced from the infinitesimal diffeomorphisms of the space-time, $x^\mu\rightarrow x^\mu+\epsilon^\mu(x)$ with $|\epsilon^\mu|\ll 1$, namely,

$$\begin{aligned}\delta h_{\mu\nu}(x)&=[\bar{g}_{\alpha\nu}(x)+h_{\alpha\nu}(x)]\bar{\nabla}_\mu\epsilon^\alpha(x)+[\bar{g}_{\mu\alpha}(x)+h_{\mu\alpha}(x)]\bar{\nabla}_\nu\epsilon^\alpha(x)+\epsilon^\alpha(x)\bar{\nabla}_\alpha h_{\mu\nu}(x)\ ,\\\delta\varphi(x)&=\epsilon^\alpha(x)\bar{\nabla}_\alpha[\bar{\varphi}(x)+\varphi(x)]\ .\end{aligned} \tag{3}$$

To fix the symmetry (3) we adopt the so-called de Donder gauge fixing,[1]

$$\mathscr{L}_{\rm GF}=-\tfrac{1}{2}\bar{g}_{\alpha\beta}C^{\alpha}C^{\beta}\ ,$$
$$C^{\alpha}\equiv(-\bar{g})^{1/4}(\bar{\nabla}_{\nu}h^{\nu}{}_{\mu}-\tfrac{1}{2}\bar{\nabla}_{\mu}h^{\alpha}{}_{\alpha}-\varphi\bar{\nabla}_{\mu}\bar{\varphi})t^{\mu\alpha}\ , \tag{4}$$

where $t^{\mu\alpha}$ is the square root of the tensor

$$\sum_{\alpha} t^{\mu\alpha}t^{\nu\alpha}=\bar{g}^{\mu\nu}\ .$$

Thus, the Faddeev-Popov ghost Lagrangian is obtained by subjecting C^{α}, defined in Eq. (4), to the transformations (3) with now the gauge parameters $\epsilon^{\mu}(x)$ set as

$$\epsilon^{\mu}(x)=\rho\eta^{\mu}(x)\ ,$$

where ρ is an anticommuting c number independent of x and $\eta^{\mu}(x)$ is an anticommuting vector-valued field known as the Faddeev-Popov ghost field. We find that the Faddeev-Popov Lagrangian is given by

$$\mathscr{L}_{\rm ghost}=(-\bar{g})^{1/2}\eta^{*}_{\mu}\bar{g}^{\mu\nu}[(\bar{g}^{\alpha\beta}\bar{\nabla}_{\alpha}\bar{\nabla}_{\beta})\bar{g}_{\nu\sigma}-\bar{R}_{\nu\sigma}-\bar{\nabla}_{\nu}\bar{\varphi}\bar{\nabla}_{\sigma}\bar{\varphi}]\eta^{\sigma}\ , \tag{5}$$

where η^{*}_{μ} is an independent ghost field; we may think of η^{*}_{μ} as the Hermitian conjugate of η_{μ}. At this point, we split the ghost η^{μ} into a classical background (or odd Grassmann variable) $\bar{\eta}^{\mu}$ and a quantum part $\tilde{\eta}^{\mu}$, i.e.,

$$\eta^{\mu}=\bar{\eta}^{\mu}+\tilde{\eta}^{\mu}\ .$$

An analogous decomposition is understood for η^{*}_{μ}. So we find that the quadratic part of the Taylor functional expansion of (5) about the background ghost fields is given by, in the more general case of complex matter fields ϕ,

$$\begin{aligned}\mathscr{L}_{\rm 2ghost}=(-\bar{g})^{1/2}\{&h_{\alpha\beta}(\tfrac{1}{2}\bar{\eta}^{*}_{\mu}\bar{J}^{\mu\alpha\beta}_{\nu}-\bar{\eta}^{*}_{\mu}\bar{M}^{\mu\alpha\beta}_{\nu})\tilde{\eta}^{\nu}+\tilde{\eta}^{\nu}\bar{g}_{\gamma\rho}\bar{N}^{\mu\sigma\gamma\rho}_{\nu}\bar{\nabla}_{\sigma}\tilde{\eta}^{*}_{\mu}+\varphi[\bar{\nabla}_{\mu}(\bar{\eta}^{\mu}\bar{\nabla}^{\nu}\bar{\varphi}^{*})]\tilde{\eta}^{*}_{\nu}+\varphi^{*}[\bar{\nabla}_{\nu}(\bar{\eta}^{*}_{\mu}\bar{\nabla}^{\mu}\bar{\varphi})]\tilde{\eta}^{\nu}\\&+h_{\alpha\beta}(\tfrac{1}{2}\bar{\eta}^{\nu}\bar{J}^{\mu\alpha\beta}_{\nu}-\bar{\eta}^{\nu}\bar{M}^{\mu\alpha\beta}_{\nu})\tilde{\eta}^{*}_{\mu}+\varphi[\bar{\nabla}^{\mu}(\bar{\eta}^{\nu}\bar{\nabla}_{\nu}\bar{\varphi}^{*})]\tilde{\eta}^{*}_{\mu}+\varphi^{*}[\bar{\nabla}_{\nu}(\bar{\eta}^{\nu}\bar{\nabla}^{\mu}\bar{\varphi})]\tilde{\eta}^{*}_{\mu}\}+\mathscr{L}^{\rm HV}_{\rm ghost}\ ,\end{aligned} \tag{6}$$

where we have defined

$$\bar{J}^{\mu\alpha\beta}_{\nu}\equiv\bar{g}^{\mu\tau}(\bar{\nabla}_{\gamma}\bar{N}^{\gamma\alpha\beta}_{\nu\tau}+\bar{N}^{\pi\delta\rho}_{\nu\tau}\bar{N}^{\gamma\alpha\beta}_{\delta\rho}\bar{g}_{\pi\gamma})\ , \tag{7a}$$

$$\bar{N}^{\mu\nu\gamma}_{\alpha\beta}\equiv\bar{g}^{\mu\pi}\bar{\Gamma}^{\nu}_{\pi(\alpha}\delta^{\gamma}_{\beta)},\quad \bar{N}^{\beta\mu\nu\gamma}_{\alpha}\equiv\bar{g}^{\beta\tau}\bar{N}^{\mu\nu\gamma}_{\alpha\tau}\ , \tag{7b}$$

$$\bar{M}^{\rho\alpha\beta}_{\sigma}\equiv\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}(\delta^{\rho}_{\sigma}\bar{\nabla}_{\mu}\bar{\nabla}_{\nu}-\bar{N}^{\rho\tau\chi}_{\pi\mu}\bar{N}^{\pi\delta\eta}_{\sigma\nu}\bar{g}_{\tau\delta}\bar{g}_{\chi\eta})\ , \tag{7c}$$

$$\mathscr{L}^{\rm HV}_{\rm 2ghost}\equiv(-\bar{g})^{1/2}(\tilde{\eta}^{*}_{\rho}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\tilde{\eta}^{\rho}-\tilde{\eta}^{*}_{\mu}\bar{R}_{\nu}{}^{\mu}\tilde{\eta}^{\nu}-\tilde{\eta}^{*}_{\mu}\bar{\nabla}^{\mu}\bar{\varphi}^{*}\bar{\nabla}_{\nu}\bar{\varphi}\tilde{\eta}^{\nu})\ . \tag{7d}$$

We note that $\bar{\Gamma}^{\nu}_{\pi\alpha}$ is the Christoffel symbol due to the background metric $\bar{g}_{\mu\nu}$ and $\mathscr{L}^{\rm HV}_{\rm ghost}$ is the ghost contribution already obtained by 't Hooft and Veltman.[1] We can rewrite Eq. (6) in terms of real-valued fields setting

$$\sqrt{2}\,\eta_{\nu}=\eta^{(1)}_{\nu}+i\eta^{(2)}_{\nu}\ , \tag{8a}$$

$$\sqrt{2}\,\phi=\phi^{(1)}+i\phi^{(2)}\ , \tag{8b}$$

so that Eq. (6) becomes in matrix form $(i,j=1,2)$

$$\mathscr{L}_{\rm 2ghost}=(\tilde{\eta}_{(i)}\cdot A^{ij}\cdot\tilde{\eta}_{(j)}+\varphi_{(i)}\cdot X^{ij}\cdot\tilde{\eta}_{(j)}+h_{(i)}\cdot D^{ij}\cdot\tilde{\eta}_{(j)})(-\bar{g})^{1/2}, \tag{9}$$

where we have used the "doubling trick"[5] for the gravitational fluctuations $h_{\alpha\beta}$ and defined

$$[A^{ij}]^{\mu\nu}=[A^{ij}_{1}]^{\mu\nu}+[A^{ij}_{2}]^{\mu\nu}\ ,$$
$$[A^{ij}_{1}]^{\mu\nu}\equiv\tfrac{1}{2}[I_{d}+\sigma_{y}]^{ij}\bar{g}^{\mu\nu}\,\bar{\Box},\quad \bar{\Box}\equiv\bar{g}^{\alpha\beta}\bar{\nabla}_{\alpha}\bar{\nabla}_{\beta}\ ,$$
$$[A^{ij}_{2}]^{\mu\nu}\equiv-\tfrac{1}{2}[I_{d}+\sigma_{y}]^{ij}(\bar{R}^{\mu\nu}+\bar{\nabla}^{\mu}\bar{\varphi}^{*}\bar{\nabla}^{\nu}\bar{\varphi})\ ,$$
$$[X^{ij}]^{\nu}\equiv\{[\sigma_{z}]^{ij}{\rm Re}(B^{\nu})-[\sigma_{x}]^{ij}{\rm Im}(B^{\nu})\}+\{[I_{d}]^{ij}{\rm Re}(E^{\nu})+i[\sigma_{y}]^{ij}{\rm Im}(E^{\nu})\}\ ,$$
$$[D^{ij}]^{\alpha\beta\nu}=[D^{ij}_{1}]^{\alpha\nu}\bar{\nabla}^{\beta}+[D^{ij}_{2}]^{\alpha\beta\nu}\ ,$$
$$[D^{ij}_{1}]^{\alpha\nu}\equiv\tfrac{1}{2}[I_{d}+\sigma_{x}-i\sigma_{y}+\sigma_{z}]^{ij}{\rm Re}(\bar{\nabla}^{\alpha}\bar{\eta}^{\nu})-\tfrac{1}{2}[I_{d}+\sigma_{x}+i\sigma_{y}-\sigma_{z}]^{ij}{\rm Im}(\bar{\nabla}^{\alpha}\bar{\eta}^{\nu})\ ,$$
$$[D^{ij}_{2}]^{\alpha\beta\nu}\equiv\tfrac{1}{2}[I_{d}+\sigma_{x}-i\sigma_{y}+\sigma_{z}]^{ij}{\rm Re}(D^{\alpha\beta\nu})-\tfrac{1}{2}[I_{d}+\sigma_{x}+i\sigma_{y}-\sigma_{z}]^{ij}{\rm Im}(D^{\alpha\beta\nu})\ ,$$

where

$$[I_{d}]^{ij}=\delta^{ij},\quad [\sigma_{x}]^{ij}=\begin{bmatrix}0&1\\1&0\end{bmatrix},$$
$$[\sigma_{y}]^{ij}\equiv\begin{bmatrix}0&-i\\-i&0\end{bmatrix},\quad [\sigma_{z}]^{ij}\equiv\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$
$${\rm Re}(A+iB)\equiv A,\quad {\rm Im}(A+iB)\equiv B\ ,$$
$$B^{\nu}\equiv\bar{\nabla}^{\mu}(\bar{\eta}^{*}_{\mu}\bar{\nabla}^{\nu}\bar{\varphi})\ ,$$
$$E^{\nu}\equiv\bar{\nabla}^{\nu}(\bar{\eta}^{\mu}\bar{\nabla}_{\mu}\bar{\varphi}^{*})\ ,$$
$$D^{\alpha\beta\nu}\equiv\tfrac{1}{2}\bar{\eta}^{*}_{\mu}\bar{g}^{\sigma\nu}(\bar{\nabla}^{\eta}\bar{N}^{\mu\alpha\beta}_{\sigma\eta}-\bar{N}^{\mu\tau x}_{\gamma\sigma}\bar{N}^{\gamma\alpha\beta}_{\tau x})\ .$$

It is more convenient to rewrite the ghost Lagrangian (9) in compact form

$$\begin{aligned}\mathscr{L}_{\rm 2ghost}=\{&[\Phi_{(i)}]^{T}_{a}\cdot[\mathscr{H}^{ij}]^{ab}\cdot[\Phi_{(j)}]_{b}\\&+2[\Phi_{(i)}]^{T}_{a}\cdot[\mathscr{N}^{ij}\bar{\nabla}]^{ab}\cdot[\Phi_{(j)}]_{b}\\&+[\Phi_{(i)}]^{T}_{a}\cdot[\mathscr{M}^{ij}]^{ab}\cdot[\Phi_{(j)}]_{b}\}(-\bar{g})^{1/2}\ .\end{aligned} \tag{10}$$

Here, the contraction between world indices must be understood and the matrices $\mathscr{H}$, $\mathscr{N}$, and $\mathscr{M}$ are of the form

$$[\mathscr{H}^{ij}]^{ab} \equiv \begin{bmatrix} [A_1^{ij}] & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} \end{bmatrix}, \quad a,b=1,2,3 \tag{11a}$$

$$[\mathscr{N}^{ij}]^{ab} \equiv \begin{bmatrix} \underset{\sim}{0} & [D_1^{ij}] \\ \underset{\sim}{0} & \underset{\sim}{0} \end{bmatrix}, \tag{11b}$$

$$[\mathscr{M}^{ij}]^{ab} \equiv \begin{bmatrix} [A_2^{ij}] & [X^{ij}] & [D_2^{ij}] \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} \end{bmatrix}, \tag{11c}$$

$$[\Phi_{(i)}]_a \equiv (\bar{\eta}_{(i)}, \varphi_{(i)}, h_{(i)})^T . \tag{11d}$$

In this formalism, the counter-Lagrangian that eliminates all one-loop divergences due to (10) is given in the dimensional regularization scheme[4] by

$$\Delta\mathscr{L}_{\text{ghost}} = -\frac{(-\bar{g})^{1/2}}{\epsilon}\text{Tr}[\tfrac{1}{2}(\mathscr{M}^{ij} - \bar{\nabla}\cdot\mathscr{N}^{ij} + \mathscr{N}^{il}\cdot\mathscr{N}^{lj} - \tfrac{1}{6}R)^2 + \tfrac{1}{12}(\bar{\nabla}\times\mathscr{N}^{il} + \mathscr{N}^{ij}\times\mathscr{N}^{lj}) + \tfrac{1}{60}(\bar{R}_{\alpha\beta}\bar{R}^{\alpha\beta} - \tfrac{1}{3}\bar{R}^2)] ,$$

$$\epsilon \equiv 16\pi^2(n-4) \to 0^+ , \tag{12}$$

where the parameter n is the dimensionality of the space-time analytically continued in the number of the dimensions from 4 to n complex.

After some lengthy, but trivial, calculations, one gets by Eqs. (11)

$$\Delta\mathscr{L}_{\text{ghost}} = -\frac{(-\bar{g})^{1/2}}{\epsilon}[\tfrac{17}{60}\bar{R}^2 + \tfrac{7}{30}\bar{R}_{\alpha\beta}\bar{R}^{\alpha\beta} + 32\bar{E}^2 + \tfrac{4}{3}\bar{E}_{[\mu\nu]}\bar{E}^{[\mu\nu]} + \bar{R}^{\mu\nu}(\bar{\nabla}_\mu\bar{\varphi}^*\bar{\nabla}_\nu\bar{\varphi}) + \tfrac{1}{6}\bar{R}(\bar{\nabla}_\mu\bar{\varphi}\bar{\nabla}^\mu\bar{\varphi}^*)$$
$$+ \tfrac{16}{3}\bar{E}\bar{R} + 4\bar{E}(\bar{\nabla}_\mu\bar{\varphi}^*\bar{\nabla}^\mu\bar{\varphi}) + \tfrac{1}{2}(\bar{\nabla}_\mu\bar{\varphi}^*\bar{\nabla}_\nu\bar{\varphi})^2 + (\bar{\nabla}^\mu\bar{B}_\mu{}^{\alpha*})(\bar{\nabla}^\mu\bar{B}_{\mu\alpha})$$
$$+ (\bar{\nabla}^\alpha\bar{B}_\mu{}^{\mu*})(\bar{\nabla}_\alpha\bar{B}_\mu{}^\mu) + \tfrac{1}{12}\bar{\eta}^*_\sigma\bar{R}^{\mu\nu\beta\sigma}\bar{R}_{\mu\nu\beta\rho}\bar{\eta}^\rho] . \tag{13}$$

In Eq. (13) we have defined the quantities

$$\bar{E}_{\mu\nu} \equiv \bar{\nabla}_\mu\bar{\eta}^*_\rho\bar{\nabla}_\nu\bar{\eta}^\rho = \bar{E}_{(\mu\nu)} + \bar{E}_{[\mu\nu]} ,$$
$$\bar{E}_{(\mu\nu)} \equiv \tfrac{1}{2}(\bar{E}_{\mu\nu} + \bar{E}_{\nu\mu}),$$
$$\bar{E}_{[\mu\nu]} \equiv \tfrac{1}{2}(\bar{E}_{\mu\nu} - \bar{E}_{\nu\mu}), \quad \bar{E} \equiv \bar{g}^{\alpha\beta}\bar{E}_{(\alpha\beta)} , \tag{14}$$
$$\bar{B}_\mu{}^\nu \equiv \bar{\eta}_\mu\bar{\nabla}^\nu\bar{\varphi} .$$

Remember that $\bar{\eta}^\mu$ and $\bar{\eta}^*_\mu$ are odd Grassmann c-number variables. We see that in the hypothesis $\bar{\eta}^\alpha = \bar{\eta}^*_\beta = 0$, Eq. (13) is in agreement with the result obtained by 't Hooft and Veltman.[1]. Moreover, for the case of pure gravity ($\bar{\varphi} = \bar{\varphi}^* = 0$)

$$\Delta\mathscr{L}^{(\text{PQG})}_{\text{ghost}} = -\frac{(-\bar{g})^{1/2}}{\epsilon}(\tfrac{17}{60}\bar{R}^2 + \tfrac{7}{30}\bar{R}_{\alpha\beta}\bar{R}^{\alpha\beta} + \tfrac{16}{3}\bar{R}\bar{E} + 32\bar{E}^2 + \tfrac{4}{3}\bar{E}_{[\mu\nu]}\bar{E}^{[\mu\nu]} + \tfrac{1}{12}\bar{\eta}^*_\sigma\bar{R}^{\mu\nu\beta\sigma}\bar{R}_{\mu\nu\beta\rho}\bar{\eta}^\rho) . \tag{15}$$

In the case of pure gravity, the field equations for the classical background fields derived from

$$\mathscr{L} = \mathscr{L}_E + \mathscr{L}_{\text{GF}} + \mathscr{L}_{\text{FP}}$$

with $\mathscr{L}_E$, $\mathscr{L}_{\text{GF}}$, and $\mathscr{L}_{\text{FP}}$ defined in Eq. (1), Eqs. (5a), and (5b), respectively, read[6]

$$\bar{R}_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R} = -\tfrac{1}{2}(\bar{E}_{(\mu\nu)} - \bar{g}_{\mu\nu}\bar{E}) ,$$
$$\partial_\mu[(-\bar{g})^{1/2}\bar{g}^{\mu\nu}] = 0 ,$$
$$(\bar{g}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu)\bar{\eta}^\alpha = 0 , \tag{16}$$
$$(\bar{g}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu)\bar{\eta}^*_\alpha = 0 .$$

Then, Eq. (15) can be simplified with the aid of Eqs. (16) and becomes

$$\Delta\mathscr{L}^{(\text{PQG})}_{\text{ghost}} = -\frac{(-\bar{g})^{1/2}}{\epsilon}(\tfrac{18\,859}{480}\bar{E}^2 + \tfrac{7}{120}\bar{E}_{(\mu\nu)}\bar{E}^{(\mu\nu)} + \tfrac{4}{3}\bar{E}_{[\mu\nu]}\bar{E}^{[\mu\nu]} + \tfrac{1}{12}\bar{\eta}^*_\sigma\bar{R}^{\mu\nu\beta\sigma}\bar{R}_{\mu\nu\beta\rho}\bar{\eta}^\rho) . \tag{17}$$

One recognizes that the one-loop counterterm (17) is not of a type present in the Einstein "effective" Lagrangian and is therefore of nonrenormalizable type. A similar conclusion was already obtained in Refs. 3 and 7.

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[1]G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré XX, 69 (1974).

[2]G. Curci and R. Ferrari, Nuovo Cimento 35A, 273 (1976); see also T. Kugo and I. Ojima, Suppl. Prog. Theor. Phys. 66, 1 (1979).

[3]N. Nakanishi and K. Yamagishi, Prog. Theor. Phys. 65, 1719 (1981).

[4]G. 't Hooft and M. Veltman, Nucl. Phys. B50, 318 (1972); G. 't Hooft, *ibid.* B62, 444 (1973); see also J. Honerkamp, *ibid.* B48, 269 (1972); R. Kallosh, *ibid.* B78, 293 (1974); S. Ichinose and M. Omote, *ibid.* B203, 221 (1982).

[5]The "doubling trick" consists in the introduction, in addition to the field $h_{\mu\nu}$, of the field $h'_{\mu\nu}$ which interacts with itself and the other fields in the identical way as the $h_{\mu\nu}$.

[6]N. Nakanishi, Prog. Theor. Phys. 59, 972 (1978).

[7]G. Leibbrandt, Rev. Mod. Phys. 47, 849 (1975); D. M. Capper and M. A. Namazie, Nucl. Phys. B142, 535 (1978); M. Martellini, Nuovo Cimento B54, 1 (1979).