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Proton stopping power of heavy nuclei

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Inclusive cross sections for the reactions $p+A \to p+X$ at 100 GeV are discussed in the framework of the evolution model proposed by Hwa. An exact solution to the evolution equation is found. A momentum-degradation length of 4.9 fm is inferred from the data.

Recent data[1] on the inclusive cross sections for $p+A \to p+X$, where the incident proton energy is 100 GeV and A is a heavy nucleus, have fired much interest in its implications for ultrarelativistic heavy-ion collisions. In one analysis of the data, Busza and Goldhaber[2] come to the conclusion that the proton stopping power of heavy nuclei is far greater than should be expected on the basis of conventional ideas, although still much less than a baryon cascade would predict. In a very different analysis, Hwa[3] proposed an evolution model to describe the data. He reaches the opposite conclusion, that the momentum-degradation length in nuclear matter (distance over which the proton's momentum decreases by $1/e$) is very large, perhaps 17 fm.

In an effort to understand the above disparity we found an exact analytic solution to the evolution equation. We refit the data and obtain a momentum-degradation length of 4.9 fm. The exact sources of this huge difference will be commented on at the end.

According to the evolution model[3] consider a nucleon propagating through the nucleus A. Depending on the impact parameter $\vec{s}$ this proton collides on average with

$$\overline{N}_A(\vec{s}) = \sigma_{NN} \int dz\, \rho_A(\vec{s},z) \tag{1}$$

nucleons. Here σ_{NN} is the nucleon-nucleon cross section (taken to be 40 mb for numerical purposes) and $\rho_A(\vec{r})$ is the nuclear-density distribution. The cross section for collision on N target nucleons in a line is then given by integrating the corresponding Poisson distribution over all impact parameters:[4,5]

$$\sigma_A(N) = \int d^2s \frac{1}{N!} [\overline{N}_A(\vec{s})]^N \exp[-\overline{N}_A(\vec{s})] \ . \tag{2}$$

If we neglect the surface diffuseness of the nuclei and apply a uniform density distribution of $\rho_0 = 0.17\ \text{fm}^{-3}$, Eq. (2) leads to

$$\sigma_A(N) = (N+1)\pi \left[1 - e^{-F} \sum_{j=1}^{N+1} F^j/j!\right] \Big/ 2\sigma_{NN}^2 \rho_0^2 \ ,$$
$$F = 2\sigma_{NN}\rho_0 R_A \ . \tag{3}$$

We intend to describe the momentum degradation of the nucleon propagating through the nucleus. Denote the invariant distribution function by $H(x,N)$. This is the probability that the incident nucleon has laboratory momentum fraction x after hitting N target nucleons. It is normalized to unity in invariant phase space:

$$\int_0^1 \frac{dx}{x} H(x,N) = 1 \ . \tag{4}$$

In order to determine $H(x,N)$ it is assumed[3] that the following evolution equation is satisfied:

$$H(x,N+1) = \int_x^1 \frac{dx'}{x'} H(x',N) Q(x/x') \ . \tag{5}$$

Here $Q(x)$ is the probability that the incident nucleon has momentum fraction x after a collision with one more target nucleon. Since baryon number is conserved, we assume that in high-energy collisions the incident nucleon (or its valence quarks) survives the collision with the target nucleons, so $Q(x)$ is normalized to unity. If we also assume that the first and subsequent collisions in a tube show the same behavior, $Q(x)$ can be approximated by[3]

$$Q(x) = \lambda x + \lambda' \delta(x-1) = H(x,1) \ . \tag{6}$$

The first term is a statement that for hard nucleon-nucleon collisions the longitudinal-momentum distribution is flat. The second term represents elastic and soft inelastic collisions. This seems to be a fair representation of the data in the range 6–405 GeV at least.[6,7] Due to the normalization condition on $Q(x)$, the parameters λ and λ' satisfy the relation $\lambda + \lambda' = 1$. Evidently, they may be interpreted as probabilities.

In Ref. 3 an approximate solution of Eqs. (5) and (6) is given for $N >> 1$. Its applicability is, however, questionable since even for large nuclei the average collision number in a tube is only 2–4. The complete solution of Eqs. (5) and (6) can be given analytically for any N by the following simple formula:

$$H(x,N) = x \sum_{n=1}^{N} \binom{N}{n} \lambda^n \lambda'^{(N-n)} \frac{(-\ln x)^{n-1}}{(n-1)!} + \lambda'^N \delta(x-1) \ . \tag{7}$$

This immediately gives the inclusive nucleon cross section integrated over the transverse momentum as

$$\sum_N \sigma_A(N) H(x,N) \ . \tag{8}$$

We can also fit the differential cross section $E\,d^3\sigma/dp^3$ at fixed $p_\perp$ if it factorizes in $p_\perp$ and x. This, however, requires the introduction of an unknown normalization factor $g(p_\perp)$ in Eq. (8). The experiment of Ref. 1 is for $p+A \to p+X$ and $p+A \to \bar{p}+X$, with most data taken at $p_\perp = 0.3$ GeV. The evolution model as formulated so far predicts the final baryon distribution in x and integrated over all $p_\perp$. Concerning the question of the integrity of the proton we can make the following observations. First, it is known that the number of antiprotons[1] and hyperons[8] are negligible compared to the number of protons. This leaves only the neutron. An essential point in the analysis is that charge exchange in $p+A$ reactions should be suppressed relative to $p+p$ reactions.[3] The reasoning is that the first hard collision separates the valence quarks from their surrounding sea of quark-antiquark pairs. Therefore, it is much more difficult for the valence quarks to find a down quark to form a neutron, i.e., $uud + d\bar{d} \to udd + u\bar{d}$. Certainly this could be measured in the future. Concerning the question of factorization in $p_\perp$ and x, we assume that it holds for $p+A$ reactions as it does for $p+p$ reactions.

We have fit the data of Ref. 1 with a best value of $\lambda = 0.52$ and an overall normalization of $g(p_\perp = 0.3$ GeV/$c) = 0.99$ (GeV/$c)^{-2}$. See Fig. 1. The goodness of the fit lends support to our neglect of charge exchange, although an A-independent renormalization due to that effect could not be discerned from the present analysis. We also note that some of the figures in Ref. 1 plot the invariant cross section as a function of Feynman x. The difference in x's only begins to become significant (about 10%) for $x < 0.3$. Finally, we note that even though the value of λ has been fitted to $p+A$ reactions, it is consistent with $p+p$ reactions. In the latter case

$$\frac{d\sigma}{dx} = \lambda \sigma_{NN} = 20.8 \text{ mb} \ , \tag{9}$$

which compares well with the experimental values 17–22 mb for a compilation[7] of beam energies from 19 to 405 GeV.

Using the solution Eq. (7) for $H(x,N)$ the momentum degradation of an incident proton in nuclear matter can be described. If its initial momentum is $p(0)$ the expected value of the longitudinal momentum after penetrating to a depth z is

$$p(z) = p(0) \int_0^1 \frac{dx}{x} x H(x, \bar{N}(z)) = (1-\lambda/2)^{\bar{N}(z)} \ , \tag{10}$$

where

$$\bar{N}(z) = \rho_0 \sigma_{NN} z = 0.68 (z/1 \text{ fm}) \tag{11}$$

is the average number of collisions up to this depth. The distribution of Eq. (10) is an exponential with degradation

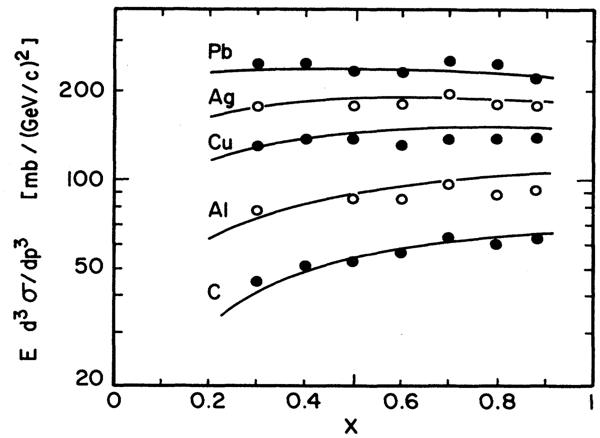

FIG. 1. Invariant differential cross sections for the inclusive reactions $p+A \to p+X$ for an incident energy of 100 GeV and at a final transverse momentum of 0.3 GeV/c. The data are from Ref. 1. The solid curves are the predictions of the evolution model with $\lambda = 0.52$.

length

$$\Lambda_p = [-\rho_0 \sigma_{NN} \ln(1-\lambda/2)]^{-1} = 4.9 \text{ fm} \ . \tag{12}$$

This is only somewhat larger than 3 mean free paths, because

$$l = 1/\rho_0 \sigma_{NN} = 1.5 \text{ fm} \ .$$

There are three differences between Hwa's analysis and ours. First, we have found an exact analytic solution to the evolution equation and hence do not need to employ a large-N approximation. Second, we calculate the probability $\sigma_A(N)$ for the proton to hit N nucleons and then sum over N. In Ref. 3 only a single mean $\bar{N}_A$ was employed. Third, we use a cross-sectional area of 40 mb for each tube. This compares with $\pi(0.8 \text{ fm})^2 = 20$ mb used in Ref. 3.

In conclusion, based upon proton-nucleus data and an exact solution to the evolution model of Hwa, we infer a momentum-degradation length of 4.9 fm for high-energy protons in nuclear matter. The primary uncertainties in the present analysis of the data seem to be the relative contribution of neutrons and the extent to which the x distribution at $p_\perp = 0.3$ GeV/c reflects the $p_\perp$-integrated distribution. We hope that future experiments will eliminate these uncertainties.

After completion of this work we received a paper by C.-Y. Wong [Phys. Rev. Lett. 52, 1393 (1984)]. In it the data of Ref. 1 are fit with a model very similar to the evolution model. The fit to the data is not quite as good as ours, especially for lead. No momentum-degradation length is quoted, but a value of 3.8 fm may be inferred.

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