Two-gluino bound states

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We estimate the branching ratio and argue that two-gluino bound states can be detected in $\Upsilon \to \gamma + {}^1S_0(\tilde{g}\tilde{g})$ if $m_{\tilde{g}} < 4.5$ GeV. The signal would be comparable to the backgrounds in magnitude. In the case of top-quarkonium decay, the signal could be much enhanced due to the exchange of a possibly light scalar top quark. We also show that two-gluino bound states can exist only in ${}^1S_0$, ${}^3P_{0,1,2}$, ${}^1D_2$, ${}^3F_{2,3,4}, \ldots$, but not in ${}^3S_1$, ${}^1P_1$, ${}^3D_{1,2,3}$, ${}^1F_3, \ldots$ states. The decay width of the $\tilde{g}\tilde{g}$ ground state is calculated. Finally, we discuss the $\tilde{g}\tilde{g}$ potential.

One of the most basic assumptions of quantum chromodynamics is the existence of an octet of colored (and confined) massless objects called gluons. A direct and decisive test of this hypothesis would be the observation of color-singlet two-gluon bound states (popularly known as glueballs). Much experimental activity is currently going on to discover such states but the situation is still not conclusive.

Recently a lot of work is being done in supersymmetry (SUSY), which predicts the existence of superpartners of quarks, leptons, and gauge bosons. In particular, SUSY models predict the existence of an octet of colored spin-$\frac{1}{2}$ partners of gluons known popularly as gluinos. The gluinos are expected to be massive due to the breakdown of SUSY. A lot of theoretical and experimental effort is going on in trying to discover them. In fact the beam-dump experiments already give a lower bound $m_{\tilde{g}} > 1$–4 GeV,[1] depending on various assumptions. There are also arguments in favor of heavier gluinos.[2] Clearly, as for gluons a decisive and direct test of gluinos would be the observation of two-gluino bound states.

The purpose of this paper is to discuss the production mechanism, the decay width, and the potential of the two-gluino bound states. We first calculate the branching ratio for the decay

$$\Upsilon \to \gamma\zeta, \quad \zeta = 1\,{}^1S_0(\tilde{g}\tilde{g}) \; . \tag{1}$$

Here ζ is the $\tilde{g}\tilde{g}$ ground state. The attractive point about this decay mode is that $\tilde{g}\tilde{g}$ bound states could be observed in the inclusive photon spectrum as a narrow peak around

$$E_\gamma = (M_\Upsilon^2 - M_\zeta^2)/2M_\Upsilon \; , \tag{2}$$

so that even though the branching ratio is small, there is a fair chance of its detection. We then show that unlike the quarkonium case, the levels $n\,{}^3S_1$, $n\,{}^1P_1$, $n\,{}^3D_{1,2,3}$, $n\,{}^1F_3, \ldots$ are *absent* in the $\tilde{g}\tilde{g}$ spectrum. Further, we calculate the decay width of the $\tilde{g}\tilde{g}$ ground state ζ and find that $\Gamma(\zeta) = 300$–47 MeV, for $1 < m_{\tilde{g}} < 4.5$ GeV. Finally we discuss the $\tilde{g}\tilde{g}$ potential.

I. PRODUCTION OF TWO-GLUINO BOUND STATES

If $m_{\tilde{g}} \leq 4.5$ GeV then $\tilde{g}\tilde{g}$ bound states could be produced through the decay mode given by Eq. (1). Since the mass of the gluino is expected[1] to be at least of the same order as that of the charm quark, the process of Eq. (1) would take place at short distances, and a perturbative evaluation of the production amplitude should be meaningful. When the gluino is much lighter, we do not have a reliable method to calculate the long-distance contribution of the nonperturbative nature. The relevant diagrams are drawn in Fig. 1(a). These are quite analogous to the lowest-order diagrams for the process

$$\Upsilon \to \gamma\eta_c, \quad \eta_c = 1\,{}^1S_0(c\bar{c}) \; . \tag{3}$$

The corresponding branching ratio has already been calculated by Guberina and Kühn.[3] Their calculation also goes through in our case provided we replace M_{η_c} by M_ζ, $R_{\eta_c}(0)$ by $R_\zeta(0)$, and the color factor $C = \frac{4}{9}$ by $\frac{1}{2} \times 6$. The reader should notice that the factor $\frac{1}{2}$ in C comes from the Majorana nature of the gluino. The final expression for the decay rate is given by [see Eq. (13) of Ref. 3]

$$\Gamma(\Upsilon \to \gamma\zeta) = \frac{2^9 \alpha_s^4 \alpha Q_b^2 |f(\xi)|^2}{M_\Upsilon^3(1-\xi)} \frac{R_\Upsilon^2(0)}{4\pi M_\Upsilon} \frac{R_\zeta^2(0)}{4\pi M_\zeta} \; , \tag{4}$$

where $\xi = M_\zeta^2/M_\Upsilon^2$ and $f(\xi)$ is a complicated loop integral which can, however, be evaluated analytically. The function $|f(\xi)|^2$ has been plotted in Ref. 3 as a function of ξ. Notice that as $m_{\tilde{g}}$ varies from 1 to 4.5 GeV, the value of $|f(\xi)|^2$ drops smoothly from 13 to about 0. The branching ratio is

$$\frac{\Gamma(\Upsilon \to \gamma\zeta)}{\Gamma(\Upsilon \to \gamma gg)} = \frac{9\alpha_s^2 |f(\xi)|^2 |R_\zeta(0)|^2}{\pi(\pi^2 - 9) M_\zeta (M_\Upsilon^2 - M_\zeta^2)} \; . \tag{5}$$

The strong coupling α_s is evaluated at the relevant scale M_ζ^2:

$$\alpha_s(M_\zeta^2) = 12\pi / [(33 - 2f)\ln(M_\zeta^2/\Lambda^2)]$$

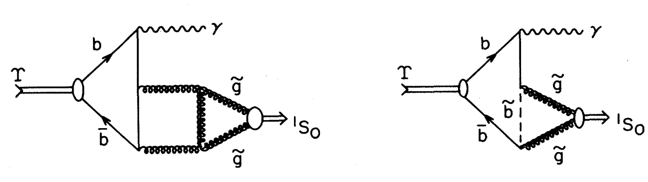

FIG. 1. Typical diagrams for the process $\Upsilon \to \gamma\,{}^1S_0(\tilde{g}\tilde{g})$ via (a) intermediate gluons or (b) scalar-quark exchanges.

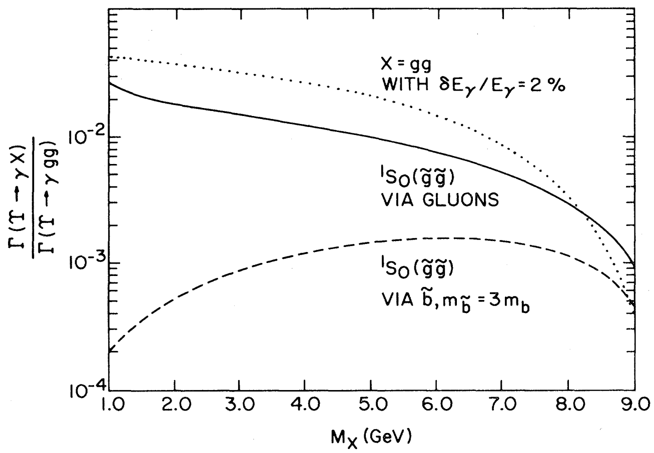

FIG. 2. The rate of $\Upsilon \rightarrow \gamma\, {}^1S_0(\tilde{g}\tilde{g})$ normalized to that of $\Upsilon \rightarrow \gamma gg$. The solid (dashed) curve is for the contributions from the intermediate gluons (the scalar-quark exchange with $m_{\tilde{b}} = 3m_b$). The backgrounds from the process $\Upsilon \rightarrow \gamma gg$ with energy resolution 2% are shown as the dotted curve.

with $\Lambda = 200$ MeV and the effective flavor number $f = 4$. The wave function $R_\zeta^2(0)$ will be studied later in this paper and its value is $R_\zeta^2(0) \simeq 0.63$–5.4 GeV^3 for $m_{\tilde{g}} \simeq 1$–4.5 GeV. We obtain the branching ratios of Eq. (5) to be 0.02 to 0.0009 for $M_\zeta = 2$–9 GeV. The result is illustrated as the solid curve in Fig. 2.

The main attractive point of this reaction from an experimental point of view is its detectable signature in the inclusive photon-energy spectrum. One would observe a narrow peak at E_γ as given by Eq. (2). In view of this simplicity, we believe that there is a fair chance of detecting the $\tilde{g}\tilde{g}$ bound states through this reaction (if $m_{\tilde{g}} < 4.5$ GeV), even though the branching ratio is so small. The main background comes from direct hard photons and their inclusive distribution has already been estimated.[4] Assuming an energy resolution of $\Delta E_\gamma / E_\gamma = 2\%$, one expects a photon signal $\Upsilon \rightarrow \gamma\zeta$ comparable to the background $\Upsilon \rightarrow \gamma gg$ (see the dotted curve in Fig. 2). For $M_\zeta < 5$ GeV, the branching ratio $\Gamma(\Upsilon \rightarrow \gamma\zeta)/\Gamma(\Upsilon \rightarrow \gamma gg)$ exceeds 10^{-2}, which requires a total of several thousand radiative Γ decays, a very large data sample, to resolve the signal.

There exist lower-order $O(\alpha\alpha_s^2)$ diagrams for $\Gamma(\Upsilon \rightarrow \gamma\zeta)$ via the exchange of the scalar quark [see Fig. 1(b)]. Following the formalism of Ref. 5, we obtain the contribution from this source,

$$\frac{\Gamma(\Upsilon \rightarrow \gamma\zeta)}{\Gamma(\Upsilon \rightarrow \gamma gg)} = \frac{16\pi R_\zeta^2(0)}{(\pi^2-9)M_\Upsilon^3}\left[\frac{m_b}{m_{\tilde{b}}}\right]^4\left[1-\frac{M_\zeta^2}{M_\Upsilon^2}\right] . \quad (6)$$

Here $m_{\tilde{b}}$ is the mass of the scalar quark. As Eq. (6) shows, the effect of the scalar exchange is usually suppressed by the large value of $m_{\tilde{b}}$ and it is not important here in the process $\Upsilon \rightarrow \gamma\zeta$, as illustrated by the dashed curve in Fig. 2 for the case $m_{\tilde{b}} > 3m_b$. However, the situation would be different for the top quarkonium decaying into $\gamma\zeta$. The scalar-quark mass $m_{\tilde{t}}$ could be close to the quark mass m_t. The signal then is much enhanced.

II. $\tilde{g}\tilde{g}$-BOUND-STATE QUANTUM NUMBERS

It is well known that the charge conjugation and parity of a given ${}^{2S+1}L_J$ state of the $q\bar{q}$ system is given by

$$C = (-1)^{L+S}, \quad P = (-1)^{L+1} , \quad (7)$$

so that in the quarkonium spectrum one has the $C = +1$ states

$$n\,{}^1S_0,\; n\,{}^3P_{0,1,2},\; n\,{}^1D_2,\; n\,{}^3F_{2,3,4},\; \ldots , \quad (8a)$$

as well as the $C = -1$ states

$$n\,{}^3S_1,\; n\,{}^1P_1,\; n\,{}^3D_{1,2,3},\; n\,{}^1F_3,\; \ldots . \quad (8b)$$

However, the situation is quite different for $\tilde{g}\tilde{g}$ states since under charge conjugation, a gluino is its own[6] antiparticle. As the generalized Pauli exclusion principle requires the overall wave function to be antisymmetric for $\tilde{g}\tilde{g}$ states, $L+S$ is always even. The quantum numbers of the $\tilde{g}\tilde{g}$ state are still given by Eq. (7) with the constraint

$$C = (-1)^{L+S} = +1 . \quad (9)$$

Thus half of the quarkonium spectrum characterized by $C = -1$ states as given by Eq. (8a) is entirely *absent* in the $\tilde{g}\tilde{g}$ spectrum. In a recent paper Zuk, Joshi, and Wignall[7] have erroneously concluded that in the $\tilde{g}\tilde{g}$ spectrum all the $C = +1$ states as given by Eq. (8a) are *absent*.

III. DECAY RATES OF TWO-GLUINO BOUND STATES

The dominant decay mode of the ground state is expected to be two gluons via the exchange of a gluino. This is analogous to the $\eta_c \rightarrow gg$ case for which Barbieri *et al.*[8] have calculated the decay rate. Their calculation goes through for our case provided one replaces M_{η_c} by M_ζ, and $R_{\eta_c}(0)$ by $R_\zeta(0)$ and multiplies their calculation by a factor of $\frac{27}{4}$ to take into account the appropriate color factors and the Majorana nature of the gluino. We thus find that

$$\Gamma(\zeta - gg) = 18\alpha_s^2 |R_\zeta(0)|^2 / M_\zeta^2 = 300\text{–}47 \text{ MeV} , \quad (10)$$

as $m_{\tilde{g}}$ varies from 1 to 4.5 GeV. As we shall see below from potential-model calculations, $|R_\zeta(0)|^2$ varies from 0.63 to 5.4 GeV^3 in that case. We thus find that the $\tilde{g}\tilde{g}$ ground state, though not as narrow as J/ψ, Υ, etc., is still quite narrow.

In order to have our prediction, Fig. 2, unaffected by the $\tilde{g}\tilde{g}$ width, it is necessary for the $\tilde{g}\tilde{g}$ state to be narrower than the photon resolution, i.e.,

$$\Gamma < \tfrac{1}{2}(M_\Upsilon^2/M_\zeta)(\delta E_\gamma/E_\gamma)(1 - M_\zeta^2/M_\Upsilon^2) .$$

Our numerical analysis shows that the above inequality holds for $M_\zeta < 6.5$ GeV in the case $\delta E_\gamma / E_\gamma = 2\%$.

The interesting thing about the $\tilde{g}\tilde{g}$ spectrum is that unlike the quarkonium case, no electromagnetic transitions are possible between the different levels.

IV. $\tilde{g}\tilde{g}$ POTENTIAL

To obtain the $\tilde{g}\tilde{g}$ spectrum one has to first know the $\tilde{g}\tilde{g}$ potential. As in the quarkonium case, at short distance the dominant part is due to one-gluon exchange. On taking the appropriate color factors into account it is not difficult to show that

$$V_{\tilde{g}\tilde{g}}^{1-\text{gluon}}(r)=\frac{9}{4}V_{q\bar{q}}^{1\text{-gluon}}(r)=-\frac{3\alpha_s}{r} \quad . \tag{11}$$

However, we have neither any information about the long-range color octet-octet force nor any about the ratio of the long-range color-octer-octet and long-range color-triplet-antitriplet force. As an educated guess we assume that this ratio is the same as that due to one-gluon exchange and assume that the entire $\tilde{g}\tilde{g}$ potential is $\frac{9}{4}$ times the quarkonium potential. Much simplification occurs if we choose an effective potential which scales with the constituent mass. Martin's power-law potential[9]

$$V(r)=\lambda r^{\nu}+C \quad , \tag{12}$$

is one such possibility. It provides impressive phenomenological fits to the $b\bar{b}$ and $c\bar{c}$ spectra with the choice

$$\nu=0.1 \quad . \tag{13}$$

We therefore assume that the $\tilde{g}\tilde{g}$ potential is $\frac{9}{4}$ times that of Eqs. (12) and (13). From dimensional considerations it can be easily shown that[10]

$$R_{\zeta}^{2}(0)=R_{\eta_c}^{2}(0)(2.25m_{\tilde{g}}/m_c)^{3/(2+\nu)} \quad , \tag{14}$$

with $m_c=1.8$ GeV. On using

$$R_{\eta_c}^{2}=R_{J/\psi}^{2}(0)=0.46\ \text{GeV}^3 \quad , \tag{15}$$

as obtained from the leptonic width of J/ψ, we then find that $R_{\zeta}^2(0)$ varies from 0.63 to 5.4 GeV^3 for $m_{\tilde{g}}=1$ to 4.5 GeV.

Summarizing, we have suggested a clean, though rather difficult, test for detecting two-gluino bound states. We hope that this will stimulate our experimental friends to look at the feasability of looking for two-gluino bound states Υ decay or even top-quarkonium decay.

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