Radiation zeros and a test for the g value of the τ lepton

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We suggest that an experimental test for the g value of the τ lepton may be feasible. The idea is to take advantage of the radiation zero which occurs in the radiative decays $\tau \to e\nu\bar{\nu}\gamma$ and $\tau \to \mu\nu\bar{\nu}\gamma$ for $g=2$. We calculate the differential decay rate for an arbitrary g value of the τ. The results for back-to-back electron- (muon-) photon events with large electron (muon) energies are indeed sensitive to the g value of the τ. Our estimates for the expected event rates are not unreasonable and one could hope to achieve a reasonable upper bound for g, perhaps even a measured value.

It is now nine years since the discovery of the τ lepton.[1] All the experimental evidence so far is consistent with the τ being a sequential lepton with no internal structure or constituents.[2] Just as in the case of the electron and the muon, the τ interacts with the same weak and electromagnetic interactions (no strong interactions), obeys its own lepton conservation law, and has spin $\frac{1}{2}$. However, it has the astonishingly large mass

$$M = 1782\ \mathrm{MeV}/c^2 \ .$$

The following question arises: Is there any other property of the τ lepton which might be amenable to experiment and which tests e-μ-τ universality? In this Brief Report, we suggest that it might be possible to experimentally test the static g value of the τ, at least obtaining an upper bound for it, if not a measured value. Of course one expects, according to universality, that $g \simeq 2$ and the anomaly

$$a \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi} + O(\alpha^2) \ . \tag{1}$$

Here we take a very pragmatic point of view and suggest that, by looking for the radiative decays

$$\tau \to e\nu\bar{\nu}\gamma \quad \text{and} \quad \tau \to \mu\nu\bar{\nu}\gamma \tag{2}$$

in a particular region of phase space, one can obtain information about g. The idea is to take advantage of the phenomenon of radiation zeros.[3] It has been shown that the amplitude for certain processes involving one real photon vanishes[4] in a certain region of phase space called the null zone, *provided* $g=2$ *for all charged particles with spin.*[5,6] In the case of the processes in Eq. (2) the null zone is described by the following:[4,7] (i) $e\gamma$ (or $\mu\gamma$) are back-to-back in the τ c.m. frame and (ii) the energy of the e (or μ) is a maximum, i.e.,

$$x = \frac{2E}{M} = 1 \ . \tag{3}$$

Thus our procedure is to calculate the differential decay rate for the radiative decays [Eq. (2)] with an assumed τ anomaly, $a \neq 0$. Since the radiation zero will be spoiled for $a \neq 0$, we will study the differential decay rate near the null zone where we hope it will provide a test for the g value of the τ.

In the case of the process

$$d\bar{u} \to W^-\gamma \ , \tag{4}$$

this does indeed provide a sensitive test for the g value of both the W boson[3] and the quarks.[8] Because of the nonintegral quark charges, the null zone for reaction (4) is in the interior of the phase space,[9] whereas for the radiative τ decay (2), the null zone is at the edge of the phase space, as described in Eq. (3).

The differential decay rate for the radiative decay (2) has been calculated in the τ c.m. frame in the limit of zero electron (or muon) mass and can be expressed as

$$\frac{d^3\Gamma(\tau \to e\nu\bar{\nu}\gamma)}{dx\,dy\,d\Omega} = \left(\frac{\alpha}{4\pi}\right)\Gamma_{\mathrm{tot}}(\tau \to e\nu\bar{\nu})H \ , \tag{5}$$

where $x = 2E/M$ and $y = 2E_\gamma/M$ are the scaled energies of the electron and the photon, respectively, $d\Omega$ is the solid angle into which the electron is emitted, and

$$\Gamma_{\mathrm{tot}}(\tau \to e\nu\bar{\nu}) = \frac{G^2M^5}{192\pi^3} \ . \tag{6}$$

Our result for the quantity H is

$$H = A + A'\left(\frac{a}{2}\right) + A''\left(\frac{a}{2}\right)^2 \ , \tag{7}$$

where

$$\begin{aligned} yA = {} & \frac{8}{\Delta}[y^2(3-2y) + 6xy(1-y) + 2x^2(3-4y) - 4x^3] \\ & + 8[2x^3(1+2y) - xy(3-y-y^2) - x^2(3-y-4y^2)] \\ & + 2\Delta[x^2y^2(6-5y-2y^2) - 2x^3y(4+3y)] \\ & + 2\Delta^2x^3y^2(2+y) \ , \\ \frac{A'}{4} = {} & x^3y^2\Delta^2 + x^2y\Delta(2-2x-y) + 2xy(1-x-y) \ , \\ \frac{A''}{2} = {} & x^2y^2\Delta(2x+y-2) + 2x^2y(3-2y-2x) \ . \end{aligned} \tag{8}$$

The quantity

$$\Delta = 1 - \cos\theta \ , \tag{9}$$

where θ is the angle between the electron and the photon directions. Note that

$$\Delta_{\min} = 2(x+y-1)/xy \ .$$

The null zone is given by

$$\Delta = 2$$

and (10)

$$x = 1 \ .$$

If we set $\Delta = 2$, we obtain

$$A(\Delta=2) = 4(1-x)y[2x^2(1-y)+(3-x-2y)] \ ,$$
$$A'(\Delta=2) = 8xy(1-x)(1-y)(1+2x) \ , \quad (11)$$

and

$$A''(\Delta=2) = 4x^2y(1-y)(3-2x-y) \ .$$

One can see from Eq. (11) that A and A' vanish[10] in the null zone (independent of y), *but* A'' *does not.* So the τ anomaly does indeed spoil the radiation zero by an amount given by

$$H(\Delta=2, x=1) = y(y-1)^2a^2 \geq 0 \quad (12)$$

for any a and y. The positive definiteness of this result and A and A'' in Eq. (11), serves as a check on our result. Moreover, for $a=0$, we agree with previously known results.[11] Notice that, although, in general, for small y, A has the typical bremsstrahlung infrared behavior

$$A \sim \frac{1}{y} \ , \quad (13)$$

for $\Delta = 2$, $A \sim y$ and so we may integrate down to $y=0$. Since the radiation zero is present for $a=0$ independent of y, for our purposes it is appropriate to consider

$$F(x) = \int_0^1 dy\, H(\Delta=2)$$
$$= B(\Delta=2) + B'(\Delta=2)\left[\frac{a}{2}\right] + B''(\Delta=2)\left[\frac{a}{2}\right]^2 \ , \quad (14)$$

where

$$B(\Delta=2) = \tfrac{2}{3}(1-x)(2x^2-3x+5) \ ,$$
$$B'(\Delta=2) = \tfrac{4}{3}x(1-x)(1+2x) \ , \quad (15)$$
$$B''(\Delta=2) = 4x^2\left[\frac{-x}{3} + \frac{5}{12}\right] \ .$$

From Eqs. (5) and (14) our result for the differential decay rate for back-to-back electron-photon events, integrated over all photon energies, is

$$\frac{d^2\Gamma(\tau \to e\nu\bar{\nu}\gamma)}{dx\, d\Omega} \Big/ \Gamma_{\rm tot}(\tau \to e\nu\bar{\nu}) = \frac{\alpha}{4\pi}F(x) \ , \quad (16)$$

with $F(x)$ given by Eqs. (14) and (15). Figure 1 shows the behavior of $F(x)$ for $a=0$, 5, and 10. The radiation zero can be seen for $a=0$: $F(1)=0$. For $a \neq 0$, $F(1) \neq 0$ and the distribution is very different, particularly for large a, where the overall magnitude of F is much larger.

We now need to discuss the feasibility of such an experiment. For the purposes of this estimate, we will assume a luminosity $\mathcal{L} = 2\times 10^{31}$ cm^{-2}sec^{-1} and a cross section

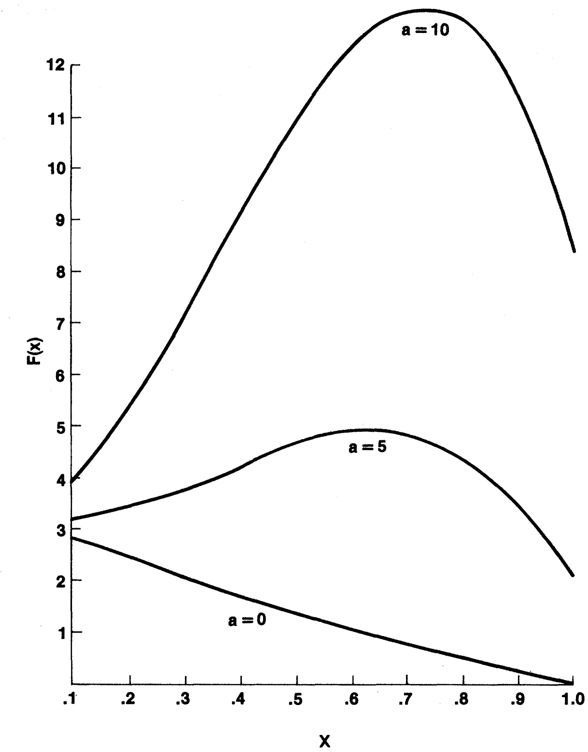

FIG. 1. $F(x)$ vs x for $a=0$, 5, and 10.

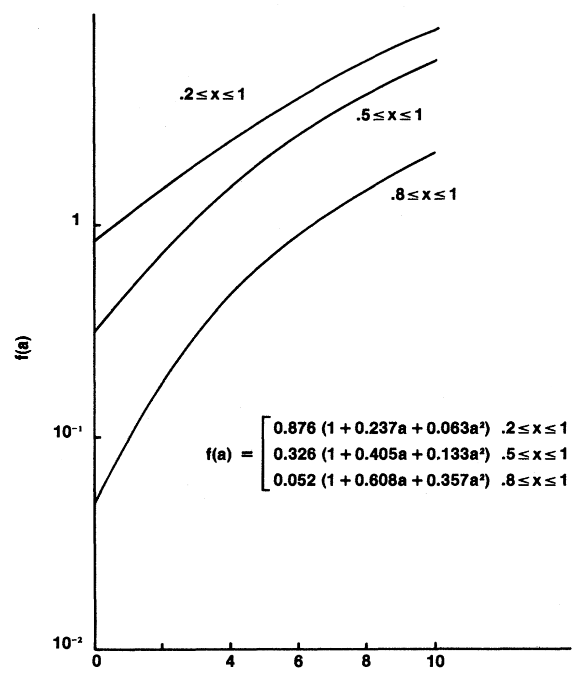

FIG. 2. $f(a)$ vs a for the events in the bins $0.2 \leq x \leq 1$, $0.5 \leq x \leq 1$, and $0.8 \leq x \leq 1$.

$\sigma_{e^+e^-\to\tau^+\tau^-} \sim 2$ nb, giving 4×10^{-2} events/sec. We will consider the events with $0.5 \leq x \leq 1$. A smaller x bin around $x=1$ would give greater sensitivity to a, but, of course, fewer events. So we define

$$f(a)=\int_{0.5}^{1} F(x)dx \quad . \tag{17}$$

A convenient result for $f(a)$ is

$$f(a)=0.326+0.132a+0.0434a^2 \quad . \tag{18}$$

Using the leptonic branching ratio

$$\frac{\Gamma(\tau\to e\nu\bar{\nu})}{\Gamma_{\text{tot}}(\tau)}=\frac{\Gamma(\tau\to\mu\nu\bar{\nu})}{\Gamma_{\text{tot}}(\tau)}=17\% \tag{19}$$

and

$$\Delta\Omega \sim 0.1(2\pi) \quad , \tag{20}$$

from Eq. (16) we find our estimate for the expected number n of events/sec to be

$$n \sim 10^{-5} f(a) \quad . \tag{21}$$

The results for $f(a)$ are shown in Fig. 2, where we also give, for comparison, the corresponding results for the larger range $0.2 \leq x \leq 1$ and the smaller range $0.8 \leq x \leq 1$. As expected, the smaller range yields more sensitivity to a, particularly for small a, but, of course, the overall rate is lower. This can also be seen from the formulas for $f(a)$ given in Fig. 2. For a possible running time of $\sim 10^7$ sec, the event rates [see Eq. (21)] are not unreasonable. One could, in fact, hope to achieve a reasonable upper bound for a, perhaps even a measured value.

Of course, before an experiment could be attempted, detailed calculations, including experimental cuts and studies of possible background, would have to be done. If one were to search for these decays in $e^+e^-\to\tau^+\tau^-$ and trigger on $e\mu\gamma$ events, background from $e^+e^-\to e^+e^-\gamma$ and $e^+e^-\to\mu^+\mu^-\gamma$ would be eliminated. One does have to worry about the radiated photons from the beam particles faking the decay photons. However, since these bremsstrahlung photons are strongly peaked in the forward direction, appropriate cuts should eliminate most of this background. Our intent here is to point out that such an experimental test of the g value of the τ lepton is, at least, feasible and to encourage experimentalists to give it serious consideration.

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