Effect of quark masses on the perturbative thrust

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We study the effect of quark masses on the thrust to $O(\alpha_s)$ in QCD. We find the mass corrections to be significant and show that they reduce the estimated nonperturbative effects. They also allow realistic limits to be placed on the parameter Λ.

I. INTRODUCTION

Jet cross sections, suitably defined, are infrared finite and hence calculable order by order in perturbative QCD.[1] In addition, some authors[2,3] have proposed variables to measure the jetlike properties of an event. The thrust[2] is one such variable. From a calculational point of view the thrust is a good variable for measuring jetlike properties, since it is defined to be linear in the momentum, which makes it infrared-insensitive[4,5] in all orders of perturbation theory. Here we study the effect of quark masses on the thrust.

The average thrust was calculated to lowest order in α_s for massless quarks by various authors[2,6] in QCD perturbation theory for e^+e^- annihilation. However, these calculations do not give a measure of the full perturbative QCD contribution to the jetlike properties of an event since they neglect the masses of the quarks which are definitely not negligible for the c and the b (and possibly heavier) quarks in the current energy range of 10–40 GeV, as we shall show below.

The $O(\alpha_s^2)$ corrections to the thrust distribution $d\sigma/dT$ have been studied by various authors.[7–10] The $O(\alpha_s^2)$ correction to the average thrust in the zero-mass limit has been calculated by Clavelli and Wyler.[11] Their calculations show that this gives a value of

$$\langle 1-T\rangle = 1.05\alpha_s/\pi + 9.5(\alpha_s/\pi)^2 \ ,$$

which is a fairly large correction to the $O(\alpha_s)$ result. However, that conclusion depends substantially on what α_s is chosen to be in the range of energies 10–50 GeV. It has recently been argued[12,13] that the thrust is of limited usefulness in the study of perturbative QCD effects since, contrary to the data, the average value of $1-T$ exhibits no energy dependence other than that implicit in α_s, which is at best a weak dependence, whereas the data show the presence of strong power corrections, implying the dominance of fragmentation effects according to Ref. 12.

In this paper we are mainly concerned with the study of simple perturbative higher-twist effects due to quark masses. Our study shows that when we include the masses of the quarks in the calculation to $O(\alpha_s)$, a sharp falloff in the region 10–25 GeV is found, reminiscent of the DESY PLUTO data which show a similar behavior. This encourages us to believe that perturbation theory alone includes at least some of the effects evident in the data.

In addition, Field has very recently emphasized[14] the ambiguity in the determination of α_s, or equivalently Λ, in various Monte Carlo studies. He argues that one should compare the data directly with analytic calculations to set bounds on Λ from perturbation theory.

We find here that choosing $\Lambda = 300$ MeV makes the data for $\langle 1-T\rangle$ lie along our curve calculated from perturbation theory. This, of course, represents the uppermost bound on Λ from perturbation theory alone. In addition, if we take nonperturbative effects to go as $a_{NP}/\sqrt{s}$, as suggested by the PLUTO collaboration, then by choosing $\Lambda \approx 100$ MeV, we get a rough fit to the data in the region of $a_{NP} \approx 0.1$ GeV. The PLUTO colloboration has fitted their data to the formula

$$\langle 1-T\rangle = 1.05\alpha_s/\pi + a_{NP}/\sqrt{s} \ ,$$

and obtain the value of $a_{NP} = 0.60 \pm 0.15$ GeV. Thus we notice that including the masses drastically decreases the estimate of nonperturbative effects in $\langle 1-T\rangle$. Conversely, using the PLUTO value for $a_{NP} \approx 0.6$ GeV, we obtain an estimate of Λ around 50 MeV.

We have performed an analytic calculation of $\langle T\rangle$ keeping terms of $O(m^2/W^2)$, where $W^2 = s$. We have also evaluated the average thrust numerically.

In Sec. II, we define the thrust as it is traditionally defined, and also in the way we modify it slightly to incorporate massive quarks. We show how the thrust is calculated in perturbation theory, and we discuss the numerical calculation.

In Sec. III we present a discussion of what our results imply with regard to the QCD scale parameter Λ, and hence for the running coupling constant α_s. Section IV contains the conclusion.

II. THRUST

The thrust as defined in Ref. 2 is given by

$$T = 2\,\frac{\max \sum_{i\in h} (p_i\cdot\hat{n})}{\sum_i |p_i|} \ , \tag{2.1}$$

where $\sum |p_i|$ runs over all observed particles and $\sum_{i\in h}(p_i\cdot\hat{n})$ runs over all particles in a hemisphere. $\hat{n}$ is a unit vector chosen in a direction that maximizes the numerator. This defines the jet axis.

This definition is appropriate for all massless particles since it is insensitive to collinear and infrared divergences. To include the mass effects in the definition of the thrust, we modify (2.1) slightly and write

$$T = \frac{2}{W}\max \sum_{i\in h}(p_i\cdot\hat{n}) \ . \tag{2.2}$$

We calculate $\langle 1-T\rangle$ taking into account the effect of massive virtual quarks, which we assume are not directly observed but decay into very light particles. Note that the denominator W equals $\sum |p_j|$ if the final particles p_j are all light. In addition the above definition (2.2) is independent of the decay of the massive quarks as long as it occurs in the forward hemisphere. On the other hand, decays which cross the hemispheres decrease T and hence increase $\langle 1-T\rangle$.

We calculate this quantity to $O(\alpha_s)$ in e^+e^- annihilation. From the definition of the thrust, we see that for a three-particle final state, it is given by

$$T \equiv \max[(x_1^2-\xi)^{1/2}, (x_2^2-\xi)^{1/2}, x_3] \ , \tag{2.3}$$

where $x_i = 2E_i/W$, E_i being the energy of the ith particle in the final state in the c.m. frame and $\xi = 4m^2/W^2$, m being the mass of the quarks. Also $W^2 = s$, the c.m. energy squared.

For a two-particle final state, x_1 and x_2 are both equal to 1 always, so the thrust is just $\sqrt{1-\xi}$.

The differential cross section for the real gluon emission process for a quark of mass m is given by

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dx_1dx_2} = \frac{2\alpha_s}{3\pi}\left[\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} - \frac{\xi}{2}\left(\frac{1}{(1-x_1)^2}+\frac{1}{(1-x_2)^2}+\frac{2}{(1-x_1)}+\frac{2}{(1-x_2)}\right) - \frac{\xi^2}{4}\left(\frac{1}{(1-x_1)}+\frac{1}{(1-x_2)}\right)^2\right] , \tag{2.4}$$

where $\sigma_0 = (4\pi\alpha^2/s)e_i^2$ is the total cross section for $e^+e^- \to q_i\bar{q}_i$ for $\xi \to 0$, with e_i in units of $(4\pi\alpha)^{1/2}$.

The average value of the thrust is given by

$$\langle T\rangle = \left(\int T\frac{d\sigma}{dT}\,dT\right)\Big/\left(\int \frac{d\sigma}{dT}\,dT\right) . \tag{2.5}$$

The contribution to $\langle T\rangle$ from the virtual diagram (the vertex correction) is trivially calculated since there are only two particles in the final state, and so $x_1 = x_2 = 1$ always.

The contribution to $\langle T\rangle$ from the real-gluon-emission part is less trivial. We introduce a gluon mass Λ to regulate the infrared divergences. On adding the contribution from the real and virtual diagrams, we find that the infrared divergences cancel exactly, as expected, leaving us with a quantity that is finite and has a well-defined $\xi \to 0$ limit.

We find that the numerator in (2.5) is given by [to $O(\xi)$]

$$\begin{aligned}\frac{1}{\sigma_0}\int T\frac{d\sigma}{dT} = 1 - \frac{\xi}{2} + \frac{4\alpha_s}{3\pi}\Bigg\{ & \tfrac{137}{16}\xi + \tfrac{5}{4}\xi\ln 2 - \tfrac{1}{2}\xi^{1/2} + \tfrac{7}{9} + \tfrac{1}{4}\xi\ln^2\xi - \xi\ln 2\ln\xi + \frac{\pi^2}{6} - \frac{\xi\pi^2}{6} - \tfrac{1}{8}\xi\ln\xi \\ & - \tfrac{1}{2}\xi\ln 3\ln 2 - \tfrac{9}{2}\xi\ln 3 - \ln^2 3 + \tfrac{3}{8}\ln 3 - \tfrac{1}{2}\xi\left[\mathrm{Li}_2\left(1-\xi^{1/2}+\frac{\xi}{2}\right) - \mathrm{Li}_2(\tfrac{1}{3}+\tfrac{1}{2}\xi)\right] \\ & - 2\mathrm{Li}_2(\tfrac{2}{3}-\tfrac{1}{2}\xi) + \xi\mathrm{Li}_2(\tfrac{2}{3}-\tfrac{1}{2}\xi) - \tfrac{1}{2}\xi\mathrm{Li}_2(\tfrac{1}{3}-\tfrac{1}{4}\xi) + \xi\ln^2 2\Bigg\} ,\end{aligned} \tag{2.6}$$

where $\mathrm{Li}_2(x)$ is the dilogarithm function.[15] The $\xi \to 0$ limit gives us

$$\langle T\rangle = 1 + \frac{4\alpha_s}{3\pi}\left(\frac{1}{36} + \frac{\pi^2}{6} - \ln^2 3 + \tfrac{3}{8}\ln 3 - 2\mathrm{Li}_2(\tfrac{2}{3})\right) . \tag{2.7}$$

This agrees with the result of De Rújula, Ellis, Floratos, and Gaillard,[6] and differs from Ref. 2 by a factor of 4.

The numerical value of the term within brackets is ≈ -0.05. Note that for the massless case, the change in $\langle T\rangle$ as a function of energy is purely due to the change in the running coupling constant α_s.

A plot of $\langle 1-T\rangle$ versus energy W is given in Fig. 1 for the massless case to $O(\alpha_s)$, along with the $O(\xi)$ corrections to the massless result. The difference at around 20 GeV is about 30% and at 30 GeV, approximately half that. In addition, as mentioned in the introduction, a sharp fall with energy is noted in the 10–25-GeV range, similar to that observed experimentally. We have used here a value of $\Lambda \approx 300$ MeV to calculate α_s, and calculated the 10-GeV point using only four quarks (i.e., below the b-quark threshold).

Of course, this result does not incorporate all the mass effects to this order in $O(\alpha_s)$. For that purpose we do a numerical calculation of the average thrust.

The result of doing the numerical calculation, after extracting the infrared divergences, is summarized in Table I where $A(\xi)$ is defined by

$$\int (1-T)\frac{d\sigma}{dT}\,dT = \sigma_0\left(1 + \frac{4\alpha_s}{3\pi}A(\xi)\right) \text{ with } \sigma_0 = \frac{4\pi\alpha^2}{s}e_i^2 . \tag{2.8}$$

We plot the values of $\langle 1-T\rangle$ as a function of the energy in Fig. 1. All three curves in Fig. 1 are, of course, weighted with the proper charges of the quarks and normalized by dividing by the total cross section. Λ is chosen to be 300 MeV.

We again see the sharp drop in the region 10–25 GeV, but the difference between the massless case and the massive case has been reduced to about 15% around 20 GeV and about 10% around 30 GeV. It seems fairly safe to conclude that beyond about 50 GeV, the mass effects are no longer very important (barring, of course, the t quark). However, below about 30 GeV they start becoming important and show strong power corrections behavior as suggested by the data.

Later on we discuss how our plot of $\langle 1-T\rangle$ changes as we use different values of Λ and incorporating massless $O(\alpha_s^2)$ effects, and what we may conclude from them.

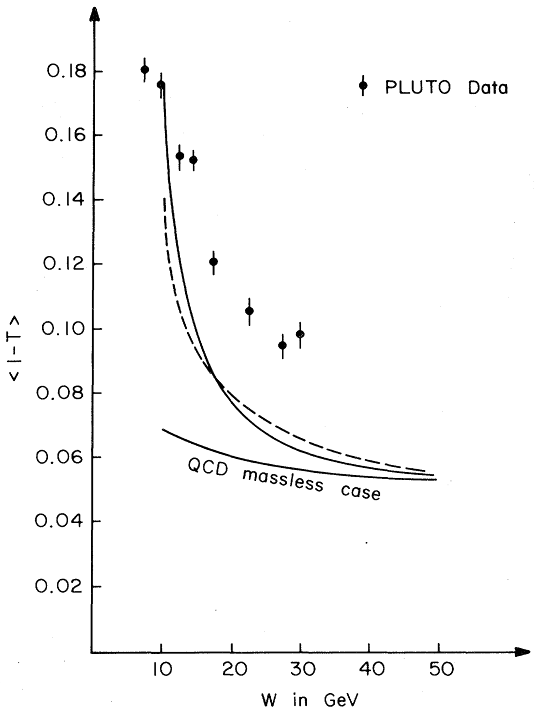

FIG. 1. The thrust for massive quarks. The dashed curve is the $O(m^2/W^2)$ approximation. The massless case is given for comparison. $\Lambda = 300$ MeV.

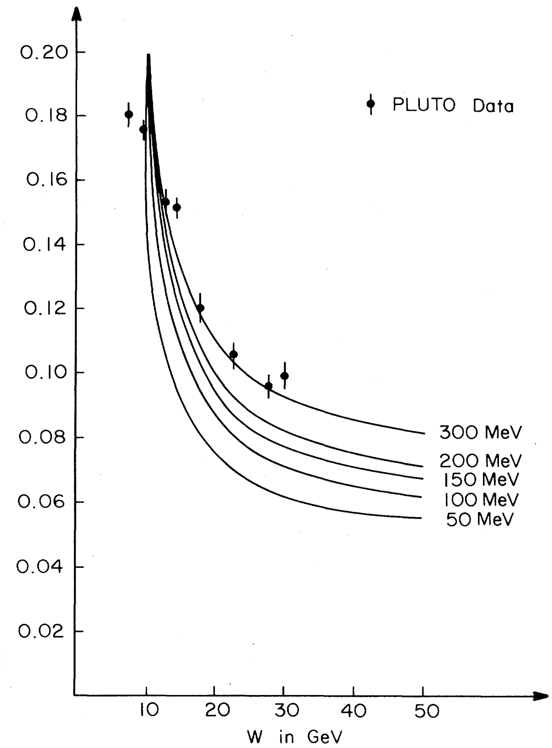

FIG. 2. The thrust as a function of c.m. energy W for different Λ [including $O(\alpha_s^2)$ massless correction].

III. QCD SCALE PARAMETER

In Fig. 2 we have plotted $\langle 1-T\rangle$ versus energy for various values of Λ. We have included in the plots, apart from the mass-dependent terms, the contribution from the $O(\alpha_s^2)$ process as calculated by Clavelli and Wyler.[11] This contributes a factor 9.5 $(\alpha_s/\pi)^2$ to $\langle 1-T\rangle$.

We notice that for $\Lambda = 300$ MeV, our plot is in fairly good agreement with the data from PLUTO. However, this is not to be taken seriously since fragmentation effects have not been taken into account at all. It is, however, safe to say that 300 MeV is the very upper limit for Λ from purely perturbative considerations, and the actual value of Λ is, of course, less.

If we assume that nonperturbative effects are approximat-

TABLE I. Values of $A(\xi)$ for various values of ξ.

ξ	$A(\xi)$
0.02	0.7612
0.04	0.7028
0.06	0.6755
0.08	0.6533
0.10	0.6351
0.15	0.6048
0.20	0.5944
0.25	0.6034

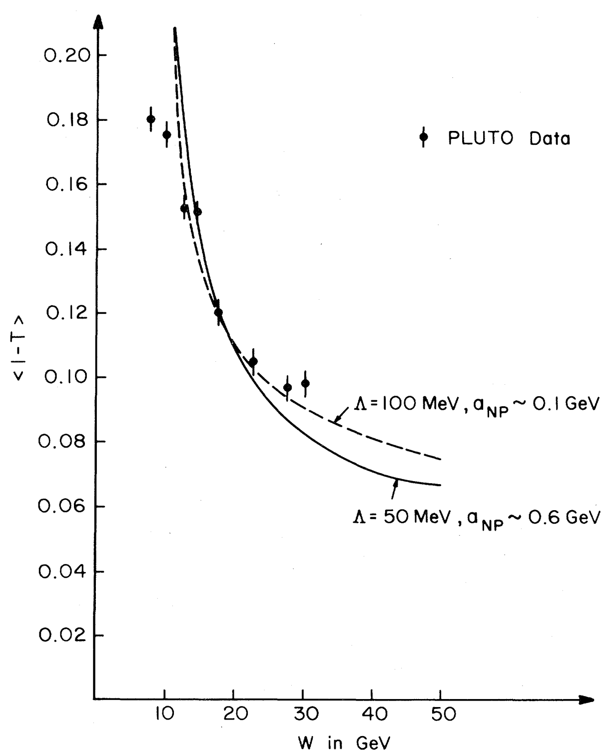

FIG. 3. The thrust as a function of W for two different combinations of Λ and a_{NP}. $a_{\mathrm{NP}} \approx 0.6$ suggested by PLUTO collaboration.

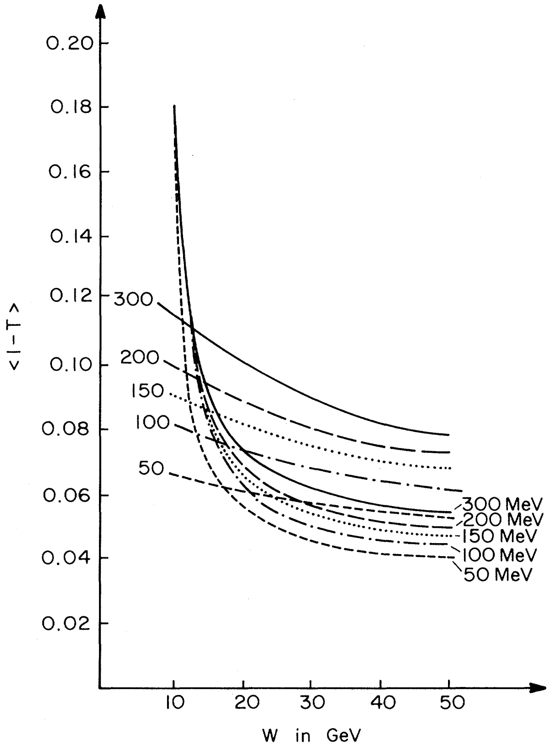

FIG. 4. Comparison of the magnitude of the $O(\alpha_s^2)$ massless correction (labeled on the left) and $O(\alpha_s)$ massive case, different Λ.

ed by $\langle 1-T\rangle \approx a_{NP}/\sqrt{s}$ as in Ref. 13, where a_{NP} is found to be 0.6 ± 0.15 GeV, then we can add this to our perturbative result and see what effect it has. If we do so, we find that a value of Λ around 50 MeV gives a fair fit to the data. This is in the low range of suggested Λ values. Conversely if we take a typical value of Λ at about 100 MeV and plot the curve for $\langle 1-T\rangle$ including mass effects and $O(\alpha_s^2)$ corrections, we find that we need $a_{NP} \approx 0.1$ GeV to give a rough agreement with the PLUTO data. These are shown in Fig. 3. Thus as we said before, including the effects due to mass reduces the estimated size of the nonperturbative corrections.

Finally we look at the relative magnitudes of the $O(\alpha_s^2)$ terms compared to the mass effects at $O(\alpha_s)$. We find that the crossover point at which the $O(\alpha_s^2)$ term surpasses the mass correction is around 12–17 GeV for Λ in the range 50–300 MeV. This is demonstrated in Figs. 4 and 5. This

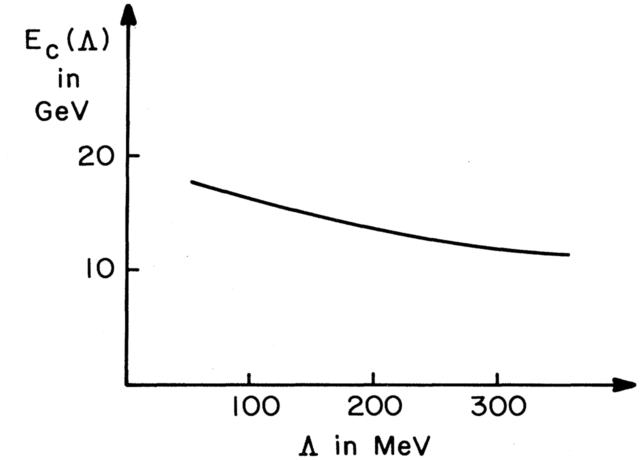

FIG. 5. Plot of $E_c(\Lambda)$, the energy at which the $O(\alpha_s)$ massive-quark contribution surpasses the $O(\alpha_s^2)$ massless case, as a function of Λ.

is a fairly high energy range, and it increases as Λ decreases, which again bolsters our contention that masses are important in this energy range.

IV. CONCLUSION

We have shown explicitly in this paper that it is necessary to incorporate the masses of quarks in order to be able to study and compare data for $\langle 1-T\rangle$ with the theoretical predictions of perturbative QCD. We have also studied the dependence of our results on Λ and shown that mass effects tend to reduce the estimate of nonperturbative corrections.

It is important to understand the higher-twist effects of masses in perturbation theory and thereby distinguish them from the effects of fragmentation models built into Monte Carlo programs. This would be particularly apt in the case of the, until now elusive, t quark since its charge is 2/3.

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