Composite model with three confining hypercolor groups

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A preon model of composite quarks, leptons, and weak bosons is constructed based on the hypercolor group $G_{\rm HC} = {\rm SU}(2)_{\rm TC} \times {\rm SU}(2)_L \times {\rm SU}(2)_R$ with a chiral SU(4) hyperflavor symmetry. Two of the hypercolor forces confine the preons into composite fermions which are hypercolor singlets and technifermions belonging to four-, three-, and two-dimensional representations of technicolor. The most-attractive-channel hypothesis leads to technifermion condensates which (1) break the hyperflavor symmetry *in stages* providing 't Hooft anomaly matching at the first stage, (2) determine the generation structure through a resulting discrete axial $Z(m)_A$ group, and (3) lead to light-fermion masses through techniboson-exchange graphs. At most three generations of standard quarks and leptons plus two "mirror" generations arise in the model with Q, B, and (optionally) L conserved quantum numbers.

I. INTRODUCTION

The notion that quarks and leptons may not be elementary objects has received widespread attention[1] over the past several years. Although no completely satisfactory composite model has emerged to date, various models have been proposed which address the issues of repetitive family structure and mass generation, which remain largely unsolved in the simpler versions of grand unified theories[2] (GUT's) where the quarks and leptons are treated as elementary objects and the strong, weak, and electromagnetic interactions are unified at some high energy scale. However attractive the GUT scenario and despite the remarkable success of the electroweak $\sin^2\theta_W$ mixing-angle prediction,[3] the hierarchy problem[4] concerning the widely disparate mass scales for the GUT and ${\rm SU}(2)_L \times {\rm U}(1)$ symmetry breakings remain a mystery in that context. Moreover, at the time of this writing, proton decay has not been observed,[5] in disagreement with the simplest SU(5) model in the standard GUT scenario.

In contrast to the vast GUT desert spanning many orders of magnitude above 1 TeV, where no new particles or interactions are predicted, composite models in general predict new constituent or preon interactions at energies above the confining hypercolor (HC) scale, typically of the order of $1-10^3$ TeV. Below $\Lambda_{\rm HC}$ the presently observed quarks and leptons exist as preon bound states. If the weak bosons themselves are composite, more massive excited states are also expected. Only the supersymmetry scenario,[6] where superpartners of the observed quarks, leptons, and gauge bosons must exist in the mass range of 100—1000 GeV, is expected to exhibit a richer spectrum of particles. Various authors have even recently attempted to combine the features of supersymmetry and compositeness to construct supersymmetric preon models.[7]

Composite quarks and leptons emerge as natural extensions of the idea to replace fundamental Higgs scalars by fermion-pair bound states, where the fermions carry a new kind of color [technicolor[8] (TC)] which becomes confining at a scale of order 0.3—1.0 TeV. It may then happen that the elementary preons can form both hypercolor-singlet composite quarks and leptons as well as technifermions which carry technicolor as part of the hypercolor confining force.

The composite nature of the weak bosons W and Z is somewhat more speculative. Past experience suggests that interactions which are mediated by massive bosons (e.g., strong interactions and ρ,ω bosons) are residual interactions[9] of a more elementary force (quantum chromodynamics) which is transmitted by massless gauge fields (gluons). On the other hand, the standard Glashow-Salam-Weinberg model[10] of electroweak interactions appears in good shape[11] in that the charged W and neutral Z bosons appear to have approximately the correct masses predicted for them—up to and including radiative corrections. Nevertheless, Kögerler, Schildknecht, and Fritzsch[12] have shown that if one invokes W dominance for the least massive weak boson, one can suppress the effects of the isoscalar- and higher-isovector-boson contributions so as to explain the "low-energy" electroweak effects in a satisfactory alternative manner: only at higher energies will predictions of the residual "weak"-interaction picture depart radically from the standard gauge-model scenario. The composite-model builder is therefore free at this time to consider preon models with either elementary or composite weak bosons.

Other sources of departure for composite-model construction concern the origin of parity violation in weak interactions. In some models which are inherently chiral asymmetric,[13] the number of left-handed preons is chosen to be greater than the number of right-handed preons. In other cases,[14] the left-handed quarks and leptons are taken to be composite while the right-handed ones are fundamental. Another favorite scheme[15] is to make the model left-right symmetric, but include a mechanism that breaks the left-right symmetry at a suitable energy scale.

In this paper the author has attempted to construct a preon model which is left-right symmetric, involves composite weak bosons, and which includes a technicolor mechanism among its strong confining forces. Preliminary results of this investigation have already been reported by the author.[16] This work is an outgrowth of composite models developed by Abbott and Farhi, and other authors of Ref. 14, and more recently by Schrempp and Schrempp.[17] In the Abbott-Farhi model, the $SU(2)_L$ gauge force is strong and confining, rather than weak as in the standard model. With elementary fermions and scalar fields which are doublets under $SU(2)_L$, composite left-handed quarks and leptons can be constructed which exhibit a residual (weak) SU(2) global symmetry.[18] The right-handed quarks and leptons are regarded as elementary and do not participate in the weak interactions.

Schrempp and Schrempp[17] have recently extended the Abbott-Farhi model[14] by replacing the fundamental scalars by fermionic preons which transform as doublets under the strong confining hypercolor product group $SU(2)_L \times SU(2)_R$. Both left-handed and right-handed quarks and leptons are now composite fermions which exhibit a residual $SU(2)_L \times SU(2)_R$ global symmetry. The left-right symmetry is broken by assigning different confinement scales to the $SU(2)_L$ and $SU(2)_R$ hypercolor groups. Their model admits one generation of standard quarks and leptons, although more generations can be recovered through suitable enlargement[19] of the model.

Here we extend their model by considering three confining groups: $G_{\rm HC} = SU(N)_{\rm TC} \times SU(2)_L \times SU(2)_R$, one of which can be identified with technicolor. With proper breaking of the hyperflavor symmetry through technifermion condensates, 't Hooft's anomaly conditions[20] can be matched so as to yield n standard generations of quarks and leptons and $n-1$ generations of "mirror" quarks and leptons, where the weak interaction enters through the appearance of a global $SU(2)_L \times SU(2)_R$ symmetry which did not exist at the preon level. All quarks and leptons of one generation are kept massless on the scale of $\Lambda_{\rm HC}$ by a conserved $U(1)_A$ symmetry; a discrete axial symmetry labels the different generations and prevents mass generation through pairing off between standard and "mirror" generations. Three conserved vectorial U(1) symmetries are identified with I_3, $B-L$, and $B+L$, so the correct charges can be assigned and quark-lepton transitions forbidden. The problems associated with proton decay occurring rapidly through preon rearrangement are thus alleviated.

With the presence of a confining technicolor group, the standard quarks and leptons can acquire light masses through techniboson exchange and technifermion condensates. No extended technicolor (ETC) mechanism[21] is introduced or required. A possible mechanism for lowering some of the weak-boson masses is also present through technifermion condensates.

In short, the strong $SU(2)_L \times SU(2)_R$ forces confine the preons into three-preon bound states which are technifermions or technicolor singlets (quarks and leptons). The technicolor force in turn plays three roles: it breaks the conserved chiral symmetry in stages through technifermion condensates with anomaly matching provided in the first stage at the $\Lambda_{2L,R}$ confining scales; the particular condensate(s) which acquire a nonzero vacuum expectation value determines the surviving discrete symmetry which distinguishes different generations of quarks and leptons; and masses for the light fermions are generated through techniboson exchange and technifermion condensates.[22]

Unfortunately, since the hypercolor dynamics is not well understood, we can at most present a plausible scenario for the symmetry-breaking and mass-generation features of the model. In this regard, the most-attractive-channel hypothesis of Raby, Dimopoulos, and Susskind[23] plays a crucial role in evolving the picture from the high-energy (10^3 TeV) preon interaction region down through the technicolor (1–100 TeV) and low-energy (1–100 GeV) region. Nevertheless, we find that scalar-condensate formation and concomitant symmetry breaking go hand in hand with the mass-generation mechanisms to yield a plausible scenario. We do not claim that the model is completely realistic, but many of the desired features may persist in a similar form in the real world.

The outline of our work is as follows. The preon model is formulated and composite spectrum deduced in Sec. II. The role of technifermion condensates and the most-attractive-channel hypothesis is explored in Sec. III. Electroweak interactions are discussed in Sec. IV, and mass-generation mechanisms are presented in Sec. V. We conclude with a summary of the features of the model in Sec. VI.

II. PREON MODEL AND COMPOSITE SPECTRUM

A. Preons and hypercolor/hyperflavor symmetries

Our starting point is a set of preons which transform under the product hypercolor group

$$G_{\rm HC} = SU(N)_{\rm TC} \times SU(2)_L \times SU(2)_R \ , \qquad (2.1)$$

where the confinement scales labeled $\Lambda_{\rm TC}$, Λ_{2L}, and Λ_{2R} will all be taken greater than 300 GeV but are otherwise left unspecified for the time being. The $SU(N)_{\rm TC}$ group will be identified with technicolor, and as in the Schrempp-Schrempp model,[17] $SU(2)_L \times SU(2)_R$ is chosen so as to recover a "hidden" global flavor symmetry $SU(2)_L \times SU(2)_R$ associated with the left-right-symmetric weak interactions. We treat lepton number as the fourth color in the manner of Pati and Salam[24] and assign the preons to the hypercolor group in (2.1) according to

$$(N;2,2)_L + (N;2,2)_R + (N;2,1)_L + (N;1,2)_R + 4(1;2,1)_L + 4(1;1,2)_R \ .$$

The $U(4)_L \times U(4)_R \times [U(1)]^4$ hyperflavor (HF) symmetry is broken by instanton effects[25] down to

$$G_{\rm HF} = SU(4)_L \times SU(4)_R \times U(1)_V \times U(1)_{V'} \times U(1)_A \ , \qquad (2.2)$$

in terms of which the left-handed preons appear in the representations

$$T=(\underline{N};\underline{2},\underline{2};1,1)_{1,0;2},\quad T'=(\underline{\overline{N}};\underline{2},\underline{2};1,1)_{-1,0;2}\ ,$$
$$U=(\underline{N};\underline{2},\underline{1};1,1)_{0,4;-4},\quad U'=(\underline{\overline{N}},\underline{1},\underline{2};1,1)_{0,-4;-4}\ , \tag{2.3}$$
$$V=(\underline{1};\underline{2},\underline{1};4,1)_{0,-N;-N},\quad V'=(\underline{1};\underline{1},\underline{2};1,\overline{4})_{0,N;-N}\ .$$

The hypercolor representations of (2.1) are underlined for clarity.

The U(1) values are assigned by constructing from the preon number operators three conserved generators

$$Y_1=\eta_T-\eta_{T'}\ ,$$
$$Y_2=4(\eta_U-\eta_{U'})-N(\eta_V-\eta_{V'})\ , \tag{2.4}$$
$$X_A=2(\eta_T+\eta_{T'})-4(\eta_U+\eta_{U'})-N(\eta_V+\eta_{V'})\ ,$$

which are orthogonal to the three generators broken by instantons

$$Q_{\rm TC}=2(\eta_T+\eta_{T'})+(\eta_U+\eta_{U'})\ ,$$
$$Q_L=2N(\eta_T+\eta_{T'})+N\eta_U+4\eta_V\ , \tag{2.5}$$
$$Q_R=2N(\eta_T+\eta_{T'})+N\eta_{U'}+4\eta_{V'}\ .$$

From these three broken generators we can form the orthogonal set

$$X_{B1}=2(\eta_T+\eta_{T'})+(\eta_U+\eta_{U'})\ ,$$
$$X_{B2}=N(\eta_T+\eta_{T'})-2N(\eta_U+\eta_{U'})+10(\eta_V+\eta_{V'})\ , \tag{2.6}$$
$$Y_B=N(\eta_U-\eta_{U'})+4(\eta_V-\eta_{V'})\ ,$$

corresponding to two broken axial U(1)'s and one broken vectorial U(1). The change in the broken generators can be related to the $\nu_{\rm TC}$, ν_{2L}, and ν_{2R} winding numbers[25] by

$$\Delta X_{B1}=20\nu_{\rm TC}+9N(\nu_L+\nu_R)\ ,$$
$$\Delta X_{B2}=(2N^2+40)(\nu_L+\nu_R)\ , \tag{2.7}$$
$$\Delta Y_B=(N^2+16)(\nu_L-\nu_R)\ .$$

Finally, separating the effects of the independent instantons we find

$$\Delta\left[X_{B1}-\frac{9N}{2N^2+40}X_{B2}\right]=20\nu_{\rm TC}=0,\pm 20,\pm 40,\ldots,$$
$$\Delta\left[\frac{1}{20+N^2}X_{B2}+\frac{2}{16+N^2}Y_B\right]=4\nu_L=0,\pm 4,\pm 8,\ldots, \tag{2.8}$$
$$\Delta\left[\frac{1}{20+N^2}X_{B2}-\frac{2}{16+N^2}Y_B\right]=4\nu_R=0,\pm 4,\pm 8,\ldots .$$

Although three U(1) symmetries are broken by instantons, three discrete symmetries emerge intact. In addition to the two discrete axial symmetries, the unbroken $U(1)_A$ axial symmetry in (2.2) prevents the preons from pairing off and becoming massive.

The rank of the $SU(N)_{\rm TC}$ group has been left unspecified, but N must be even to avoid the theoretical inconsistency pointed out by Witten[26] which arises when an odd number of SU(2) doublets are present. If we further impose the requirement of asymptotic freedom[27] on each of the hypercolor forces, we require both

$$5N+4<22 \text{ and } 12<11N\ . \tag{2.9}$$

Hence we are led to $N=2$ as the sole possibility. We shall not immediately impose this restriction but shall later return to it for purposes of illustration.

B. Anomaly matching conditions and light composite fermions

Below all three confinement scales $\Lambda_{\rm TC}$, Λ_{2L}, and Λ_{2R}, the only composite fermions and bosons which can exist are hypercolor singlets. If, for simplicity, we restrict our attention to three-preon bound states for the fermions, a complete list of the left-handed Weyl spinors is given by

$$l_1:\ V(TU')=(4,1)_{1,-(4+N);-(2+N)},\quad l_1':\ V'(T'U)=(1,\overline{4})_{-1,4+N;-(2+N)}\ ,$$
$$l_2:\ V(TU')^\dagger=(4,1)_{-1,4-N;2-N},\quad l_2':\ V'(T'U)^\dagger=(1,\overline{4})_{1,-4+N;2-N}\ ,$$
$$l_3:\ V^\dagger(T'U'^\dagger)=(\overline{4},1)_{-1,4+N;6+N},\quad l_3':\ V'^\dagger(TU^\dagger)=(1,4)_{1,-(4+N);6+N}\ , \tag{2.10}$$
$$l_4:\ V^\dagger(T'U'^\dagger)^\dagger=(\overline{4},1)_{1,-4+N;-6+N},\quad l_4':\ V'^\dagger(TU^\dagger)^\dagger=(1,4)_{-1,4-N;-6+N}\ ,$$

for even N. The l_i label the indices of each bound state, while only the chiral-SU(4) representations are indicated in parentheses.

Hyperflavor anomalies[28] which exist on the preon level must be matched on the composite level as emphasized first by 't Hooft[20] and later clarified[29] by Frishman, Schwimmer, Banks, and Yankielowicz and by Coleman and Grossman. In particular, eight nontrivial anomaly conditions must be matched (for a left-right-symmetric solution) corresponding to triangle diagrams with $[SU(4)_L]^3$, $[SU(4)_L]^2U(1)_V$, $[SU(4)_L]^2U(1)_{V'}$, $[SU(4)_L]^2U(1)_A$, $[U(1)_A]^3$, $U(1)_A[U(1)_V]^2$, $U(1)_A[U(1)_{V'}]^2$, and $U(1)_AU(1)_VU(1)_{V'}$ currents at the three vertices. It is a simple matter to check that the anomaly conditions cannot be matched for any non-negative[30] integer choice of the eight indices appearing in (2.9). This requires that saturation of the anomaly conditions involves massless Goldstone bosons in addition to any possible massless composite fermions, implying that at least part of the hyperflavor symmetry of (2.2) must be broken.[20,29]

Postponing to Sec. III a study of what condensates can form to break the chiral hyperflavor symmetry, we simply observe here that if $G_{\rm HF}$ is broken into the subgroup

$$G'_{\rm HF}={\rm SU}(3)_L\times{\rm SU}(3)_R\times{\rm U}(1)_V\times{\rm U}(1)_{V'}\times{\rm U}(1)_{V''}\times{\rm U}(1)_{A'} \tag{2.11}$$

anomaly matching can be achieved for all vertex diagrams at the composite-fermion and preon levels. In terms of this new hyperflavor group, the preons now transform according to

$$\begin{aligned} T&=(\underline{N};\underline{2},\underline{2};1,1)_{1,0,0;0},\quad T'=(\underline{\overline{N}};\underline{2},\underline{2};1,1)_{-1,0,0;0}\,,\\ U&=(\underline{N};\underline{2},\underline{1};1,1)_{0,4,0;0},\quad U'=(\underline{\overline{N}};\underline{1},\underline{2};1,1)_{0,-4,0;0}\,,\\ Q&=(\underline{1};\underline{2},\underline{1};3,1)_{0,-N,1;1},\quad Q'=(\underline{1};\underline{1},\underline{2};1,\overline{3})_{0,N,-1;1}\,,\\ L&=(\underline{1};\underline{2},\underline{1};1,1)_{0,-N,-3;-3},\quad L'=(\underline{1};\underline{1},\underline{2};1,1)_{0,N,3;-3}\,, \end{aligned} \tag{2.12}$$

where the first two ${\rm U}(1)_V$ quantum numbers are Y_1 and Y_2, as before, and the new third ${\rm U}(1)_{V''}$ and ${\rm U}(1)_{A'}$ generators are

$$Y_3=\eta_Q-\eta_{Q'}-3(\eta_L-\eta_{L'}),\quad X'=\eta_Q+\eta_{Q'}-3(\eta_L+\eta_{L'})\,, \tag{2.13}$$

respectively. The original ${\rm U}(1)_A$ is broken down to a discrete group ${\rm Z}(m)_A$, where m will later be specified, which serves to keep all preons massless if $m>4$. In general, the discrete symmetries (2.8) resulting from the instanton effects will be broken by condensates which induce the breaking of $G_{\rm HF}$ into $G'_{\rm HF}$.

The hypercolor-singlet composite states in (2.9) split into the set

$$\begin{aligned} &l_{11}: L(TU')=(1,1)_{1,-(4+N),-3;-3},\quad l'_{11}: L'(T'U)=(1,1)_{-1,4+N,3;-3}\,,\\ &l_{21}: L(TU')^\dagger=(1,1)_{-1,(4-N),-3;-3},\quad l'_{21}: L'(T'U)^\dagger=(1,1)_{1,-(4-N),3;-3}\,,\\ &l_{12}: Q(TU')=(3,1)_{1,-(4+N),1;1},\quad l'_{12}: Q'(T'U)=(1,\overline{3})_{-1,4+N,-1;1}\,,\\ &l_{22}: Q(TU')^\dagger=(3,1)_{-1,4-N,1;1},\quad l'_{22}: Q'(T'U)^\dagger=(1,\overline{3})_{1,-(4-N),-1;1}\,,\\ &l_{31}: L^\dagger(T'U'^\dagger)=(1,1)_{-1,4+N,3;3},\quad l'_{31}: L'^\dagger(TU^\dagger)=(1,1)_{1,-(4+N),-3;3}\,,\\ &l_{41}: L^\dagger(T'U'^\dagger)^\dagger=(1,1)_{1,-(4-N),3;3},\quad l'_{41}: L'^\dagger(TU^\dagger)^\dagger=(1,1)_{-1,4-N,-3;3}\,,\\ &l_{32}: Q^\dagger(T'U'^\dagger)=(\overline{3},1)_{-1,4+N,-1;-1},\quad l'_{32}: Q'^\dagger(TU^\dagger)=(1,3)_{1,-(4+N),1;-1}\,,\\ &l_{42}: Q^\dagger(T'U'^\dagger)^\dagger=(\overline{3},1)_{1,-(4-N),-1;-1},\quad l_{42}: Q'^\dagger(TU^\dagger)^\dagger=(1,3)_{-1,4-N,1;-1}\,, \end{aligned} \tag{2.14}$$

where only the representations of the broken hyperflavor group $G'_{\rm HF}$ in (2.10) are indicated. Now matching of 12 nontrivial anomaly conditions is obtained such that the restrictions on the indices reduce to one equation

$$l_A-l_B=1\,, \tag{2.15a}$$

where

$$\begin{aligned} l_A&=l_{1i}=l_{2i}=l'_{1i}=l'_{2i}\,,\\ l_B&=l_{3i}=l_{4i}=l'_{3i}=l'_{4i}\,, \end{aligned}\qquad i=1,2\,. \tag{2.15b}$$

With $l_B=0$, a global ${\rm SU}(2)_L\times{\rm SU}(2)_R$ symmetry appears for the composite fermions, as in the Schrempp-Schrempp model,[17] resulting in a global doublet structure; at the preon level this global symmetry is absent and is replaced by the confining ${\rm SU}(N)_{\rm TC}\times{\rm SU}(2)_L\times{\rm SU}(2)_R$ gauge symmetry instead. With $l_B\neq0$, the global symmetry is ${\rm SU}(2)_L\times{\rm SU}(2)_R\times{\rm SU}(2)'_L\times{\rm SU}(2)'_R$. In any case, the general solution is N independent, i.e., independent of the rank of the gauged ${\rm SU}(N)_{\rm TC}$ group.[31]

In order to associate the physical quarks and leptons with the composite states of (2.14), we identify the following quantum numbers with the Y_1, Y_2, and Y_3 hypercharges:

$$\begin{aligned} I_{3L}+I_{3R}&=\tfrac{1}{2}Y_1\,,\\ B-L&=\tfrac{1}{3}Y_3\,,\\ B+L&=-\frac{2}{N}Y_1-\frac{1}{2N}Y_2-\frac{1}{6}Y_3\,, \end{aligned} \tag{2.16}$$

so the electric charge is given by

$$\begin{aligned} Q&=I_{3L}+I_{3R}+\tfrac{1}{2}(B-L)\\ &=\tfrac{1}{2}Y_1+\tfrac{1}{6}Y_3\,. \end{aligned} \tag{2.17}$$

The quantum numbers for all preons and composite states are given in Table I, from which we identify set A with one generation of standard quarks and leptons, while set B corresponds to a generation of "mirror" quarks and leptons for the same choice of chiral color ${\rm SU}(3)_L\times{\rm SU}(3)_R$. Anomaly matching condition (2.15) then requires n standard generations and $n-1$ "mirror" generations of quarks and leptons. In terms of diagonal (vectorial) ${\rm SU}(3)_c$, the "mirror" composites behave just like another standard generation though they belong to a different global ${\rm SU}(2)_L\times{\rm SU}(2)_R$ group and have different weak bosons associated with these interactions.[32]

The states within a generation are prevented from pairing off and becoming massive by the ${\rm U}(1)_{A'}$ symmetry of (2.13). On the other hand the preons and standard and mirror generations are prevented from pairing off only by

TABLE I. Quantum numbers for the preons and composite fermions of each generation. The representations and hypercharge labels correspond to the hypercolor and hyperflavor groups of (2.1) and (2.11).

		I_3	Q	B	L
	Preons				
	$T=(\underline{N};\underline{2},\underline{2};1,1)_{1,0,0;0}$	$\frac{1}{2}$	$\frac{1}{2}$	$-1/N$	$-1/N$
	$U=(\underline{N};\underline{2},\underline{1};1,1)_{0,4,0;0}$	0	0	$-1/N$	$-1/N$
	$Q=(\underline{1};\underline{2},\underline{1};3,1)_{0,-N,1;1}$	0	$\frac{1}{6}$	$\frac{1}{3}$	0
	$L=(\underline{1};\underline{2},\underline{1};1,1)_{0,-N,-3;-3}$	0	$-\frac{1}{2}$	0	1
	$T'=(\underline{\bar{N}};\underline{2},\underline{2};1,1)_{-1,0,0;0}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$1/N$	$1/N$
	$U'=(\underline{N};\underline{1},\underline{2};1,1)_{0,-4,0;0}$	0	0	$1/N$	$1/N$
	$Q'=(\underline{1};\underline{1},\underline{2};1,\bar{3})_{0,N,-1;1}$	0	$-\frac{1}{6}$	$-\frac{1}{3}$	0
	$L'=(\underline{1};\underline{1},\underline{2};1,1)_{0,N,3;-3}$	0	$\frac{1}{2}$	0	-1
Set A composites (standard generation)					
f_L	ν_l: $L(TU')=(1,1)_{1,-4-N,-3;-3}$	$\frac{1}{2}$	0	0	1
	l: $L(TU')^\dagger=(1,1)_{-1,4-N,-3;-3}$	$-\frac{1}{2}$	-1	0	1
	u: $Q(TU')=(3,1)_{1,-4-N,1;1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	0
	d: $Q(TU')=(3,1)_{-1,4-N,-1;1}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0
f_L^c	ν_l^c: $L'(T'U)=(1,1)_{-1,4+N,3;-3}$	$-\frac{1}{2}$	0	0	-1
	l^c: $L'(T'U)^\dagger=(1,1)_{1,-4+N,3;-3}$	$\frac{1}{2}$	1	0	-1
	u^c: $Q'(T'U)=(1,\bar{3})_{-1,4+N,-1;1}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0
	d^c: $Q'(T'U)^\dagger=(1,\bar{3})_{1,-4+N,1;1}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0
Set B composites (mirror generation)					
F_L^c	$\nu_l'^c$: $L^\dagger(T'U'^\dagger)=(1,1)_{-1,4+N,3;3}$	$-\frac{1}{2}$	0	0	-1
	l'^c: $L^\dagger(T'U'^\dagger)^\dagger=(1,1)_{1,-4+N,3;3}$	$\frac{1}{2}$	1	0	-1
	u'^c: $Q^\dagger(T'U'^\dagger)=(\bar{3},1)_{-1,4+N,-1;-1}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0
	d'^c: $Q^\dagger(T'U'^\dagger)^\dagger=(\bar{3},1)_{1,-4+N,-1;-1}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0
F_L	ν_l': $L'^\dagger(TU^\dagger)=(1,1)_{1,-4-N,-3;3}$	$\frac{1}{2}$	0	0	1
	l': $L'^\dagger(TU^\dagger)^\dagger=(1,1)_{-1,4-N,-3;3}$	$-\frac{1}{2}$	-1	0	1
	u': $Q'^\dagger(TU^\dagger)^\dagger=(1,3)_{1,-4-N,1;-1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	0
	d': $Q'^\dagger(TU^\dagger)^\dagger=(1,3)_{-1,4-N,1;-1}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0

the discrete symmetry $Z(m)_A$ into which the original $U(1)_A$ symmetry is broken, provided $m>4$. This discrete symmetry, in turn, can be used to distinguish different generations. We shall return to this issue later when we discuss possible condensates which effect the hyperflavor symmetry breaking.

C. Other composite fermions and bosons

The list of hypercolor-singlet bosons include the scalar and vector states presented in Table II. These boson states are unprotected by the G_{HF} chiral symmetry and get heavy with typical masses of the (largest) confinement scale Λ_{TC}, Λ_{2L}, or Λ_{2R}. The only exceptions are pseudo-

TABLE II. List of hypercolor-singlet bosons for $SU(2)_{TC}$. Only the hyperflavor representations of (2.2) are indicated.

Spin 0		
	$TT'=(1,1)_{0,0;4}$	$(TT')^\dagger=(1,1)_{0,0;-4}$
	$VV=(6,1)_{0,-4;-4}$	$(VV)^\dagger=(6,1)_{0,4;4}$
	$V'V'=(1,6)_{0,4;-4}$	$(V'V')^\dagger=(1,6)_{0,-4;4}$
Spin 1		
	$TT'^\dagger=(1,1)_{2,0;0}$	$T'T^\dagger=(1,1)_{-2,0;0}$
	$TT^\dagger=(1,1)_{0,0;0}$	$T'T'^\dagger=(1,1)_{0,0;0}$
	$UU^\dagger=(1,1)_{0,0;0}$	$U'U'^\dagger=(1,1)_{0,0;0}$
	$VV^\dagger=(15,1)_{0,0;0}$	$V'V'^\dagger=(1,15)_{0,0;0}$
	$VV^\dagger=(1,1)_{0,0;0}$	$V'V'^\dagger=(1,1)_{0,0;0}$

TABLE III. List of technibosons for $SU(2)_{TC}$. Only the technicolor and hyperflavor representations of (2.2) are indicated.

Spin 0		
	$TT=(\underline{3};1,1)_{2,0;4}$	$(TT)^{\dagger}=(\underline{3};1,1)_{-2,0;-4}$
	$TT'=(\underline{3},1,1)_{0,0;4}$	$(TT')^{\dagger}=(\underline{3};1,1)_{0,0;-4}$
	$T'T'=(\underline{3};1,1)_{-2,0;4}$	$(T'T')^{\dagger}=(\underline{3};1,1)_{2,0;-4}$
	$UV=(\underline{2};4,1)_{0,2;-6}$	$(UV)^{\dagger}=(\underline{2},\overline{4},1)_{0,-2;6}$
	$U'V'=(\underline{2};1,\overline{4})_{0,-2;-6}$	$(U'V')^{\dagger}=(\underline{2};1,4)_{0,2;6}$
Spin 1		
	$TT'^{\dagger}=(\underline{3};1,1)_{2,0;0}$	$T'T^{\dagger}=(\underline{3};1,1)_{-2,0;0}$
	$TT^{\dagger}=(\underline{3};1,1)_{0,0;0}$	$T'T'^{\dagger}=(\underline{3};1,1)_{0,0;0}$
	$UU^{\dagger}=(\underline{3};1,1)_{0,0;0}$	$U'U'^{\dagger}=(3;1,1)_{0,0;0}$
	$UV^{\dagger}=(\underline{2};\overline{4},1)_{0,6;-2}$	$U'V'^{\dagger}=(\underline{2};1,4)_{0,-6;-2}$
	$U^{\dagger}V=(\underline{2};4,1)_{0,-6;2}$	$U'^{\dagger}V'=(\underline{2};1,4)_{0,6;2}$

Goldstone bosons[33] (PGB's) which arise from the chiral G_{HF} symmetry breaking via condensates. This is taken up in Sec. III.

Other bosons can be constructed which are nonsinglets with respect to one or two of the confining gauge groups. Examples of technibosons are given in Table III for the special case of $SU(2)_{TC}$. With the exception of Sec. IV, we shall not make use of bosons which are nonsinglets under one of the confining $SU(2)_{L,R}$ groups and do not list them explicitly.

Also of special interest are the technifermions which are singlets under the confining $SU(2)_L \times SU(2)_R$. These states can be classified into several types according to their hyperflavor transformation properties: TUU', TUV', and TVV'. A complete list is presented in Table IV, again for $SU(2)_{TC}$. Included among the TUV' type are conventional techniquarks and technileptons,[8] but all others have exotic Q, B, and L assignments.

III. TECHNIFERMION CONDENSATES

A. Spontaneous symmetry breaking

We now attempt to identify fermion condensates which can develop nonzero vacuum expectation values and dynamically break the chiral symmetry from G_{HF} in (2.2) down to G'_{HF} in (2.11). Various authors[34] have shown that multipreon condensates occur in more attractive channels than two-preon condensates. This suggests that we explore the possibility that composite hypercolor-nonsinglet fermions, e.g., technifermions, form six-preon condensates.

The symmetry breaking $G_{HF} \to G'_{HF}$ can occur all in one step or through a series of steps. Consider first the one-step process suggested in our earlier work[16] on this model. There the condensate

$$\langle (TVV')(T'V^{\dagger}V'^{\dagger})\rangle = (15,15)_{0,0;4} \qquad (3.1)$$

is presumed to develop a nonzero vacuum expectation value which breaks the chiral SU(4) symmetry down to chiral SU(3) with the $U(1)_A$ symmetry broken to $Z(4)_A$. New $U(1)_{V''}$ and $U(1)_{A'}$ symmetries appear so the $G_{HF} \to G'_{HF} \times Z(4)_A$ directly. Alternatively the separate condensates

$$\langle TV(TV)^{\dagger}\rangle = (15,1)_{0,0;0}\ , \qquad (3.2a)$$

$$\langle T'V'(T'V')^{\dagger}\rangle = (1,15)_{0,0;0}\ , \qquad (3.2b)$$

TABLE IV. List of technifermions for $SU(2)_{TC}$. Only the technicolor and hyperflavor representations of (2.2) are indicated.

Type TUU'		
	$TUU'=(\underline{4};1,1)_{1,0;-6}$	$T'U'U=(\underline{4};1,1)_{-1,0;-6}$
	$T(UU')^{\dagger}=(\underline{4};1,1)_{1,0;10}$	$T'(U'U)^{\dagger}=(\underline{4};1,1)_{-1,0;10}$
	$T^{\dagger}(UU'^{\dagger})=(\underline{4};1,1)_{-1,8;-2}$	$T'^{\dagger}(U'U^{\dagger})=(\underline{4};1,1)_{1,-8;-2}$
	$T^{\dagger}(UU'^{\dagger})^{\dagger}=(\underline{4};1,1)_{-1,-8;-2}$	$T'^{\dagger}(U'U^{\dagger})^{\dagger}=(\underline{4};1,1)_{1,8;-2}$
	$TUU'=(\underline{2};1,1)_{1,0;-6}$	$T'U'U=(\underline{2};1,1)_{-1,0;-6}$
	$T(UU')^{\dagger}=(\underline{2};1,1)_{1,0;10}$	$T'(U'U)^{\dagger}=(\underline{2};1,1)_{-1,0;10}$
	$T^{\dagger}(UU'^{\dagger})=(\underline{2};1,1)_{-1,8;-2}$	$T'^{\dagger}(U'U^{\dagger})=(\underline{2};1,1)_{1,-8;-2}$
	$T^{\dagger}(UU'^{\dagger})^{\dagger}=(\underline{2};1,1)_{-1,-8;-2}$	$T'^{\dagger}(U'U^{\dagger})^{\dagger}=(\underline{2};1,1)_{1,8;-2}$
Type TUV'		
	L_1: $VTU'=(\underline{3};4,1)_{1,-6;-4}$	$V'T'U=(\underline{3};1,\overline{4})_{-1,6;-4}$
	L_2: $V(TU')^{\dagger}=(\underline{3};4,1)_{-1,2;0}$	$V'(T'U)^{\dagger}=(\underline{3};1,\overline{4})_{1,-2;0}$
	L_3: $V^{\dagger}(T'U'^{\dagger})=(\underline{3};\overline{4},1)_{-1,6;8}$	$V'^{\dagger}(TU^{\dagger})=(\underline{3};1,4)_{1,-6;8}$
	L_4: $V^{\dagger}(T'U'^{\dagger})^{\dagger}=(\underline{3};\overline{4},1)_{1,-2;-4}$	$V'^{\dagger}(TU^{\dagger})^{\dagger}=(\underline{3};1,4)_{-1,2;-4}$
	l_{S1}: $VT'U'=(\underline{3};4,1)_{-1,-6;-4}$	$V'TU=(\underline{3},1,\overline{4})_{1,6;-4}$
	l_{S2}: $V(T'U')^{\dagger}=(\underline{3};4,1)_{1,2;0}$	$V'(TU)^{\dagger}=(\underline{3};1,\overline{4})_{-1,-2;0}$
	l_{S3}: $V^{\dagger}(TU'^{\dagger})=(\underline{3};\overline{4},1)_{1,6;8}$	$V'^{\dagger}(T'U^{\dagger})=(\underline{3};1,4)_{-1,-6;8}$
	l_{S4}: $V^{\dagger}(TU'^{\dagger})^{\dagger}=(\underline{3};\overline{4},1)_{-1,-2;-4}$	$V'^{\dagger}(T'U^{\dagger})^{\dagger}=(\underline{3};1,4)_{1,2;-4}$
Type TVV'		
	l_{N1}: $TVV'=(\underline{2};4,\overline{4})_{1,0;-2}$	$T'V'V=(\underline{2};4,\overline{4})_{-1,0;-2}$
	l_{N2}: $T(VV')^{\dagger}=(\underline{2};\overline{4},4)_{1,0;6}$	$T'(V'V)^{\dagger}=(\underline{2};\overline{4},4)_{-1,0;6}$
	l_{N3}: $T^{\dagger}(VV'^{\dagger})=(\underline{2};4,4)_{-1,-4;-2}$	$T'^{\dagger}(V'V^{\dagger})=(\underline{2};\overline{4},\overline{4})_{1,4;-2}$
	l_{N4}: $T^{\dagger}(VV'^{\dagger})^{\dagger}=(\underline{2};\overline{4},\overline{4})_{-1,4;-2}$	$T'^{\dagger}(V'V^{\dagger})^{\dagger}=(\underline{2};4,4)_{1,-4;-2}$

and

$$\langle (TU^\dagger U'^\dagger)(T'U^\dagger U'^\dagger)\rangle = (1,1)_{0,0;20} \tag{3.2c}$$

could effect the desired breaking $G_{\rm HF} \to G'_{\rm HF} \times Z(20)_A$ in this case. The latter scheme involving hyperboson condensates is less probable[34] than the former technifermion condensate since it involves just four rather than six preons in the condensate. Also technibosons are not protected by a chiral symmetry from developing large masses and becoming sterile with respect to condensate formation.

We now consider a more appealing condensation scenario based on the most-attractive-channel (MAC) hypothesis of Raby, Dimopoulos, and Susskind.[23] According to the MAC hypothesis, the relative binding strength of two technifermions can be estimated by considering the one technigluon exchange graph and its corresponding coupling coefficient and quadratic Casimir operator $g^2(\mu)C_2(R)$, where R is the dimension of the technicolor representation of the technifermions and μ is the energy scale in question. At a given energy scale μ, the binding is greater the larger the quadratic Casimir operator. On the other hand, it is conventionally assumed[35] that a vacuum condensate will be formed whenever $\alpha(\mu_R)C_2(R) \simeq 1$. This implies that technifermions with larger C_2's will condense at higher energy scales than those with smaller C_2's which become strong enough to condense only at lower energy scales, provided $\alpha(\mu)$ rises as μ decreases. This will be the case for asymptotically free theories or for nonasymptotically free theories if the β function is negative in the relevant region, such as beyond the ultraviolet fixed point.[36]

Given the above pattern of hypercolor condensation in the most attractive channels, the $G_{\rm HF}$ hyperflavor symmetry can be broken down to $G'_{\rm HF}$ in two steps with the first condensates involving type-TUU' technifermions breaking the $U(1)_A$ symmetry. Examples of such condensates are

$$\langle (T^\dagger UU'^\dagger)(T'^\dagger U^\dagger U')\rangle = (1,1)_{0,0;-4} , \tag{3.3a}$$

$$\langle (TUU')(T'U^\dagger U'^\dagger)\rangle = (1,1)_{0,0;4} , \tag{3.3b}$$

$$\langle (TUU')(T^\dagger UU'^\dagger)\rangle = (1,1)_{0,8;-8} , \tag{3.3c}$$

$$\langle (TU^\dagger U'^\dagger)(T^\dagger UU'^\dagger)\rangle = (1,1)_{0,8;8} , \tag{3.3d}$$

$$\langle (TUU')(T'UU')\rangle = (1,1)_{0,0;-12} , \tag{3.3e}$$

$$\langle (TU^\dagger U'^\dagger)(T'U^\dagger U'^\dagger)\rangle = (1,1)_{0,0;20} , \tag{3.3f}$$

where only the $G_{\rm HF}$ hyperflavor representations are indicated. In the respective cases, the $U(1)_A$ continuous symmetry is broken into the discrete axial subgroups $Z(4)_A$, $Z(8)_A$, $Z(12)_A$, or $Z(20)_A$ accordingly, and the $U(1)_{V'}$ symmetry is conserved or may be broken down to $Z(8)_V$ for (3.3c) and (3.3d). The discrete symmetries resulting from the instanton interactions are broken completely. We, of course, have no handle over which breaking pattern occurs but simply note the two extremes corresponding to the largest, $Z(20)_A$, and the smallest subgroup, $Z(4)_A$, both of which keep $U(1)_{V'}$ unbroken.

It is of interest to remark that once the $U(1)_A$ group is broken by one (or more) of the condensates in (3.3), anomaly matching can be achieved for the three remaining nontrivial vertex diagrams of the form $[SU(4)_L]^3$, $[SU(4)_L]^2U(1)_V$, and $[SU(4)_L]^2U(1)_{V'}$. This suggests that although the confinement scales may be ordered according to $\Lambda_{\rm TC} \ll \Lambda_{2L} \lesssim \Lambda_{2R}$, the effective scale where the type-TUU' technifermions confine is of the same order of magnitude as the Λ_{2L} and Λ_{2R} scales.

In addition to the hypercolor-singlet set (2.10) of fermions with indices $l_A - l_B = 1$ as in (2.15), type-TUV' and type-TVV' technifermions can appear at this stage. The restrictions on the indices required to satisfy anomaly matching at the preon and composite levels for an N-independent solution are

$$\begin{aligned} 0 &= L_1 - L_3 = L_2 - L_4 , \\ 0 &= l_{S1} - l_{S3} = l_{S2} - l_{S4} , \\ 0 &= l_{N1} - l_{N2} . \end{aligned} \tag{3.4}$$

These states are protected from pairing off by the discrete $Z(m)_A$ symmetry provided $m > 4$.

Condensates of the type-TUV' technifermions can then occur via the N^2-1, $N(N+1)/2$, or $N(N-1)/2$ dimensional representations of technicolor at a lower energy scale than the first-stage condensates of the type-TUU' technifermions in (3.3). These condensates are numerous in number and serve to break the chiral $SU(4)_L \times SU(4)_R$ symmetry. Among the possibilities are

$$\langle (VTU')(V^\dagger T'U'^\dagger)\rangle = (15,1)_{0,0;4} , \tag{3.5a}$$

$$\langle (VTU')(V'T'U)\rangle = (4,\bar{4})_{0,0;-4-2N} , \tag{3.5b}$$

$$\langle (VT^\dagger U'^\dagger)(V'T'^\dagger U^\dagger)\rangle = (4,\bar{4})_{0,0;4-2N} . \tag{3.5c}$$

With (3.5a) and its primed counterpart, the chiral hyperflavor group is broken to

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_{V'} \times U(1)_{V''} \times U(1)_{A'} \times Z(4)_A . \tag{3.6a}$$

With (3.5b), $N=2$ and $m=12$ or 20 according to the condensates of (3.3), the surviving group is only

$$SU(4)_c \times U(1)_V \times U(1)_{V'} , \tag{3.6b}$$

whereas with $N=2$, (3.5c) and any m or (3.5b) and $m=4,8$ the surviving group is

$$SU(4)_c \times U(1)_V \times U(1)_{V'} \times Z(m)_A \tag{3.6c}$$

as the discrete symmetry remains unbroken. In the above, $SU(4)_c$ is the Pati-Salam four-color symmetry group. If the axial symmetry breaks down to $Z(4)_A$ or completely, the TUV'-type and TVV'-type technifermions are no longer protected by the discrete axial symmetry, and all can pair off and get massive.

Finally we note that if any type-TVV' technifermions survive the hyperflavor symmetry breaking as light objects, they can condense at the $\Lambda_{\rm TC}$ scale and break the symmetry down to

$$SU(3)_c \times U(1)_V \times U(1)_{V'} \times U(1)_{V''} . \tag{3.7}$$

For this purpose, the condensate

$$\langle (TQQ')(T'LL')\rangle = (3,\bar{3})_{0,0,0;-4} \quad (3.8a)$$

in the case of (3.6a) or the condensate

$$\langle (TVV')(T'UU')\rangle = (15)_{0,0} \quad (3.8b)$$

in the case of (3.6b) and (3.6c) are suitable. Alternatively, one can simply gauge the $SU(3)_c \times U(1)_{em}$ subgroup of the surviving global symmetry in (3.6). In any event, all the technifermions get massive, while only the HC-singlet fermions of the type listed in (2.14) remain relatively light. They themselves can pick up small masses through techniboson exchange diagrams with the help of technifermion condensates as discussed in Sec. V. No spectators need be introduced if only $SU(3)_c \times U(1)_{em}$ is gauged.

B. Pseudo-Goldstone bosons

Weinberg[33] has clearly demonstrated that to each broken generator of a chiral symmetry group corresponds one (pseudo-) Goldstone boson. Consider first the one-step $G_{HF} \to G'_{HF}$ symmetry-breaking condensate of (3.1).

The full G_{HF} group of (2.2) has 33 generators which transform according to

$$\begin{aligned} &VV^\dagger = (15,1)_{0,0;0}, \quad V'V'^\dagger = (1,15)_{0,0;0}\,, \\ &Y_1 \sim \frac{1}{\sqrt{2}}(TT^\dagger - T'T'^\dagger) = (1,1)_{0,0;0}\,, \\ &Y_2 \sim \frac{1}{(8N^2+32)^{1/2}}[4(UU^\dagger - U'U'^\dagger) \\ &\qquad - N(VV^\dagger - V'V'^\dagger)] = (1,1)_{0,0;0}\,, \\ &X_A \sim \frac{1}{(8N^2+40)^{1/2}}[2(TT^\dagger + T'T'^\dagger) - 4(UU^\dagger + U'U'^\dagger) \\ &\qquad - N(VV^\dagger + V'V'^\dagger)] = (1,1)_{0,0;0}\,, \end{aligned} \quad (3.9)$$

where only the hyperflavor symmetry is specified. Under the breaking of G_{HF} down to G'_{HF} only 20 of the 33 generators are conserved, while 13 are broken as follows:

Conserved generators:

$$\begin{aligned} &Q^\dagger Q = (8,1), \quad Q'^\dagger Q' = (1,8)\,, \\ &Y_1, Y_2\,, \\ &Y_3 \sim \frac{1}{\sqrt{24}}[Q^\dagger Q - Q'^\dagger Q' - 3(L^\dagger L - L'^\dagger L')] = (1,1)\,, \\ &X_{A'} \sim \frac{1}{\sqrt{24}}[Q^\dagger Q + Q'^\dagger Q' - 3(L^\dagger L + L'^\dagger L')] = (1,1)\,. \end{aligned} \quad (3.10)$$

Broken generators:

$$\begin{aligned} &L^\dagger Q = (3,1)_{0,0,4;(0),4}, \quad Q^\dagger L = (\bar{3},1)_{0,0-4;(0),-4}\,, \\ &L'^\dagger Q' = (1,\bar{3})_{0,0-4;(0),4}, \quad Q'^\dagger L' = (1,3)_{0,0,4;(0),-4}\,, \\ &X_A = (1,1)_{0,0,0;(0),0}\,. \end{aligned} \quad (3.11)$$

The appropriate symmetry breaking occurs with the vacuum condensate

$$\langle TT'VV'(VV')^\dagger \rangle = (15,15)_{0,0;4} \quad (3.12a)$$

in the one-step process where the nonzero element is chosen to be

$$\langle TT'LL'(LL')^\dagger \rangle = (1,1)_{0,0,0;(4),0} \neq 0 \quad (3.12b)$$

such that the 12 pseudo-Goldstone bosons transform as

$$\begin{aligned} &TT'QL^\dagger L'L'^\dagger = (3,1)_{0,0,4;(4),4}\,, \\ &TT'Q^\dagger LL'L'^\dagger = (\bar{3},1)_{0,0,-4;(4),-4}\,, \\ &TT'LL^\dagger Q'L'^\dagger = (1,\bar{3})_{0,0,-4;(4),4}\,, \\ &TT'LL^\dagger L'Q'^\dagger = (1,3)_{0,0,4;(4),-4}\,, \end{aligned} \quad (3.13)$$

and the one true Goldstone boson transforms as

$$\mathrm{Im}\, TT' = (1,1)_{0,0,0;(4),0}\,. \quad (3.14)$$

The broken generator value is indicated in parentheses. The 12 pseudo-Goldstone bosons carry chiral color and will be confined by the QCD force; the true Goldstone boson can, in fact, represent the Peccei-Quinn axion,[37] if we identify $U(1)_{PQ} \equiv U(1)_A$. The surviving discrete axial symmetry is just $Z(4)_A$.

We now turn to the two-step symmetry-breaking process via technifermions discussed earlier in some detail. The first step involves the breaking

$$G_{HF} \to SU(4)_L \times SU(4)_R \times U(1)_V \times U(1)_{V'} \times Z(m)_A$$

via one of the condensates in (3.5). The true Goldstone boson then transforms as

$$\begin{aligned} &\mathrm{Im}\, TT' = (1,1)_{0,0;(4),0}\,, \\ &\mathrm{Im}\, TT'UUU'U' = (1,1)_{0,0;(-12),0}\,, \\ &\mathrm{Im}\, TT'U^\dagger U^\dagger U'^\dagger U'^\dagger = (1,1)_{0,0;(20),0}\,, \end{aligned} \quad (3.15)$$

according to whether the surviving discrete axial symmetry group is $Z(4)_A$, $Z(12)_A$, or $Z(20)_A$. The other possible condensates in (3.5) leading to $Z(8)_A$ break the $U(1)_{V'}$ symmetry.

The chiral-SU(4)-symmetry breaking in (3.6a) occurring at the second stage leads to the 13 broken generators listed in (3.11). The corresponding pseudo-Goldstone bosons which appear must have exactly the same quantum numbers for the remaining conserved symmetries. Hence only PGB's of the following type can appear:

$$\begin{aligned} &QTU'L^\dagger T'U'^\dagger = (3,1)_{0,0,4;(4),4}\,, \\ &LTU'Q^\dagger T'U'^\dagger = (\bar{3},1)_{0,0,-4;(4),-4}\,, \end{aligned} \quad (3.16)$$

as well as their right-handed counterparts. The discrete axial symmetry $Z(m)_A$ surviving the first-stage $U(1)_A$ breaking is broken down to $Z(4)_A$ for $m=12$ or 20. As a result of this breaking, the technifermions are no longer protected from acquiring a mass and will, in general, pair off and get massive.

In the case of (3.6b) and (3.6c), where the chiral SU(4) symmetry breaks down to $SU(4)_c$, only 17 generators are conserved and 15 are broken. The 15 broken generators are all axial and diagonal in character and transform as

$$V^\dagger V + V'^\dagger V' = (15,1) + (1,15) = (15)_A\,. \quad (3.17)$$

The corresponding pseudo-Goldstone bosons transform as

$$(VTU')(V^{\dagger}T'U'^{\dagger})+(V'T'U)(V'^{\dagger}TU^{\dagger})=(15)_A\ . \tag{3.18}$$

While all the PGB's in (3.16) carry chiral color and will be confined, only 14 of the 15 broken generators of (3.17) carry $SU(3)_c$ color; the remaining one is a color singlet like the true Goldstone boson of (3.12). Hence two axions are present in the symmetry-breaking schemes of (3.6b) and (3.6c).

C. Generation labels

Different composite models admit the possibility of more than one generation of quarks and leptons through different mechanisms such as a continuous U(1) symmetry[38] or higher, or through discrete symmetries[39] arising from instanton effects. In the model considered here, the discrete symmetry arising from the breakdown of the continuous $U(1)_A$ symmetry serves as a useful generation-labeling mechanism.

We have already noted that the anomaly matching conditions reduce to the single equation (2.15) which is satisfied for any $l_A=1,2,3,\ldots$ with $l_B=l_A-1$. In order to allow the possibility that either index is greater than unity, we must be able to distinguish composite objects having the same chiral and hypercharge properties with some new label. The broken generator label, corresponding to the $Z(m)_A$ discrete symmetry arising from $U(1)_A$ breaking by condensate formation, is at our disposal. From our discussion of technifermion condensates in Sec. III A, we have noted that the surviving discrete symmetry is $Z(4)_A$, $Z(8)_A$, $Z(12)_A$, or $Z(20)_A$. We shall discuss each in turn.

In the case of a surviving $Z(4)_A$ symmetry, all X_A labels for the composite states in (2.9) are identical (for all even N), and the A and B sets of states are unprotected from pairing off when $U(1)_A$ is broken and anomaly matching can first be achieved. Hence the only possibility is $l_A=1$, $l_B=0$ corresponding to one generation of standard quarks and leptons. For a Z(8) symmetry, the conclusion is the same since one cannot introduce two generations of type A composites both of which are protected from pairing off with the single B type composites by the Z(8) symmetry.

The situation is different with a surviving $Z(12)_A$ or $Z(20)_A$ symmetry. In the former case, we can introduce two generations of type A and one of type B which are protected by the discrete $Z(12)_A$ and chiral SU(4) symmetries. Although the particular choice is not unique, once one selects X_A labels for the A states, the B state labels are uniquely determined. One possible choice for $Z(12)_A$ symmetry is given in Table V with $N=2$. For the case of $Z(20)_A$ symmetry, one can choose three generations of type A and two of type B which are labeled by this discrete chiral symmetry. One example, again for $N=2$, is given in Table VI.

It is interesting that while the anomaly matching condition (2.15) permits any number of standard generations and as many "mirror" generations less one, the technifermion condensates in (3.5) at our disposal yield at most a $Z(20)_A$ symmetry, and hence at most three standard generations and two mirror generations of composite quarks and leptons.

With regard to the technifermions, any generation pattern must be consistent with the anomaly matching conditions of (3.4). For a $Z(4)_A$ symmetry, all technifermions can pair off and get massive, so that no generation of type-TUV' and type-TVV' technifermions exist. In the case of a $Z(12)_A$ generation symmetry, at most one generation of type-TUV' and type-TVV' technifermions can occur. With a $Z(20)_A$ symmetry, at most two generations of technifermions exist. Hence only in the latter two cases is the mechanism of type-TUV' technifermion condensation definitely available to break the chiral SU(4) hyperflavor symmetry down to chiral SU(3) or $SU(4)_c$.

TABLE V. List of two standard and one mirror generations for $Z(12)_A$ with one choice of broken X_A labels.

			X_A		
A_1:	$V(TU')$	$=(4,1)$	8	$V'(T'U)$	$=(1,\bar{4})$
	$V(TU')^{\dagger}$	$=(4,1)$	0	$V'(T'U)^{\dagger}$	$=(1,\bar{4})$
A_2:	$V(TU')(TT')$	$=(4,1)$	0	$V'(T'U)(TT')$	$=(1,\bar{4})$
	$V(TU')^{\dagger}(TT')$	$=(4,1)$	4	$V'(T'U)^{\dagger}(TT')$	$=(1,\bar{4})$
B_1:	$V^{\dagger}(T'U'^{\dagger})$	$=(\bar{4},1)$	8	$V'^{\dagger}(TU^{\dagger})$	$=(1,4)$
	$V^{\dagger}(T'U'^{\dagger})^{\dagger}(TT')^2$	$=(\bar{4},1)$	4	$V'^{\dagger}(TU^{\dagger})^{\dagger}(TT')^2$	$=(1,4)$

TABLE VI. List of three standard and two mirror generations for $Z(20)_A$ with one choice of broken X_A labels.

			X_A		
A_1:	$V(TU')$	$=(4,1)$	16	$V'(T'U)$	$=(1,\bar{4})$
	$V(TU')^{\dagger}$	$=(4,1)$	0	$V'(T'U)^{\dagger}$	$=(1,\bar{4})$
A_2:	$V(TU')(TT')$	$=(4,1)$	0	$V'(T'U)(TT')$	$=(1,\bar{4})$
	$V(TU')^{\dagger}(TT')$	$=(4,1)$	4	$V'(T'U)^{\dagger}(TT')$	$=(1,\bar{4})$
A_3:	$V(TU')(TT')^3$	$=(4,1)$	8	$V'(T'U)(TT')^3$	$=(1,\bar{4})$
	$V(TU')^{\dagger}(TT')^3$	$=(4,1)$	12	$V'(T'U)^{\dagger}(TT')^3$	$=(1,\bar{4})$
B_1:	$V^{\dagger}(T'U'^{\dagger})$	$=(\bar{4},1)$	8	$V'^{\dagger}(TU^{\dagger})$	$=(1,4)$
	$V^{\dagger}(T'U'^{\dagger})^{\dagger}(TT')^2$	$=(\bar{4},1)$	4	$V'^{\dagger}(TU)^{\dagger}(TT')^2$	$=(1,4)$
B_2:	$V^{\dagger}(T'U'^{\dagger})(TT')^2$	$=(\bar{4},1)$	16	$V'^{\dagger}(TU^{\dagger})(TT')^2$	$=(1,4)$
	$V^{\dagger}(T'U'^{\dagger})^{\dagger}(TT')^4$	$=(\bar{4},1)$	12	$V'^{\dagger}(TU^{\dagger})^{\dagger}(TT')^4$	$=(1,4)$

IV. ELECTROWEAK INTERACTIONS

A. Weak isovector and isoscalar bosons

We have noted earlier that in the case of just one standard generation of quarks and leptons, a global $SU(2)_L\times SU(2)_R$ symmetry appears at the composite level which did not exist at the preon level. This symmetry can be identified with the left-right extension of the usual weak interactions. The basic difference is that the weak bosons here are bound states of hypercolor rather than fundamental gauge fields. As such, the weak force is a manifestation of the residual hypercolor interactions.[9]

The weak isovector bosons are four-preon objects which are formed from two scalars as

$$W_L^+ = H\mathscr{D}_\mu H = (1,1)_{2,-8;-4} \,,$$
$$W_L^0 = H^\dagger \overleftrightarrow{\mathscr{D}}_\mu H = (1,1)_{0,0;0} \,, \qquad (4.1a)$$
$$W_L^- = H^\dagger \mathscr{D}_\mu H^\dagger = (1,1)_{-2,8;4} \,,$$

for the $SU(2)_L$ transitions and

$$W_R^+ = K^\dagger \mathscr{D}_\mu K^\dagger = (1,1)_{2,-8;4} \,,$$
$$W_R^0 = K^\dagger \overleftrightarrow{\mathscr{D}}_\mu K = (1,1)_{0,0;0} \,, \qquad (4.1b)$$
$$W_R^- = K\mathscr{D}_\mu K = (1,1)_{-2,8;-4} \,,$$

for the $SU(2)_R$ transitions, all with $B = L = 0$ and $I_3 = 1$, 0, or -1;

$$H = TU' = (\underline{1};\underline{2},\underline{1};1,1)_{1,-4;-2} \,,$$
$$K = T'U = (\underline{1};\underline{1},\underline{2};1,1)_{-1,4;-2} \,, \qquad (4.2)$$

and their Hermitian conjugates are the composite *scalars* which form the vector bosons. Since the discrete symmetry in the case of just one generation is $Z(4)_A$, the W_L^i and W_R^i bosons have the same quantum numbers and will mix in general. We explore the possible consequences in Sec. V.

When $l_A > 1$ so that at least one "mirror" (B-type) generation appears, the global symmetry is actually $SU(2)_L \times SU(2)_R \times SU(2)'_L \times SU(2)'_R$ and the additional vector bosons transform as

$$W_L'^{+} = H'\mathscr{D}_\mu H' = (1,1)_{2,-8;-4} \,,$$
$$W_L'^{0} = H'^\dagger \overleftrightarrow{\mathscr{D}}_\mu H' = (1,1)_{0,0;0} \,, \qquad (4.3a)$$
$$W_L'^{-} = H'^\dagger \mathscr{D}_\mu H'^\dagger = (1,1)_{-2,8;4} \,,$$

and

$$W_R'^{+} = K'^\dagger \mathscr{D}_\mu K'^\dagger = (1,1)_{2,-8;4} \,,$$
$$W_R'^{0} = K'^\dagger \overleftrightarrow{\mathscr{D}}_\mu K' = (1,1)_{0,0;0} \,, \qquad (4.3b)$$
$$W_R'^{-} = K'\mathscr{D}_\mu K' = (1,1)_{-2,8;-4} \,,$$

where

$$H' = T'^\dagger U'(TT') = (\underline{1};\underline{2},\underline{1};1,1)_{1,-4;-2} \,,$$
$$K' = T^\dagger U(TT') = (\underline{1};\underline{1},\underline{2};1,1)_{-1,4;-2} \,, \qquad (4.4)$$

and their Hermitian conjugates are the composite *vector* constituents of the corresponding vector bosons. The L and R nature of these new weak bosons are assigned so they have the same charge and X_A quantum numbers as the W_L and W_R bosons.

We thus see that the dynamics of the binding mechanisms will be different for the two types of weak bosons: the standard generation transitions involve two-preon scalar constituents, while the mirror generation transitions involve four-preon vector constituents. Mixing will occur among the charged vector bosons and also among the neutral vector bosons, so the mass eigenstates will, in general, be linear combinations of the weak eigenstates.

The isoscalar boson corresponding to the above isovector objects

$$Y_L^0 = \frac{1}{\sqrt{2}}(H^\dagger \mathscr{D}_\mu H + H\mathscr{D}_\mu H^\dagger) \,,$$
$$Y_R^0 = \frac{1}{\sqrt{2}}(K^\dagger \mathscr{D}_\mu K + K\mathscr{D}_\mu K^\dagger) \,,$$
$$Y_L'^{0} = \frac{1}{\sqrt{2}}(H'^\dagger \mathscr{D}_\mu H' + H'\mathscr{D}_\mu H'^\dagger) \,, \qquad (4.5)$$
$$Y_R^0 = \frac{1}{\sqrt{2}}(K'^\dagger \mathscr{D}_\mu K' + K'\mathscr{D}_\mu K'^\dagger) \,,$$

violate Bose statistics (p-wave coupling of isospin-symmetric scalars) and are expected to be particularly massive.[40] Among the HC-singlet isovector bosons listed in Table II, only the

$$y_L^0 = VV^\dagger \rightarrow LL^\dagger \text{ and } QQ^\dagger \,,$$
$$y_L'^{0} = V'V'^\dagger \rightarrow L'L'^\dagger \text{ and } Q'Q'^\dagger \,, \qquad (4.6)$$

states couple directly to the light composite quarks and leptons. These isoscalar bosons are unprotected by the chiral symmetry and tend to pick up masses corresponding to the largest binding scalar involved.

B. Weak and electromagnetic interactions

The four-preon composite isovector bosons effect weak charged transitions between the standard (A) generation up and down quarks and neutrinos and charged leptons, as illustrated in Fig. 1. The eight-preon composite isovector bosons, on the other hand, effect the weak charged transitions between mirror (B) quarks and leptons for both the $Z(12)_A$ and $Z(20)_A$ cases with the assignments given in Tables V and VI. It is clear from the diagrams that the effective weak couplings

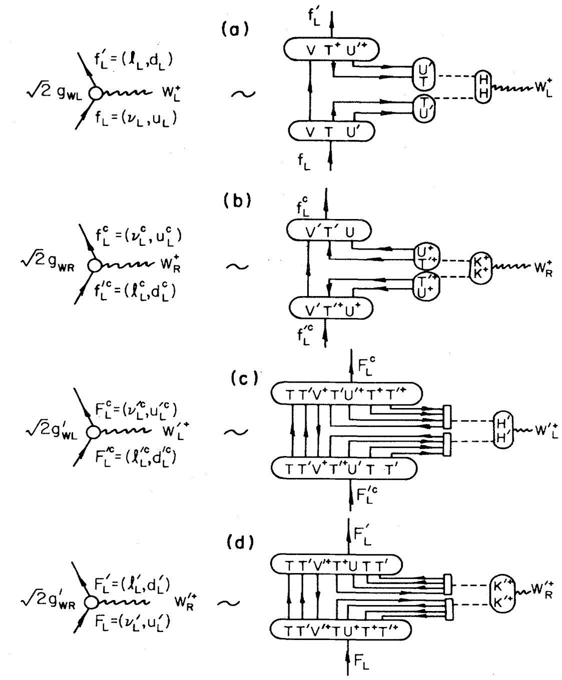

FIG. 1. Residual interaction couplings of the global $SU(2)_L$ and $SU(2)_R$ weak bosons to the charged light-fermion currents.

$$g_{W\nu l}=g_{Wud}=\cdots=g_W \tag{4.7}$$

are universal to the extent that only the hypercolor interactions are taken into account. Some renormalization effects may occur when the hyperflavor (gluon and photon) interactions are turned on, unless a Ward identity serves to guarantee the continued universality of the weak couplings.[41] This is a sticky point with all composite-weak-boson models, and we have not explored this issue in detail.

The neutral weak bosons couple to the leptons and quarks in a similar fashion, but they also mix with the photon which couples directly to the charged T or T' preon constituents of the W^0's. The photon also interacts directly with the V (charged Q or L) or V' preons, effectively mixing with the isoscalar bosons y_L^0 and $y_L'^0$. The photon—light-fermion interactions are pictured in Fig. 2. With g_W the universal weak coupling of (4.7) for the W boson, g_y the corresponding (universal) isoscalar coupling to the light fermions, λ_W and λ_y the γ-W^0 and γ-y^0 mixing parameters, respectively, we have the compositeness conditions[42] of Kögerler and Schildknecht, and Chen and Sakurai:

$$\begin{aligned} g_W\lambda_W &= g_W'\lambda_W' = e\ , \\ g_y\lambda_y &= g_y'\lambda_y = e\ , \end{aligned} \tag{4.8}$$

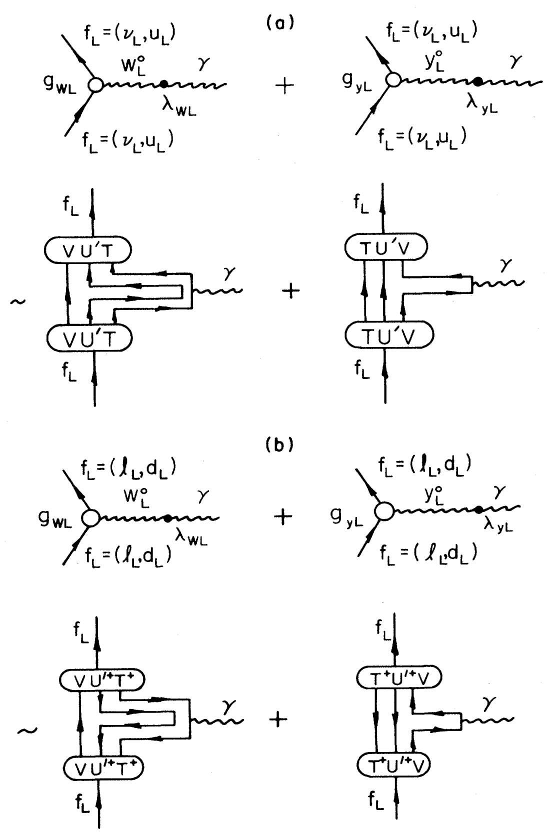

FIG. 2. Electromagnetic interactions of the light fermions through direct couplings of the photon to the weak isovector and isoscalar bosons.

where only the lowest-lying composite isovector and isoscalar bosons are considered. The standard form of the electroweak interactions is obtained provided the W-dominance approximation is a valid one, i.e., the lightest composite W is much lower in mass than the next heavier one (by a factor of 5 or more), and the isoscalar mass is much greater than the lightest W mass. In this approximation, one can identify

$$\lambda_W=\sin\theta_W\ . \tag{4.9}$$

C. Flavor-changing neutral currents

Since different generations of quarks and leptons have been distinguished by the X_A quantum number, i.e., the number of TT' pairs attached [modulo 3 for $Z(12)_A$ and 5 for $Z(20)_A$], flavor-changing neutral currents will exist and effect generation transitions among quarks and leptons if TT' pairs can be emitted or absorbed via $\phi_T \equiv TT'$ (pseudo) scalar-boson exchange. Such hypercolor-singlet bosons are tightly bound by all three confining forces and are expected to acquire masses of the order of the largest binding scale.

To effect transitions among the different generations explicitly labeled in Tables V and VI; one, two, or three ϕ_T bosons must be exchanged. Since the mass of ϕ_T is expected to be very large, multi-ϕ_T exchange is expected to be very suppressed. Examples of the basic two types of flavor-changing neutral current interactions are given in Fig. 3 corresponding to K^0-$\bar{K}^0$ mixing ($s+\bar{d}\to\bar{s}+d$) and

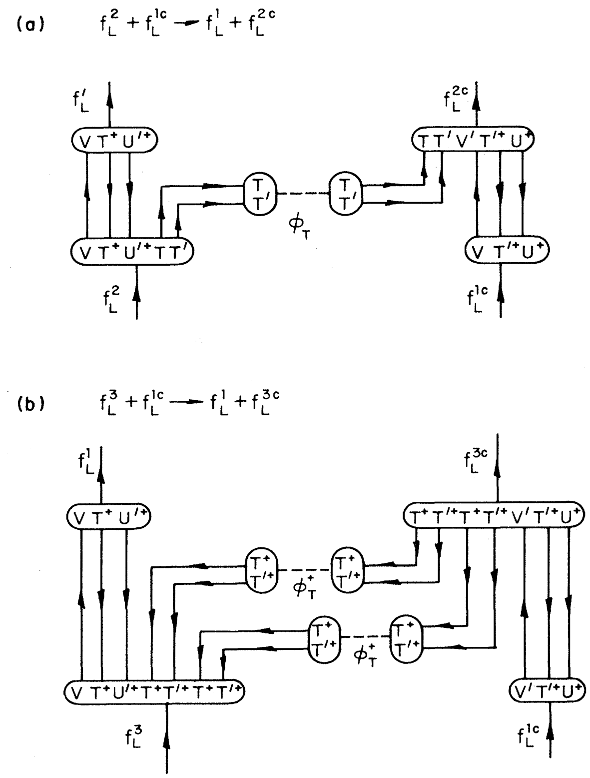

FIG. 3. Horizontal transitions involving one or two $\phi_T = TT'$ exchanges resulting in flavor-changing neutral currents.

$K^- \to \pi e \mu$ decay ($s \to de\mu$). Of course, until the mass spectrum is accurately calculated, we do not know which fermion generations in the model to associate with the generations appearing in nature.

V. ENERGY SCALES AND MASS GENERATION

A. Energy scales and $SU(2)_{TC}$ forces

We now address the issue of the ordering of the energy scales Λ_{TC}, Λ_{2L}, and Λ_{2R} at which the respective $SU(N)_{TC}$, $SU(2)_L$, and $SU(2)_R$ hypercolor forces become strong enough to confine the preons into composite hypercolor singlets. One useful clue concerning this scale ordering arises in connection with chiral symmetry breaking and anomaly matching. We have noted that anomalies associated with hyperflavor can be matched only if the original $U(1)_A$ group is broken. Condensates of type-TUU' technifermions can break this symmetry at a relatively high scale. This suggests that we choose the ordering

$$\Lambda_{TC} \ll \Lambda_{2L} \lesssim \Lambda_{2R} \ . \tag{5.1}$$

The $SU(2)_{L,R}$ forces can then be used to form technifermions and technibosons, as well as TC singlets. We allow for the possibility that the $SU(2)_L$ scale is possibly lower than the $SU(2)_R$ scale to aid in L/R symmetry breaking of the global $SU(2)_L \times SU(2)_R$ chiral hyperflavor symmetry.[17] The TC scale is taken sufficiently low that condensates of the type-TUU' technifermions can occur by the MAC hypothesis at the effective Λ_{2L}, Λ_{2R} scales for anomaly matching.

To ensure asymptotic freedom for the $SU(2)_{L,R}$ forces at the preon level, we require $N=2$ as deduced earlier from (2.9). With $SU(2)_{TC}$ as the third confining hypercolor, the three types of technifermions are just TUU', TUV', and TVV' with four- (and two-), three-, and two-dimensional representations of TC. From Table IV we then observe that given all the technifermion possibilities, the TC force is not asymptotically free at the composite level. But the possibility exists that below $\Lambda_{2L} \lesssim \Lambda_{2R}$, technicolor is born in the region beyond the ultraviolet fixed point where the β function is negative. Thus, although TC is not asymptotically free in the composite region, the TC force becomes stronger as the energy scale decreases, a possibility originally suggested by Holdom.[36] The MAC scenario for TC condensation sketched earlier in Sec. III A is a legitimate possibility.

In particular, we are led by the requirement of anomaly matching and the MAC scenario to adopt

$$\Lambda_{TC} \equiv \Lambda_{TC}(\underline{2}) \ll \Lambda_{TC}(\underline{3}) \ll \Lambda_{TC}(\underline{4}) \lesssim \Lambda_{2L} \lesssim \Lambda_{2R} \ . \tag{5.2}$$

This follows from the observation that the SU(2) quadratic Casimir operators take on the values[43] $C_2(2)=\frac{3}{4}$, $C_2(3)=2$, $C_2(4)=\frac{15}{4}$. By the assumption that the product $C_2(R)\alpha_{TC}(\mu)$ is similar for all TC condensate formation,[35] the corresponding TC couplings at the different confining scales are related by

$$5\alpha_{TC}(\mu_4) \sim \tfrac{8}{3}\alpha_{TC}(\mu_3) \sim \alpha_{TC}(\mu_2) \ . \tag{5.3}$$

The corresponding mass scales where the different confining TC forces become strong cannot be determined in

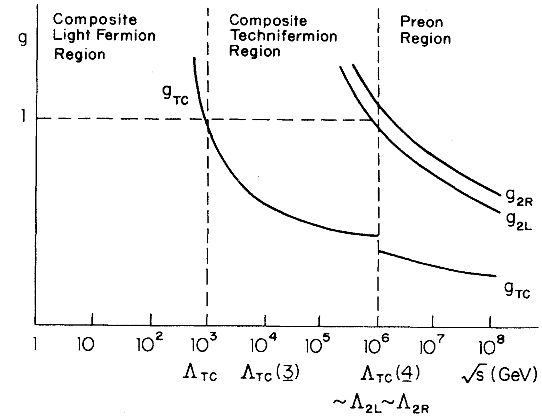

FIG. 4. Running hypercolor coupling constants g_{TC}, g_{2L}, and g_{2R} according to the MAC scenario sketched in Sec. V A.

practice, since the β function is unknown for couplings larger than the ultraviolet fixed point. For numerical estimates in the next sections, we shall adopt

$$\begin{aligned}\Lambda_{TC}(\underline{2}) &\sim 1 \text{ TeV}\\ &\ll \Lambda_{TC}(\underline{3}) \sim 10 \text{ TeV}\\ &\ll \Lambda_{TC}(\underline{4}) \sim 1000 \text{ TeV}\\ &\sim \Lambda_{2L} \sim \Lambda_{2R} \ ,\end{aligned} \tag{5.4}$$

in keeping with (5.2). We illustrate the running hypercolor couplings in Fig. 4.

B. Fermion mass generation

Many mass-generation mechanisms have been discussed in the literature. Among them are instanton contributing effects,[39] techniboson exchange with technifermion condensation,[44] and higher-order corrections[45] via pseudo-Goldstone-boson exchange or emission and reabsorption of photons. All four mechanisms are possible in this model, but the most attractive from our point of view appears to arise through TC interactions, and we shall explore that mechanism in some detail here.

We begin with the general form of the light-fermion mass matrix. With $Z(4)_A$ symmetry and just one generation of quarks and leptons, this has the simple form[46]

$$m = \begin{bmatrix} M & D \\ D^T & M' \end{bmatrix} \tag{5.5}$$

in the basis $B_4 = \{f_L, f_L^c\}$ of fermion and conjugate fermion left-handed Weyl spinors. The off-diagonal D terms are standard Dirac terms, while the diagonal M and M' contributions are of the Majorana-type and nonvanishing only for neutrinos.

With $Z(12)_A$ symmetry and two standard generations and one mirror generation of quarks and leptons present, we adopt the basis

$$B_{12} = \{f_L{}^1, f_L{}^2, f_L{}^{1c}, f_L{}^{2c}, F_L, F_L^c\}$$

and write down the general mass matrix for the light fermions according to

$$m=\begin{bmatrix} M & D & m & d \\ D^T & M' & d' & m' \\ m^T & d'^T & M'' & D' \\ d^T & m'^T & D'^T & M''' \end{bmatrix} . \tag{5.6}$$

Here D, M and M' are 2×2 matrices connecting standard-generation objects; d, d', m, and m' are 2×1 matrices connecting standard and mirror generations; and D', M'', and M''' are 1×1 elements connecting only the mirror-generation spinors. With $Z(20)_A$ symmetry, the basis is enlarged to

$$B_{20}=\{f_L{}^1, f_L{}^2, f_L{}^3, f_L{}^{1c}, f_L{}^{2c}, f_L{}^{3c}, F_L{}^1, F_L{}^2, F_L{}^{1c}, F_L{}^{2c}\}$$

and the mass matrix is 10×10 with each submatrix of (5.6) enlarged accordingly.

We now consider the Dirac mass terms

$$D_{ij} f_L^{icT} C f_L^j + D'_{\alpha\beta} F_L^{\alpha cT} C F_L^\beta + d'_{i\beta} f_L^{icT} C F_L^\beta + d_{\alpha j} F_L^{\alpha cT} C f_L^j + \text{H.c.} = D_{ij} \bar f^i{}_R f^j_L + D'_{\alpha\beta} \bar F^\alpha_R F^\beta_L + d'_{i\beta} \bar f^i{}_R F^\beta_L + d_{\alpha j} \bar F^\alpha_R f^j_L + \text{H.c.} \tag{5.7}$$

which can appear in the second and third stages of hyperflavor symmetry breaking. When the type-TUV' technifermions condense via $\underline{3}_{\text{TC}}$ forces at the scale $\Lambda_{\text{TC}}(\underline{3})$, d terms can appear when G_{HF} is broken into the chiral SU(3) channel of (3.6a), D and d terms appear in the $\text{SU}(4)_c$ channel of (3.6b) if $Z(m)_A$ is broken completely, while only D terms appear in the $\text{SU}(4)_c$ channel of (3.6c) if $Z(m)_A$ remains unbroken when $m=12$ or 20.

The mechanism of techniboson exhange with technifermion condensation can generate mass terms much like the extended technicolor[21] concept proposed several years ago. Examples of this mechanism when the chiral SU(4) symmetry is broken to chiral SU(3) are shown in Fig. 5, where the technifermion condensates of type (3.5a) can occur in the s or u channels. The Dirac d terms are generated between fermion and mirror fermion of (5.7). One or two generations of technifermions exist and can contribute if $m=12$ or 20, respectively. Since the technibosons are born at the $\Lambda_{2L,R}$ scales and pick up masses $\sim 10^3$ TeV, whereas we have assumed the condensation scale is $\Lambda_{\text{TC}}(\underline{3})\sim 10$ TeV, some of the light fermions can acquire masses of order

$$m \sim g_{\text{TC}}{}^2(\mu_3)\frac{[\Lambda_{\text{TC}}(\underline{3})]^{3+d}}{\Lambda_2{}^{2+d}} \lesssim 1 \text{ GeV} \tag{5.8}$$

Here d is the anomalous dimension associated with the number of technifermions existing at the $\Lambda_{\text{TC}}(\underline{3})$ scale.

When the chiral SU(4) symmetry is broken to $\text{SU}(4)_c$, technifermion condensates of types (3.5b) or (3.5c) can form in the u channel as illustrated in Fig. 6. This mech-

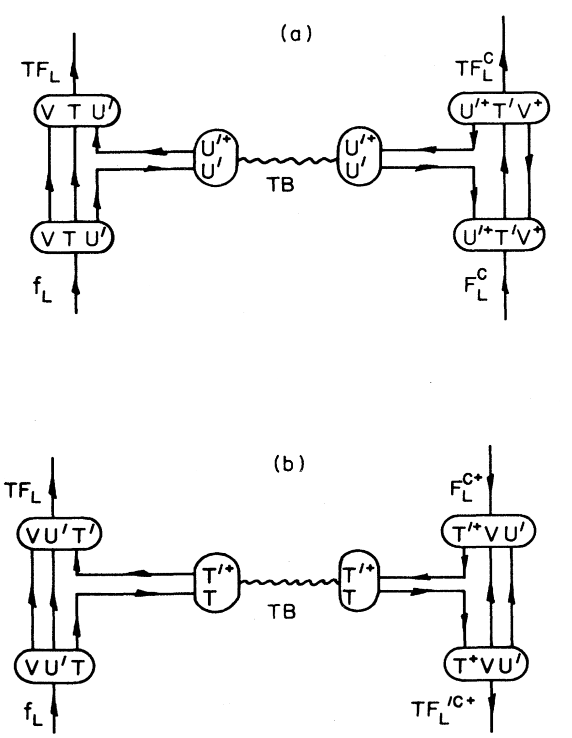

FIG. 5. Techniboson-exchange graphs with Dirac d-type mass terms resulting from type-TUV'-technifermion condensation in the (a) s channel and (b) u channel breaking the chiral SU(4) symmetry down to chiral SU(3).

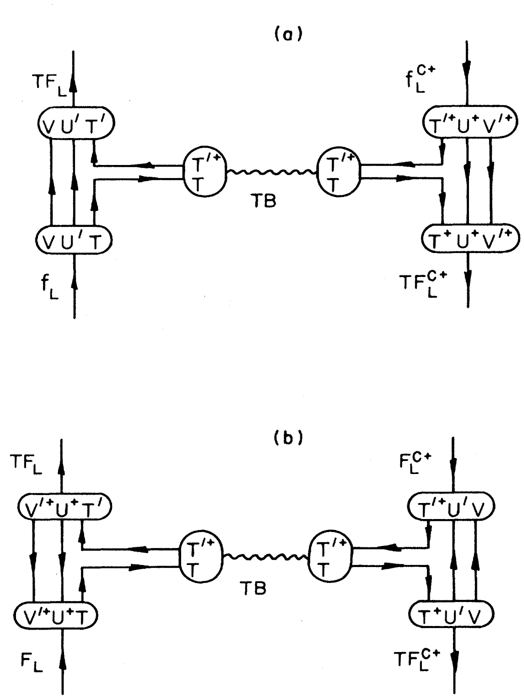

FIG. 6. Techniboson exchange graphs with Dirac D-type mass terms for (a) standard and (b) mirror-generation fermions resulting from type-TUV'-technifermion condensation in the u channel breaking the chiral SU(4) symmetry down to $\text{SU}(4)_c$.

anism leads to Dirac mass terms of the D type generated between standard and conjugate fermions or mirror and conjugate mirror fermions. If the mirror technifermion condensate of Fig. 6(b)

$$\langle (V^{\dagger}TU'^{\dagger})(V'^{\dagger}T'U^{\dagger})\rangle = (\bar{4},4)_{0,0;16}\ , \tag{5.9}$$

is somewhat larger than the technifermion condensate of Fig. 6(a)

$$\langle (VT'U')(V'TU)\rangle = (4,\bar{4})_{0,0;-8}\ , \tag{5.10}$$

one has a ready mechanism to give the mirror fermions larger masses than the fermions belonging to the standard A-type generations.

Finally we note that those fermions which escape acquiring a mass by type-TUV' technifermion condensates can pick up a mass through condensation of type-TVV' technifermions at the $\Lambda_{TC}(\underline{2})$ scale. The acquired mass is of the order of

$$m \sim g_{TC}{}^2(\mu_2)\frac{[\Lambda_{TC}(\underline{2})]^{3+d}}{\Lambda_2{}^{2+d}} \lesssim 1\ \text{MeV}\ . \tag{5.11}$$

An example of this mechanism is given in Fig. 7, where a $U'V'^{\dagger}$ vector techniboson is exchanged and a technifermion condensate of (3.8) is involved.

Majorana mass terms in the mass matrix of (5.6) arise only for neutrinos. They have the form

$$\begin{aligned} M_{ij}\nu_L^{iT}C\nu_L^{j} &+ M'_{ij}\nu_L^{icT}C\nu_L^{jcT} \\ &+ M''_{\alpha\beta}\nu_L'^{\alpha T}C\nu_L'^{\beta} + M'''_{\alpha\beta}\nu_L'^{\alpha cT}C\nu_L'^{\beta c} \\ &+ (m_{i\beta}\nu_L^{iT}C\nu_L'^{\beta} + m'_{i\beta}\nu_L^{icT}C\nu_L'^{\beta c} + \text{H.c.})\ , \end{aligned} \tag{5.12}$$

where the notation is used as given in Table I. M-type mass terms can appear via $\underline{3}_{TC}$ forces at the scale $\Lambda_{TC}(\underline{3})$ via condensates of the type

$$\langle (L'T'U)(L'T'U)\rangle = (1,1)_{-2,12,6;-6} \tag{5.13}$$

as illustrated in Fig. 8(a). This condensate carries $\Delta Q = \Delta B = 0$, and $\Delta L = 2$ and hence preserves charge and baryon number but breaks lepton number (as well as I_{3R}). The resulting Goldstone boson can be identified as the Majoron of Chikashige, Mohapatra, and Peccei.[47] The possibility exists that only right-handed (or ν_L^c) neutrinos acquire a mass, but the mass is only expected to be of the order of 1 GeV as in (5.8). m-type mass terms of the order of 1 MeV as in (5.11) can appear via $\underline{2}_{TC}$ forces at Λ_{TC} via condensates of the type

$$\langle (L'T'L^{\dagger})(T'UU'^{\dagger})\rangle = (1,1)_{-2,12,6;0} \tag{5.14}$$

as in Fig. 8(b).

We have thus seen that the hypercolor-singlet fermions can acquire masses of ~ 1 GeV or so through TC condensation and techniboson exchange. These masses are much below the scales $\Lambda_{TC}(\underline{4})$, $\Lambda_{TC}(\underline{3})$, or $\Lambda_{TC}(\underline{2})$ at which the condensation takes place. On the other hand, the technifermions which participate in the condensation get massive at the same scale. All other technifermions can pair off and get massive once the discrete symmetry $Z(m)_A$ is broken down to $Z(4)_A$. Hence we find $m_{TF} \gtrsim \Lambda_{TC} \simeq 1$ TeV for the illustrated choice of scales.

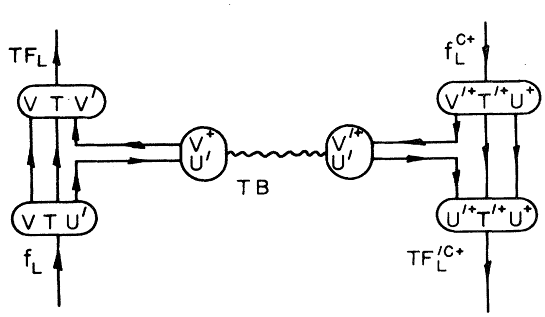

FIG. 7. Techniboson exchange graph with Dirac D-type mass term resulting from technifermion condensate (3.8) in the u channel breaking $SU(4)_c$ down to $SU(3)_c$.

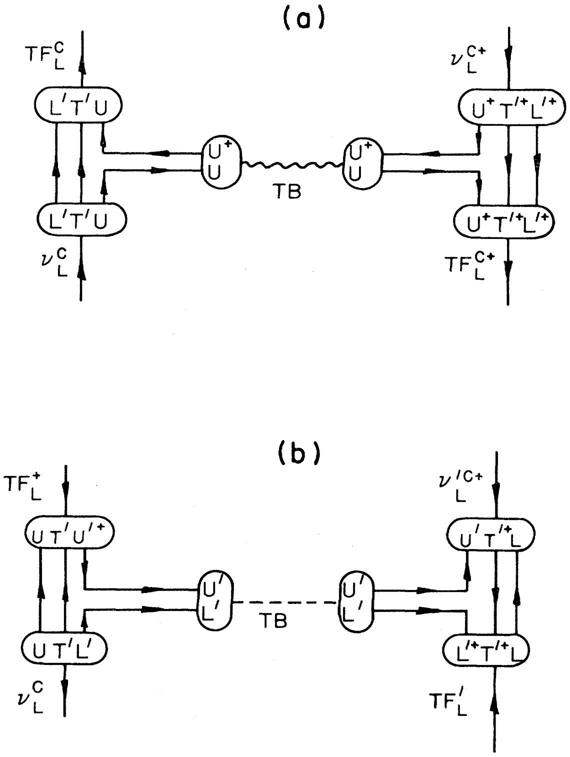

FIG. 8. Techniboson exchange graphs leading to (a) M-type and (b) m-type Majorana mass terms resulting from technifermion condensates (5.13) and (5.14), respectively, in the u channel.

C. Boson mass generation

We have already noted that all technibosons acquire masses at the preon binding scales $\Lambda_{2L,R}$. A puzzling issue common to all preon models in which the weak bosons are composite is why the weak bosons do not also acquire masses of the order of the confinement scale $\Lambda_{2L,R}$ here. No completely satisfactory answer has been given in the literature, and we do not pretend to give one here. However, we do have at our disposal a mechanism somewhat peculiar to this model.

Although the weak bosons W_L, W_R, W_L', and W_R' are "born" at the $\Lambda_{2L,R}$ scales, and should acquire masses comparable to these scales, mixing effects can occur among these weak bosons via preon exchange diagrams. The outcome of the mixing is that some masses will be driven down and others up. For example, in the case of the $Z(4)_A$ symmetry where only W_L and W_R exist, a diagram such as Fig 9(a) can occur, where the technifermion condensates

$$\langle (T^\dagger UU'^\dagger)(T'^\dagger U^\dagger U')\rangle = (1,1)_{0,0;-4} \tag{5.15a}$$

and

$$\langle (TUU')(T'UU')\rangle = (1,1)_{0,0;-12} \tag{5.15b}$$

both appear at the Λ_2 scale. With the W mass matrix elements all comparable

$$M^2 \sim g_2{}^2 \begin{bmatrix} \Lambda_2{}^2 & \Lambda_2{}^2 \\ \Lambda_2{}^2 & \Lambda_2{}^2 \end{bmatrix} \tag{5.16}$$

in the $\{W_L, W_R\}$ basis, the eigenvalues of the rank-1 tensor are just $m_W \simeq 0$ and $2\Lambda \simeq 2\times 10^3$ TeV.

In the case of the discrete symmetry groups $Z(12)_A$ or $Z(20)_A$ arising at the Λ_2 scale through type-TUU' technifermion condensation, both condensates in (5.15) cannot appear at this scale and W_L-W_R mixing will not occur at that stage. However, W_L-W_L' mixing can occur via the diagram shown in Fig. 9(b). There the type-TUU' technifermions of two different generations can form condensates

$$\langle (TUU')(T'U^\dagger U'^\dagger T^\dagger T'^\dagger)\rangle = (1,1)_{0,0;0} , \tag{5.17a}$$

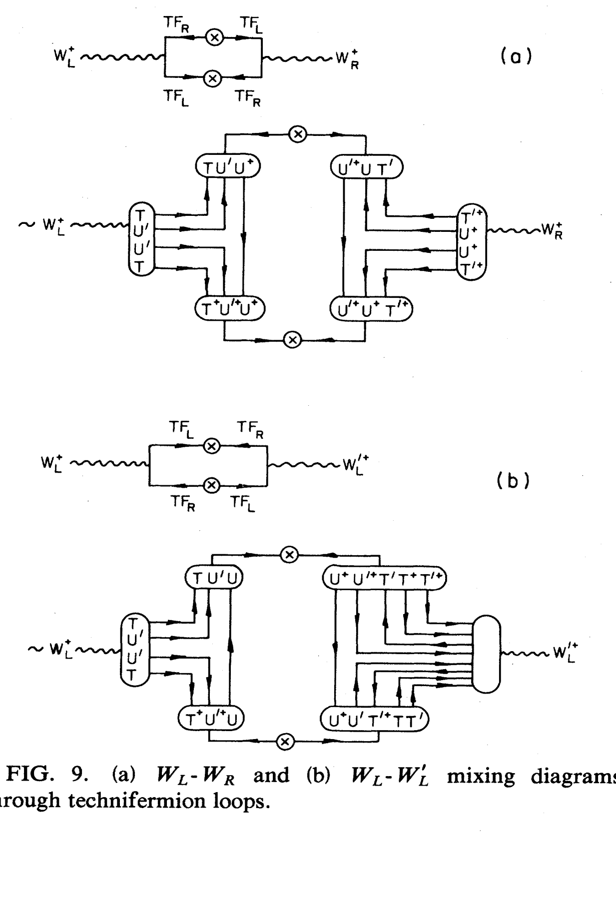

FIG. 9. (a) W_L-W_R and (b) W_L-W_L' mixing diagrams through technifermion loops.

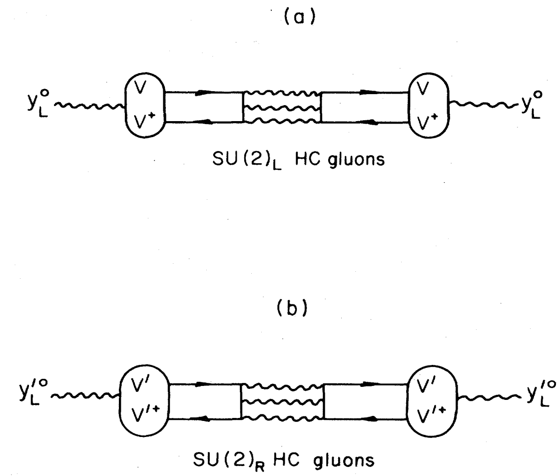

FIG. 10. Mass contributions to the weak isoscalar bosons (a) y_L^0 and (b) $y_L'^0$ through hypercolor-gluon exchange in the annihilation channels.

$$\langle (T^\dagger UU'^\dagger)(T'^\dagger U^\dagger U'TT')\rangle = (1,1)_{0,0;0} , \tag{5.17b}$$

via $\underline{4}_{TC}$ forces without further breaking the $Z(12)_A$ or $Z(20)_A$ symmetries. Again if W_L-W_L' mixing is nearly complete, one of the "left-handed" weak bosons will be driven down to a low mass as in (5.16). No such simple diagram is found for W_L-W_R' mixing.

As a result of the W_L-W_L' mixing where one of the weak-boson-mass eigenstates is driven low in mass, both the standard f_L quarks and leptons and the "mirror" F_L^c fermions will couple to this low-mass boson with impunity if the mirror states are heavy. [An analogous coupling of f_L and f_L^c states to the same light boson in (5.16) with $Z(4)_A$ symmetry would be disastrous.] We do not understand, however, why the same type of mechanism would not drive one of the "right-handed" weak bosons also low in mass.

With regard to the isoscalar bosons y_L^0 and $y_L'^0$ in (4.5) already discussed earlier, we have noted that they pick up a mass of order of the binding scales $\Lambda_{2L,R}$. Moreover, the exchange of HC gluons in the annihilation channel shown in Fig. 10 ensures that they remain massive.[40] The corresponding diagram connecting y_L^0 and $y_L'^0$ cannot appear since the exchanged HC gluons must be of the $SU(2)_L$-type or the $SU(2)_R$-type, whereas y_L^0 communicates with only the former, $y_L'^0$ only with the latter. The isoscalar bosons should thus remain much heavier than the lowest-mass-isovector weak boson, as required by the W-dominance approximation to ensure that conditions (4.8) and (4.9) apply.

VI. SUMMARY

By starting with a simple set of preons bound together by three gauged, confining hypercolor forces exhibiting an $SU(2)_{TC}\times SU(2)_L \times SU(2)_R$ symmetry, we have shown how one can obtain a rich composite model of quarks and leptons with a number of interesting features. In the simplest version the light composite fermions consist of just one standard generation of quarks and leptons. More generally, n standard and $n-1$ mirror generations of quarks and leptons emerge, where the limitation $n=1$, 2, or 3 is realized through the breaking of a conserved $U(1)_A$ symmetry to a discrete subgroup $Z(m)_A$ with $m=4$, 12, or 20, respectively. The quarks and leptons have the standard $I_{3L,R}$, B, and L quantum numbers which are all conserved, so proton decay is prohibited via simple preon rearrangement graphs. The weak bosons are composite fields which transmit the weak force through residual hypercolor interactions with a global hidden symmetry manifested only on the composite level.

In the scheme presented, the two strong $SU(2)_L \times SU(2)_R$ hypercolor forces can form three-preon light hypercolor-singlet fermions as well as technifermions transforming according to two-, three-, or four-dimensional representations of the $SU(2)_{TC}$ technicolor group. According to the most attractive channel scenario of Raby, Dimopoulos, and Susskind, the technifermions can form condensates in decreasing order of the technicolor representation as the energy decreases, breaking part of the chiral symmetry at each stage. In fact, we find that the technicolor interactions play three different roles: by breaking the originally conserved $U(1)_A$ symmetry, the $\underline{4}_{TC}$ technifermion condensates permit the 't Hooft conditions to be satisfied at the $\Lambda_{2L,R}$ confinement scales. Which particular condensate forms determines the surviving discrete axial symmetry and, in turn, the number of standard (and mirror) generations at this first stage of chiral-symmetry breaking. Technifermion condensates at the second and third stages can then generate light-fermion masses through techniboson-exchange graphs.

For this scenario to make sense we must order the relevant scales according to (5.2) and demand that the discrete $Z(m)_A$ symmetry be $Z(12)_A$ or $Z(20)_A$, i.e., that two or three standard generations of quarks and leptons exist, in order that one or two generations of (massless) technifermions survive for the mass-generation mechanism at the second and third stages. With these assumptions, we note that dynamical chiral-symmetry breaking occurs *in stages* and that 't Hooft anomaly matching is achieved through light composite fermions without the disastrous possibility that the full chiral symmetry is broken all at once leading simply to a spectrum of massless Goldstone bosons. Technicolor must be regarded as one of the hypercolor forces here and not as a hyperflavor interaction as usually assumed.

Despite the many attractive features of the model, we note the decided shortcoming that we have not succeeded in unifying the strong hypercolor forces in the preon interaction region. Also no detailed calculation of the fermion mass spectrum has been attempted since other mass-generation mechanisms beside technicolor interactions can play major roles in the model as discussed at the beginning of Sec. V B. Moreover, the mechanism suggested in Sec. V to keep some of the weak composite bosons light is admittedly very speculative Nevertheless, we feel that some of the features expanded upon may persist in even more attractive models of the real world.

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