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Intermediate mass scales and electron–muon–tau-lepton universality

Fayyazuddin
Physics Department, King Saud University, Riyadh, Saudi Arabia

Riazuddin
Physics Department, University of Petroleum and Minerals, Dhahran, Saudi Arabia
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Intermediate mass scales $\tilde{m}$ and m_R associated, respectively, with the violation of electron–muon–tau-lepton universality and the parity restoration within the framework of Pati-Salam groups $SU_L(2)\times SU_R(2)\times SU_C(4)$ and $SU_L(2)\times SU_R(2)\times[SU_C(4)]^4$ are discussed, assuming that $\tilde{m}\le m_R\le \tilde{m}_X$, where $\tilde{m}_X$ is the usual partial-unification mass scale for Pati-Salam groups. It is shown that, with $\tilde{m}\simeq 8\times 10^5$ GeV (a lower limit from the K_L-K_S mass difference), $m_R\approx\tilde{m}_X=8\times 10^9$ GeV for the first case and $m_R\simeq\tilde{m}_X\approx 9\times 10^7$ GeV or $m_R\simeq\tilde{m}=8\times 10^5$ GeV, $\tilde{m}_X=10^{11}$ GeV for the second case.

I. INTRODUCTION

If there are no gauge forces except those associated with the group $SU_L(2)\times U(1)\times SU_C(3)$, then there are only two mass scales: the mass scale of the electroweak unification $m_L\approx 100$ GeV and the grand unification mass scale $m_X\approx 10^{15}$ GeV. There are two important questions: Is there a "desert" between these two mass scales or are there intermediate mass scales? If there are such mass scales, how they can manifest themselves?

The parity violation characteristic of low-energy weak processes may not hold up to the grand unification mass scale of 10^{15} GeV. The parity restoration may occur at an intermediate mass scale $m_R \ll m_X$. If this is so, then the weak group is enlarged[1] to $SU_L(2)\times SU_R(2)\times U_{B-L}(1)$; the gauge forces associated with this group will give both $V-A$ and $V+A$ currents. The vector bosons associated with $V+A$ currents will be very massive and this interaction will be effectively suppressed at low energies. Now, $-\Delta I_{3R}=\frac{1}{2}\Delta(B-L)$ implies that the breakdown of parity and the breaking of local $(B-L)$ symmetry are related and occur at the mass scale of m_R. In such a case, this mass scale may manifest itself in the $\Delta B=2$ and $\Delta L=2$ transitions, in particular n-$\bar{n}$ oscillations.[2] For free n-$\bar{n}$ oscillations, $\tilde{t}_{n\bar{n}}\ge 10^7$ sec implies intermediate mass scale $m_R\ge 10^5$ GeV.

If the Abelian group U(1) is embedded in $SU_C(4)$ as in the Pati-Salam group,[3] then one can define a partial-unification mass $\tilde{m}_X$ associated with the partial-unification group $G_{PS}=SU_L(2)\times SU_R(2)\times SU_C(4)$. With fractionally charged quarks, the proton is stable in this model. The model gives[4,5] $\tilde{m}_X\ge 10^{12}$ GeV for $\sin^2\theta_W\le 0.25$. As such, this model has no observable consequences. However, if the residual color symmetry is $[SU_C(3)]^2=SU_{CL}(3)\times SU_{CR}(3)$, then it is natural to extend[6] G_{PS} to

$$G_{PS}=SU_L(2)\times SU_R(2)\times[SU_C(4)]^2 .$$

Recently we examined[7] this model. In this model, we get $300\le m_R\le 10^7$ GeV for $5\times 10^{12}\ge\tilde{m}_X\ge 3\times 10^{10}$ GeV. Such a model has observable consequences.

So far we have assumed that e-μ-τ universality holds up to $\tilde{m}_X\approx 10^{12}$ GeV. It is conceivable that this universality may not hold up to such a high energy. In this case, we have another intermediate mass scale associated with the violation of this universality. The purpose of this paper is to consider such a possibility. For this purpose, we consider the partial-unification groups $G_{PS}=G_{wk}\times SU_C(4)$ or $G_{PS}=G_{wk}\times[SU_C(4)]^2$, where

$$G_{wk}=\prod_i[\underset{g_{Li}}{SU_L(2)}\times\underset{g_{Ri}}{SU_R(2)}]_i$$

and i is the generation index[8] ($i=e,\mu,\tau,\ldots$ or $1,2,3,\ldots$). We shall assume the discrete symmetries

$$e\leftrightarrow\mu\leftrightarrow\tau ,$$

$$g_{Le}=g_{L\mu}=g_{L\tau} ,$$

$$g_{Re}=g_{R\mu}=g_{R\tau} ,$$

and $L\leftarrow R$ symmetry

$$g_{Li}=g_{Ri}=g_2 \quad \text{for each } i .$$

The partial-unification symmetry is spontaneously broken by the following chains:

$$\text{Model A:}\quad G_{PS}\underset{\tilde{m}_X}{\rightarrow}\prod_i[SU_L(2)\times SU_R(2)]_i\times\underset{g_1}{U_{B-L}(1)}\times SU_C(3)$$

$$\underset{m_R}{\rightarrow}\prod_i[SU_L(2)\times\underset{g_{1i}}{U(1)}]_i\times SU_C(3)$$

$$\underset{\tilde{m}}{\rightarrow}[SU_L(2)\times\underset{g'}{U'(1)}]\times SU_c(3)\underset{m_L}{\rightarrow}U_{EM}(1)\times SU_C(3) .$$

Model B: $$G_{\rm PS} \underset{\tilde{m}_X}{\longrightarrow} \prod_i [{\rm SU}_L(2)\times {\rm SU}_R(2)]_i \times {\rm U}_{B-L}(1)\times [{\rm SU}_C(3)]^2$$

$$\underset{m_R}{\longrightarrow} \prod_i [{\rm SU}_L(2)\times {\rm U}(1)]_i \times [{\rm SU}_C(3)]^2$$

$$\underset{\tilde{m}}{\longrightarrow} [{\rm SU}_L(2)\times {\rm U}'(1)]\times [{\rm SU}_C(3)]^2$$

$$\underset{m_L}{\longrightarrow} {\rm U}_{\rm EM}(1)\times [{\rm SU}_C(3)]^2 \ .$$

The mass scale for the violation of e-μ-τ universality is determined by the flavor-changing currents associated with the Y vector bosons (see Sec. III). The K_L-K_S mass difference gives the following bound on m_Y (m_c being the mass of the charm quark):

$$G_F \frac{m_{W_L}{}^2}{m_Y{}^2} < \frac{G_F(G_F m_c{}^2)}{16\pi^2} ,$$

$$\frac{m_Y}{m_{W_L}} > 4\times 10^3 .$$

It is therefore reasonable to take $\tilde{m}\approx 8\times 10^5$ GeV. With this value of $\tilde{m}$, we find the following results (numbers of generations $f=3$):

For model A: $(\tilde{m}_X = m_R)\approx 7.8\times 10^9$ GeV .

For the model B: $(\tilde{m}_X = m_R)\approx 8.7\times 10^7$ GeV .

Another interesting possibility is to take $m_R\approx\tilde{m}=8\times 10^5$ GeV. This is possible for model B only since for model A, this choice would make $\tilde{m}_X$ much larger than the Planck mass. Thus for model B, there is the possibility that

$$m_R \simeq \tilde{m}\approx 8\times 10^5 \text{ GeV}, \quad \tilde{m}_X\approx 10^{11} \text{ GeV} ,$$

independent of the number of flavors.

II. INTERMEDIATE MASS SCALES

Using the renormalization-group equations, as shown in the Appendix, we have for model A

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=-2C_1{}^2\beta_3\ln\frac{\tilde{m}_X}{m_L}+2\beta_1\ln\frac{\tilde{m}_X}{m_R}$$
$$-2\sum_i(\beta_{2i}-\beta_{1i})\ln\frac{m_R}{\tilde{m}}-2(\beta_2-\beta')\ln\frac{\tilde{m}}{m_L} , \qquad (1)$$

where

$$C_1{}^2=\tfrac{2}{3} ,$$
$$\beta_2=\frac{1}{4\pi}(-\tfrac{22}{3}+\tfrac{4}{3}f) ,$$
$$\sum_i\beta_{2i}=\frac{1}{4\pi}(-\tfrac{22}{3}f+\tfrac{4}{3}f) ,$$
$$\sum_i\beta_{1i}=\beta'=\frac{1}{4\pi}(\tfrac{4}{3}f\tfrac{5}{3}) , \qquad (2)$$
$$\beta_1=\frac{1}{4\pi}(\tfrac{4}{3}f\tfrac{2}{3}) ,$$
$$\beta_3=\frac{1}{4\pi}(-\tfrac{33}{3}+\tfrac{4}{3}f) .$$

With respect to the third equation above, note that for each generation (i.e., for each i)

$$\beta_{2i}=\frac{1}{4\pi}(-\tfrac{22}{3}+\tfrac{4}{3})$$

and we take $\sin^2\theta_W=0.23$, $\alpha^{-1}(m_L)=128$, $\alpha_s(m_L)=0.14$, $m_L=80$ GeV, θ_W being the weak mixing angle. α is the fine-structure constant, α_s is a similar constant for QCD, and f is the number of generations.

From Eqs. (1) and (2), we obtain

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=\frac{11}{3\pi}\left[\ln\frac{\tilde{m}_X m_R{}^f}{m_L{}^2\tilde{m}^{f-1}}\right] . \qquad (3)$$

This is the basic formula for intermediate mass scales for model A. We shall use it for the case $\tilde{m}_X=m_R$. In this case, we have one intermediate scale $\tilde{m}$ related to violation of e-μ-τ universality and partial unification occurs at the same mass scale at which parity restoration is set in. This is an interesting case. For this case

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=\frac{11}{3\pi}\left[\ln\frac{\tilde{m}_X{}^{f+1}}{\tilde{m}^{f-1}m_L{}^2}\right] . \qquad (4)$$

For model B (cf. the Appendix), the mass scales are given by

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=2(\beta_1-2C_1{}^2\beta_3')\ln\frac{\widetilde{m}_X}{m_L}+\frac{11}{3\pi}\left[\ln\frac{m_R^f}{\widetilde{m}^{f-1}m_L}\right], \quad (5)$$

where

$$\beta_3'=\frac{1}{4\pi}(-\tfrac{33}{3}+\tfrac{2}{3}f)\,. \quad (6)$$

From Eqs. (5) and (6), we have

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=\frac{11}{3\pi}\left[\ln\frac{\widetilde{m}_X{}^2 m_R{}^f}{\widetilde{m}^{f-1}m_L{}^3}\right]. \quad (7)$$

Again for the case $\widetilde{m}_X=m_R$, we have

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=\frac{11}{3\pi}\left[\ln\frac{\widetilde{m}_X{}^{f+2}}{\widetilde{m}^{f-1}m_L{}^3}\right]. \quad (8)$$

The results obtained from Eqs. (4) and (8) for $\widetilde{m}_X/m_L$ for models A and B are given below with $\widetilde{m}=8\times10^5$ GeV. For model A,

$$f=3:\ (\widetilde{m}_X=m_R)\approx7.8\times10^9\ \text{GeV}\,,$$
$$f=4:\ \widetilde{m}_X\approx1.23\times10^9\ \text{GeV}\,, \quad (9)$$
$$f=5:\ \widetilde{m}_X\approx3.64\times10^8\ \text{GeV}\,.$$

For model B, we get

$$f=3:\ (\widetilde{m}_X=m_R)\approx8.7\times10^7\ \text{GeV}\,,$$
$$f=4:\ \widetilde{m}_X\approx4\times10^7\ \text{GeV}\,, \quad (10)$$
$$f=5:\ \widetilde{m}_X\approx2.3\times10^7\ \text{GeV}\,.$$

If, however, we take $m_R=\widetilde{m}=8\times10^5$ GeV, Eq. (7) gives $\widetilde{m}_X\approx10^{11}$ GeV.

III. INTERACTION LAGRANGIAN AND SYMMETRY BREAKING

The fermions in this model are assigned to the following representations of the group $G_{\text{PS}}=G_{\text{wk}}\times\text{SU}_C(4)$:

$$F_L^{(1)}=\begin{bmatrix}u_1 & u_2 & u_3 & \nu_e\\ d_1 & d_2 & d_3 & e^-\end{bmatrix}_L(2,1,1,1,1,1,\bar{4})\,,$$

$$F_L^{(2)}=\begin{bmatrix}c_1 & c_2 & c_3 & \nu_\mu\\ s_1 & s_2 & s_3 & \mu^-\end{bmatrix}_L(1,1,2,1,1,1,\bar{4})\,,$$

$$F_L^{(3)}\begin{bmatrix}t_1 & t_2 & t_3 & \nu_\tau\\ b_1 & b_2 & b_3 & \tau^-\end{bmatrix}_L(1,1,1,1,2,1,\bar{4})\,.$$

Similar expressions hold for right-handed multiplets except that $\nu\to N$. We have exhibited the fermions for three families of generations. In this section we will confine our discussion to families of three generations only.

The interaction Lagrangian for this group (omitting the QCD terms which remain unchanged), is given by

$$\mathscr{L}_{\text{int}}=\frac{g_2}{\sqrt{2}}\sum_i(J_{L\mu}^{+(i)}W_{L\mu}^{(i)}+J_{R\mu}^{+(i)}W_{R\mu}^{(i)}+\text{H.c.})+\frac{g_2}{2}\sum_i(J_{3\mu}^{(i)}W_{3L\mu}^{(i)}+J_{3R\mu}^{(i)}W_{3R\mu}^{(i)})$$
$$-\frac{g}{2\sqrt{6}}i\sum_{n,i}(\bar{u}_n^{(i)}\gamma_\mu u_n^{(i)}+\bar{d}_n^{(i)}\gamma_\mu d_n^{(i)})B_\mu+\frac{3g}{2\sqrt{6}}\sum_i(\bar{\nu}_L^{(i)}\gamma_\mu\nu_L^{(i)}+\bar{N}_R^{(i)}\gamma_\mu N_R^{(i)}+\bar{e}^{(i)}\gamma_\mu e^{(i)})B_\mu$$
$$-\frac{g}{\sqrt{2}}\sum_{n,i}(u_{nL}^{(i)}\gamma_\mu\nu_L^{(i)}X_{n\mu}+\bar{u}_{nR}^{(i)}\gamma_\mu N_R^{(i)}X_{n\mu}+\bar{d}_n^{(i)}\gamma_\mu e^{(i)}X_{n\mu}+\text{H.c.})\,. \quad (11)$$

Here i is the generation index and n is the color index.

Dropping the subscripts for the moment, we define the physical weak vector bosons as

$$\vec{\text{W}}=\frac{1}{\sqrt{3}}(\vec{\text{W}}^{(1)}+\vec{\text{W}}^{(2)}+\vec{\text{W}}^{(3)}),\ \ \vec{\text{Y}}=\frac{1}{\sqrt{2}}(\vec{\text{W}}^{(1)}-\vec{\text{W}}^{(2)}),\ \ \vec{\text{Y}}'=\frac{1}{\sqrt{6}}[(\vec{\text{W}}^{(1)}-\vec{\text{W}}^{(2)})-(\vec{\text{W}}^{(2)}-\vec{\text{W}}^{(3)})]\,. \quad (12)$$

In terms of the above vector bosons, the weak-interaction Lagrangian is given by

$$\mathscr{L}_{\text{int}}^W=\frac{1}{\sqrt{3}}\frac{g_2}{\sqrt{2}}\left[\sum_i(J_{L\mu}^{+(i)}W_{L\mu}+J_{R\mu}^{+(i)}W_{R\mu}+\text{H.c.})\right]+\frac{g_2}{2\sqrt{3}}\left[\sum_i(J_{3L\mu}^{(i)}W_{3L\mu}+J_{3R\mu}^{(i)}W_{3R\mu})\right]$$
$$-g_1\left[\tfrac{1}{6}\sum_{n,i}(\bar{u}_n^{(i)}\gamma_\mu u_n^{(i)}+\bar{d}_n^{(i)}\gamma_\mu d_n^{(i)})-\tfrac{1}{2}\sum_i(\bar{\nu}_L^{(i)}\gamma_\mu\nu_L^{(i)}+\bar{N}_R^{(i)}\gamma_\mu N_R^{(i)}+\bar{e}^{(i)}\gamma_\mu e^{(i)})\right]B_\mu$$
$$+\frac{g_2}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[(J_{L\mu}^{+(e)}-J_{L\mu}^{+(\tau)})Y_{L\mu}+(L\to R)+\text{H.c.}\right]+\frac{g_2}{2}\frac{1}{\sqrt{2}}[(J_{3L\mu}^{(e)}-J_{3L\mu}^{(\tau)})Y_{3L\mu}+(L\to R)]$$

$$+\frac{g_2}{\sqrt{2}}\frac{1}{\sqrt{6}}[(J_{L\mu}^{+(e)}-2J_{L\mu}^{+(\mu)}+J_{L\mu}^{+(\tau)})Y'_{L\mu}+(L\to R)+\text{H.c.}]$$

$$+\frac{g_2}{\sqrt{2}}\frac{1}{\sqrt{6}}[(J_{3L\mu}^{(e)}-2J_{3L\mu}^{(\mu)}+J_{3L\mu}^{(\tau)})Y'_{3L\mu}+(L\to R)] \ . \tag{13}$$

Here we have put $g_1{}^2=\frac{3}{2}g^2$ and

$$J_{L\mu}^{+(i)}=i\left[\sum_n u_{nL}^{(i)}\gamma_\mu d_{nL}^{(i)}+\bar{v}_{eL}^{(i)}\gamma_\mu e_L^{(i)}\right] ,$$

$$J_{3L\mu}^{(i)}=i\left[\sum_n (u_{nL}^{(i)}\gamma_\mu u_{nL}^{(i)}-d_{nL}^{(i)}\gamma_\mu d_{nL}^{(i)})+(\nu_L^{(i)}\gamma_\mu \nu_L^{(i)}-e_L^{(i)}\gamma_\mu e_L^{(i)})\right] ,$$

$i=e,\mu,\tau$, and similar expressions hold for the other currents.

Redefining $g_2/\sqrt{3}=g_L$, the first three terms give the usual weak-interaction Lagrangian in a left-right-symmetric model involving $W_L^\pm$, $W_R^\pm$, Z_L, and Z_R. The terms involving Y,Y' bosons violate e-μ-τ universality. These are new terms.

We now briefly discuss the spontaneous symmetry breaking of G_{PS}. For model A, we introduce a set of Higgs bosons as follows.

Case (i) (Majorana neutrinos):

$$S_R^{(1)}=(1,3,1,1,1,1,10) ,$$

$$S_R^{(2)}=(1,1,1,3,1,1,10) ,$$

$$S_R^{(3)}=(1,1,1,1,1,3,10) ,$$

$$\langle S_{R,44}^{+(i)}\rangle=\frac{v}{\sqrt{2}} ,$$

and similar representations for $S_L^{(i)}$, with $\langle S_{L,44}^{+(i)}\rangle\approx 0$:

$$\phi^{(1)}=(2,2,1,1,1,1,1) ,$$

$$\phi^{(2)}=(1,1,2,2,1,1,1) ,$$

$$\phi^{(3)}=(1,1,1,1,2,2,1) ,$$

$$\langle\phi^{(i)}\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}\kappa & 0\\ 0 & \kappa'\end{bmatrix}, \quad \kappa'\ll\kappa .$$

The above Higgs system is the usual one[9] for G_{PS} generalized here to

$$G_{\text{wk}}=\prod_i [\text{SU}_L(2)\times\text{SU}_R(2)]_i .$$

We now introduce a new Higgs multiplet $\Sigma_L^{(i)},\Sigma_R^{(i)}$ belonging to following representation of G_{PS}:

$$\Sigma_L^{(1)}=(2,1,2,1,1,1,1) ,$$

$$\Sigma_L^{(2)}=(2,1,1,1,2,1,1) ,$$

$$\Sigma_L^{(3)}=(1,1,2,1,2,1,1) ,$$

$$\langle\Sigma_L^{(i)}\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}\nu & 0\\ 0 & \nu\end{bmatrix}, \quad \langle\Sigma_R^{(i)}\rangle\approx 0 .$$

In this way, we get the masses of vector bosons as

$$m_{W_L}^{\pm\,2}\sim g_2{}^2\kappa^2, \quad m_{Y_R}{}^2\sim g_2{}^2v^2 ,$$

$$m_{W_R}^{\pm\,2}\sim g_2{}^2v^2 ,$$

$$\tilde{m}_X{}^2\sim g^2v^2, \quad m_{Y_L}{}^2\sim g_2{}^2\nu^2 .$$

Thus we have the symmetry-breaking pattern as in Sec. II:

$$(\tilde{m}_X\approx m_R)\gg\tilde{m}\gg m_L$$

with $v^2\gg\nu^2\gg\kappa^2$ ($\tilde{m}$ here refers to the lightest of Y's, i.e., Y_L).

For case (ii), viz., Dirac neutrino, replace $S_{R,L}^{(i)}$ by $\Phi_{R,L}^{(i)}$, where $\Phi_{R,L}^{(i)}$ belong to the following representation of G_{PS}:

$$\Phi_R^{(1)}=(1,2,1,1,1,1,\bar{4})$$

and similar expressions for $\Phi_R^{(2)}$, $\Phi_R^{(3)}$, and $\Phi_L^{(i)}$ with

$$\langle\Phi_R^{(i)}\rangle=v/\sqrt{2}, \quad \langle\Phi_L^{(i)}\rangle\approx 0 .$$

The other Higgs fields remain unchanged.

For model B, we introduce the following Higgs system:[10]

$$\Sigma=(1,1,1,1,1,1,\bar{4},4), \quad \langle\Sigma_{44}\rangle=c ;$$

$$S_R^{(i)}=S_R^{(i)}=(1,3,1,1,1,1,1,10)$$

and similar expressions, for $S_R^{(2)}$ and $S_R^{(3)}$ with $\langle S_{R,44}^{+(i)}\rangle=v$ and $\langle S_{L,44}^{+(i)}\rangle\approx 0$;

$$\mathcal{M}^{(1)}=(2,2,1,1,1,1,\bar{4},4)$$

and similar expressions for $\mathcal{M}^{(2)}$ and $\mathcal{M}^{(3)}$ with

$$\mathcal{M}^{(i)}\to\frac{1}{\sqrt{2}}\kappa I\otimes\begin{bmatrix}M^{(i)} & 0\\ 0 & I\end{bmatrix}$$

and

$$\Sigma_L^{(1)}=(2,1,2,1,1,1,1,1)$$

and similar expressions for $\Sigma_L^{(2)}$, $\Sigma_L^{(3)}$, and $\Sigma_R^{(i)}$ with

$$\langle\Sigma_L^{(i)}\rangle=\frac{\nu}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad \langle\Sigma_R^i\rangle\approx 0 .$$

This Higgs-field system will reproduce the symmetry-breaking pattern for model B discussed in Sec. II. For the possibility $\tilde{m}_R \simeq \tilde{m}$ in this case, we can make $\tilde{m}_X$ higher by making $c^2 \gg v^2$.

IV. CONCLUSION

We conclude that by taking into account the possible violation of e-μ-τ universality, it is possible to reduce the partial-unification mass scale $\tilde{m}_X$. But the mass scale m_R is greater than or equal to $\tilde{m}$, viz., $m_R \geq 8\times 10^5$ GeV. On the other hand, m_R can be made as low[7] as 300 GeV for model B, if it is assumed that e-μ-τ universality holds all the way to $\tilde{m}_X \approx 5\times 10^{12}$ GeV.

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APPENDIX

In this appendix we give the derivation of Eqs. (1) and (5) of the text. Note first that in the symmetry limit we have the independent coupling constants

$$g_4 = g,\; g_{Li},\; g_{Ri}\;, \tag{A1}$$

where for the latter two we shall use the discrete symmetries mentioned in the text. Now note that we have the following relations for the electromagnetic coupling constant e:

$$\alpha^{-1}(m_R) = \sum_i \alpha_{Li}^{-1}(m_R) + \sum_i \alpha_{Ri}^{-1}(m_R) + \alpha_1^{-1}(m_R)\;, \tag{A2}$$

$$\alpha^{-1}(\tilde{m}) = \sum_i \alpha_{Li}^{-1}(\tilde{m}) + \sum_i \alpha_{1i}^{-1}(\tilde{m})\;, \tag{A3}$$

$$\alpha^{-1}(m_L) = \alpha_L^{-1}(m_L) + \alpha'^{-1}(m_L) \tag{A4}$$

with

$$\sum_i \alpha_{Ri}^{-1}(m_R) + \alpha_1^{-1}(m_R) = \sum_i \alpha_{1i}^{-1}(m_R)\;, \tag{A5}$$

$$\sum_i \alpha_{1i}^{-1}(\tilde{m}) = \alpha'^{-1}(\tilde{m})\;, \tag{A6}$$

$$\sum_i \alpha_{Li}^{-1}(\tilde{m}) = \alpha_L^{-1}(\tilde{m})\;, \tag{A7}$$

where

$$\alpha = \frac{e^2}{4\pi},\quad \alpha_{Li} = \frac{g_{Li}^2}{4\pi},\quad \alpha_{Ri} = \frac{g_{Ri}^2}{4\pi}\;,$$
$$\alpha_L = \frac{g_L^2}{4\pi},\quad \alpha_1 = \frac{g_1^2}{4\pi},\quad \alpha' = \frac{g'^2}{4\pi}\;.$$

Using the discrete symmetries mentioned in the text, Eqs. (A2) and (A5) become

$$\alpha^{-1}(m_R) = 2f\alpha_2^{-1}(m_R) + \alpha_1^{-1}(m_R)\;, \tag{A8}$$

$$\sum_i \alpha_{1i}^{-1}(m_R) = f\alpha_2^{-1}(m_R) + \alpha_1^{-1}(m_R)\;, \tag{A9}$$

where f is the number of generations. Now the known couplings are

$$\alpha_L^{-1}(m_L) = \alpha^{-1}(m_L)\sin^2\theta_W(m_L) = \alpha^{-1}\sin^2\theta_W\;,$$
$$\alpha'^{-1}(m_L) \equiv \alpha^{-1}\cos^2\theta_W\;, \tag{A10}$$

the relations implied by Eq. (A4). To relate the unknown couplings in Eq. (A3) with the known ones, we make use of the renormalization-group equations

$$\alpha_{Li}^{-1}(\tilde{m}) = \alpha_2^{-1}(m_R) + 2\beta_{2i}\ln\frac{m_R}{\tilde{m}}\;, \tag{A11}$$

$$\alpha_L^{-1}(m_L) = \alpha_L^{-1}(\tilde{m}) + 2\beta_2\ln\frac{\tilde{m}}{m_L}\;,$$
$$\alpha'^{-1}(m_L) = \alpha'^{-1}(\tilde{m}) + 2\beta'\ln\frac{\tilde{m}}{m_L}\;, \tag{A12}$$

where β functions are given in Eqs. (2) of the text. Equations (A12) and (A10) then give

$$\alpha^{-1}(\sin^2\theta_W - \cos^2\theta_W) = \alpha_L^{-1}(\tilde{m}) - \alpha'^{-1}(\tilde{m}) + 2(\beta_2 - \beta')\ln\frac{\tilde{m}}{m_L}\;. \tag{A13}$$

Now from Eqs. (A7) and (A11) we have

$$\alpha_L^{-1}(\tilde{m}) = \sum_i \alpha_{Li}^{-1}(\tilde{m}) = f\alpha_2^{-1}(m_R) + 2\sum_i \beta_{2i}\ln\frac{m_R}{\tilde{m}}\;, \tag{A14}$$

while Eq. (A6) gives

$$\alpha'^{-1}(\tilde{m}) = \sum_i \alpha_{1i}^{-1}(\tilde{m}) = \sum_i \alpha_{1i}^{-1}(m_R) + 2\sum_i \beta_{1i}\ln\frac{m_R}{\tilde{m}}\;.$$

Using Eq. (A9), this gives

$$\alpha'^{-1}(\tilde{m}) = f\alpha_2^{-1}(m_R) + \alpha_1^{-1}(m_R) + 2\sum_i \beta_{1i}\ln\frac{m_R}{\tilde{m}} \tag{A15}$$

so that from Eqs. (A13), (A14), and (A15) we have

$$\alpha^{-1}(\sin^2\theta_W - \cos^2\theta_W) = -\alpha_1^{-1}(m_R) + 2\sum_i (\beta_{2i} - \beta_{1i})\ln\frac{m_R}{\tilde{m}} + 2(\beta_2 - \beta')\ln\frac{\tilde{m}}{m_L}\;. \tag{A16}$$

The only unknown coupling now is $\alpha_1^{-1}(m_R)$. To eliminate this, we have for model A,

$$\alpha_s^{-1}(m_L)=\alpha_G^{-1}+2\beta_3\ln\frac{\widetilde{m}_X}{m_L}\ ,$$
$$\alpha_1^{-1}(m_R)=C_1^2\alpha_G^{-1}+2\beta_1\ln\frac{\widetilde{m}_X}{m_R}\ , \tag{A17}$$

where we have made use of the fact that in the symmetry limit $g_3(\widetilde{m}_X)=g_s(\widetilde{m}_X)=g$ and $g_1(\widetilde{m}_X)=C_1/g$. From Eqs. (A16) and (A17), we get Eq. (1) of the text.

For model B, as shown in Ref. 7, we have to simply replace C_1^2 by $2C_1^2$ and β_3 by β_3' in Eq. (A17) so that Eq. (A16) then gives in this case

$$\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]$$
$$=-4C_1^2\beta_3'\ln\frac{\widetilde{m}_X}{m_L}+2\beta_1\ln\frac{\widetilde{m}_X}{m_R}$$
$$-2\sum_i(\beta_{2i}-\beta_{1i})\ln\frac{m_R}{\widetilde{m}}-2(\beta_2-\beta')\ln\frac{\widetilde{m}}{m_L}\ . \tag{A18}$$

Substituting the values of β functions in (A18) we get Eq. (5) of the text.

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