On the validity of Harari-Freund duality

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We reexamine the two-component duality of Harari and Freund (HF) following some suggestions that it may possibly be wrong in its present form. Assuming the dominant contributions to the Pomeron come from multiparticle exchanges in the s channel, the Pomeron is constructed by the use of conformal mapping and polynomial expansions. The $P+f$ model constructed with this Pomeron and the standard Regge poles shows good agreement with the experimental data for total cross sections and the ratios of real to imaginary parts of the forward scattering amplitudes of pp, πp, and Kp. The analysis admits strong exchange degeneracy implying consistency with HF duality. This Pomeron, however, represents the totality of vacuum-exchange contributions, rather than a single exchange characterized by a simple pole in complex J plane.

I. INTRODUCTION

Recently there has been some controversy regarding the validity of Harari-Freund[1] (HF) duality. Dash, Jones, and Martin[2] in a series of papers have shown that the HF duality in its present form is wrong and should be modified to read

$$\langle \mathrm{Im} f_R \rangle = \langle \mathrm{Im} f_{\text{resonance}} \rangle + \tfrac{1}{2} \langle \mathrm{Im} f_{\text{background}} \rangle \; . \qquad (1)$$

They come to this conclusion by reanalyzing the phenomenological fits obtained by several groups of workers.[3] However, they have stressed fits which use Pomerons with monotonic energy dependence, which violate the Froissart bound. On the other hand, a pure data analysis for KN scattering by Roberts, Roy, and Gavai[4] shows that the Pomeron is most likely to be nonmonotonic and hence does conform to the hypothesis of exchange degeneracy, which, in turn, is consistent with HF duality. They have chosen different expressions for σ_P, the Pomeron contribution to the total scattering, and have shown that the one that curves up at low energy does account for the lowest χ^2. Recently the present authors[5] analyzed pp and $p\bar{p}$ data on the total cross section, the ratio of real to imaginary parts of the forward scattering amplitude, and the forward slope parameter on the basis of the $p+f$ model. To obtain the Pomeron part of the scattering amplitude, the HF conjecture that the major contribution to the Pomeron comes from the s-channel background was assumed. The s-channel background, in turn, was assumed to get dominant contribution from the s-channel branch cuts. The branch-cut contribution was obtained by using conformal mapping and a polynomial expansion. Imposing the Froissart bound on this, we obtained results which were in good agreement with the experiment. The Pomeron, so obtained, also curves up at low energy and is found to be consistent with exchange degeneracy and HF duality.

The purpose of this paper is to repeat the calculations for pp, πp, and kp processes to obtain expressions for total cross sections amplitudes and ratios of real to imaginary parts of the forward amplitudes and demonstrate that (i) HF duality is correct in its present form and (ii) the present high-energy data can still be consistent with the Froissart bound. For pp and kp we show the existence of strong exchange degeneracy, which is consistent with duality. For πp the Regge amplitudes are extrapolated to resonance region and compared with the resonance amplitudes obtained by phase-shift amplitudes. The agreement with duality predictions is very good.

In Sec. II, we derive the expression for the forward scattering amplitude and obtain expressions for σ_T and ρ. In Sec. III, numerical calculations are presented and the results are compared with experiment.

II. THE SCATTERING AMPLITUDE

We assume the spin and isopin effects to be small at high energy and normalize the scattering as

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} |f(s,t)|^2 \; , \qquad (2)$$

$$\sigma_T = \mathrm{Im} f(s,0) \; . \qquad (3)$$

We also define

$$\rho = \mathrm{Re} f(s,0) / \mathrm{Im} f(s,0) \; . \qquad (4)$$

The scattering amplitude consists of two parts—a Pomeron part and a Regge part:

$$f(s,t) = f_p(s,t) + f_R(s,t) \; . \qquad (5)$$

For $f_R(s,t)$, we adopt the form given by Collins, Gault, and Martin[3]

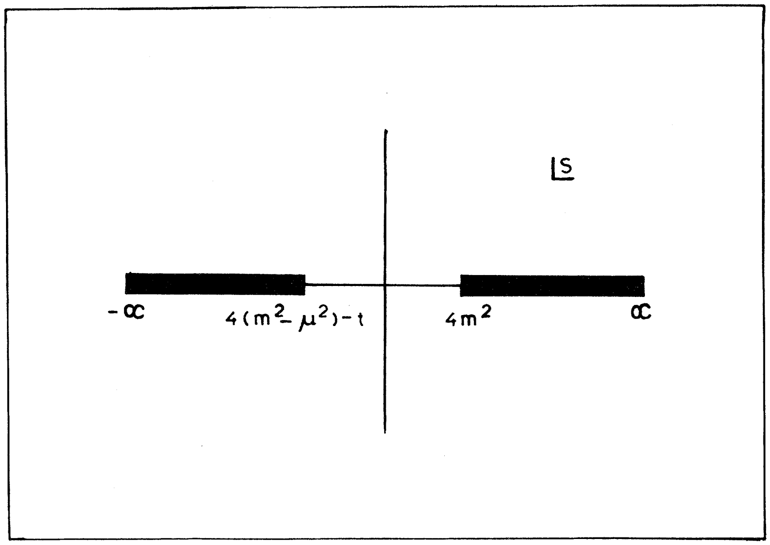

FIG. 1. s-channel cut structure for pp scattering.

$$f_R(s,t)=\sqrt{2}G_R(se^{-i\pi/2})^{\alpha_R(t)} \times e^{a_R t}[1-i\beta e^{a_3 t}(1+t/t_0)]/s \ . \tag{6}$$

This is the contribution of an exchanged pair of Regge poles. For pp and kp the desired pair is $P'-\omega$ and for πp it is P'-ρ. We also separately investigate the πp case when P' and ρ are not taken to be exchange degenerate.

As indicated in the Introduction, we obtain $f_p(s,t)$ by assuming that the Pomeron gets dominant contribution from the branch cuts in the s channel.[6] As we are interested only in the forward amplitude, we put $t=0$ and write $f(s)$ for $f(s,0)$. For pp, $f(s)$ has branch cuts from $s=4m^2$ to ∞ and from $s=4(m^2-\mu^2)$ to $-\infty$, where m and μ are masses for proton and pion, respectively (Fig. 1).

Now our method for parametrizing branch-cut contributions is to bring the whole of analyticity domain inside a regular region by a suitable conformal mapping and then expanding $f_p(s)$ in a series of suitable polynomials of the mapped variable.[7] This method has been used earlier by Frazer,[8] Levinger and Peierls,[9] and Greenberger and Margolis[10] to analyze scattering and form-factor data. These authors used the Taylor expansion, but polynomial expansion could also be used. In this connection, a general result can be stated as follows:[7] "A series of Jacobi polynomials converge in an ellipse whose foci are at ± 1 and every function which is analytic in such an ellipse can be expanded in a series of Jacobi polynomials. In the case of Lagurre polynomials the region of convergence is a parabola around the positive real axis with its focus at the origin, and in the case of Hermite polynomials the region of convergence is a strip whose central line is the real axis. In both cases the region of convergence is unbounded and an analytic function which is to be expanded must satisfy certain growth conditions in addition to being analytic in an appropriate region." For our case we know that the forward amplitude grows logarithmically at high energy and we assume that it will continue to increase saturating the Froissart bound. Such a growth rate can be incorporated by taking an expansion in Lagurre or Hermite polynomials. For the present situation, the expansion in the Hermite polynomials is more suitable. The corresponding mapping which brings the whole of the analytic domain inside a strip along the real axis (Fig. 2) is

$$z=-i\arcsin\left[\frac{s-4m^2+2\mu^2}{2\mu^2}\right] . \tag{7}$$

For large s,

$$z=\ln\left[\frac{s}{\mu^2}e^{-i\pi/2}\right] . \tag{8}$$

This shows that the present variable is closely related to the ususal Regge variable $[(s/s_0)e^{-i\pi/2}]$. With the mapping (7) we write

$$f_p(s)=\sum_{n=0}^{\infty}a_nH_n(z) . \tag{9}$$

However, the knowledge of the Froissart bound and Eq. (8) restricts us to terminate the series after $n=2$. Then

$$f_p(s)=\sum_{n=0}^{2}a_nH_n(z) . \tag{10}$$

Correct high-energy behavior can be obtained if we choose the constants as purely imaginary. Then

$$f_p(s)=i(b_0+b_1z+b_2z^2) , \tag{11}$$

where b_i's are real constants. However, we can check that $f_p(s)$, so obtained, is not real analytic. To make it real analytic, we can include additional terms which do not destroy analyticity, but vanish at high energy restoring positivity. A modified expression for $f_p(s)$ can then be written as

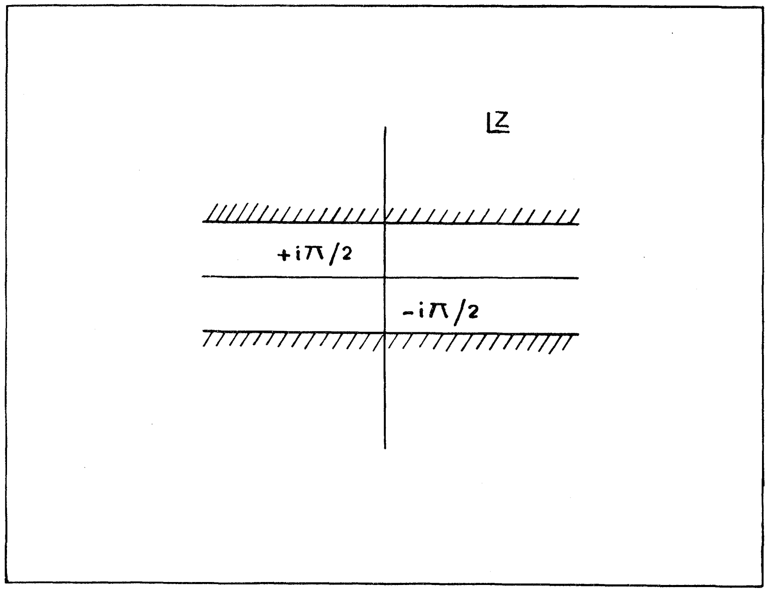

FIG. 2. z-plane cut structure for pp scattering.

$$f_p(s)=\sum_{n=0}^{2} C_n(s)(iz)^n \,. \tag{12}$$

$C_n(s)$ must obey the following properties: (i) As (iz) is real analytic $C_n(s)$ must be real analytic. (ii) For $s\to\infty$, $C_n(s)\to$ constant.

In particular, $C_2(s)$ must be dominantly imaginary at high energy. Such a function can be obtained by mapping the analyticity domain in the s-plane into a disk[8] of unit radius and then expanding it in a Taylor expansion:

$$C_n(s)=\sum_{m=0}^{\infty} C_{nm}z'^m \tag{13}$$

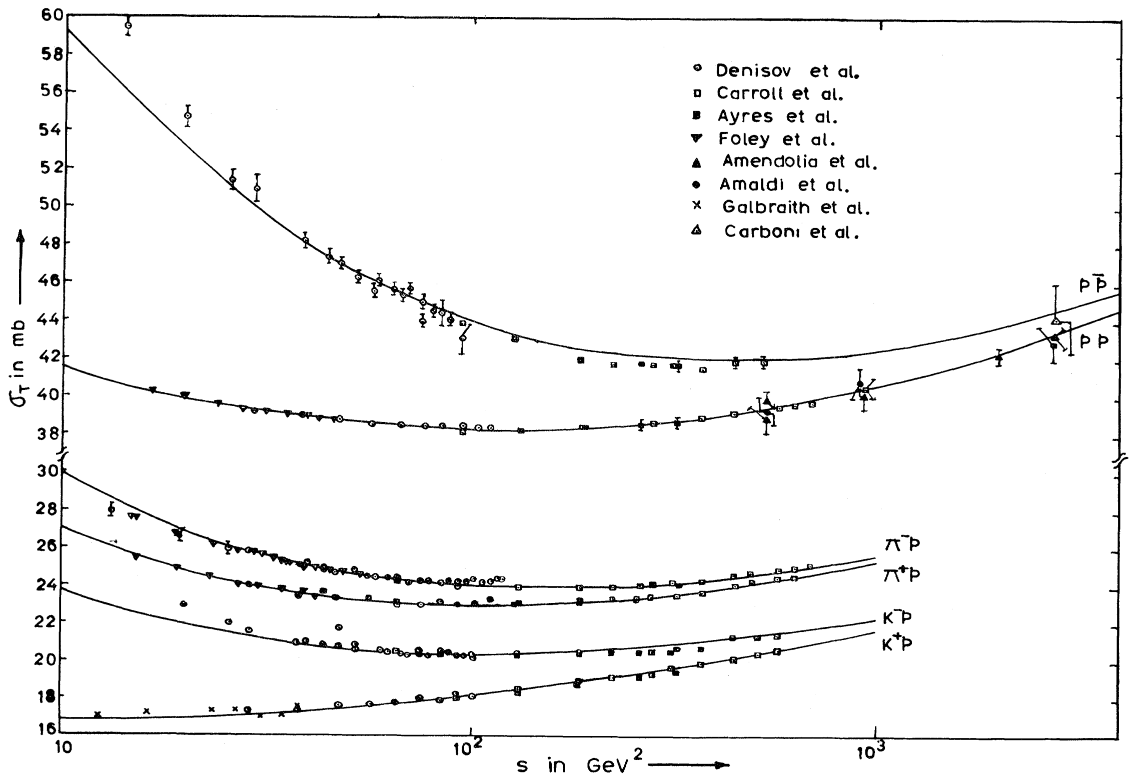

FIG. 3. Total cross sections for pp, $p\bar{p}$, $p\pi^{\pm}$, and $pK^{\pm}$.

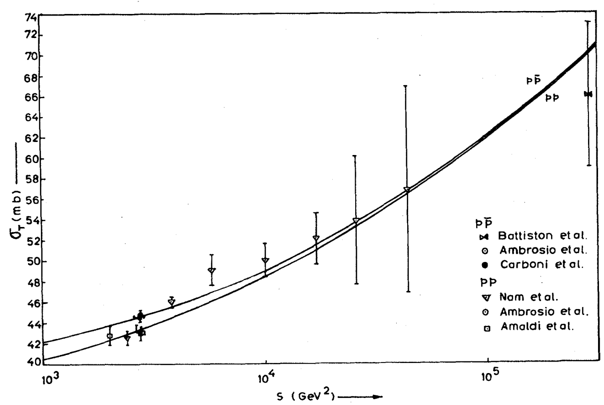

FIG. 4. Total cross section for pp and $p\bar{p}$ at high energies, including cosmic-ray and SPS energies.

with

$$z'=\frac{1}{x}+i\left[1-\frac{1}{x^2}\right]^{1/2} \tag{14}$$

and

$$x=\frac{s-4m^2+2\mu^2}{2\mu^2}\,. \tag{15}$$

The most general form for the amplitude with all the desired properties is given by

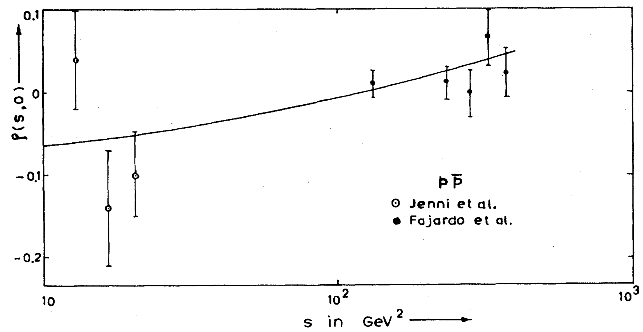

FIG. 6. Ratio of real to imaginary parts of the forward-scattering amplitude for $p\bar{p}$.

$$f_p(s)=\sum_{m\text{ odd}} C_{0m}z'^m+\left[i\sum_{m\text{ even}} C_{1m}z'^m\right]z$$

$$-\left[\sum_{m\text{ odd}} C_{2m}z'^m\right]z^2\,. \tag{16}$$

This, however, will have several parameters which will spoil the beauty of the model. Thus, we restrict to the minimum set of parameters and have

$$f_p(s)=(C_{01}-C_{21}z^2)z'+iC_{10}z\,. \tag{16a}$$

Correspondingly,

$$\sigma_T=\frac{G_R(1-\beta)}{\sqrt{s}}+\left[C_{01}-C_{21}\left[y^2-\frac{\pi^2}{4}\right]\right]\left[1-\frac{1}{x^2}\right]^{1/2}+C_{21}\frac{\pi y}{x}+C_{10}y\,, \tag{17}$$

$$\rho=\left\{-\frac{G_R(1+\beta)}{\sqrt{s}}+\left[C_{01}-C_{21}\left[y^2-\frac{\pi^2}{4}\right]\right]\Big/x-C_{21}\pi y\left[1-\frac{1}{x^2}\right]^{1/2}+C_{10}\frac{\pi}{2}\right\}\Big/\sigma_T\,, \tag{18}$$

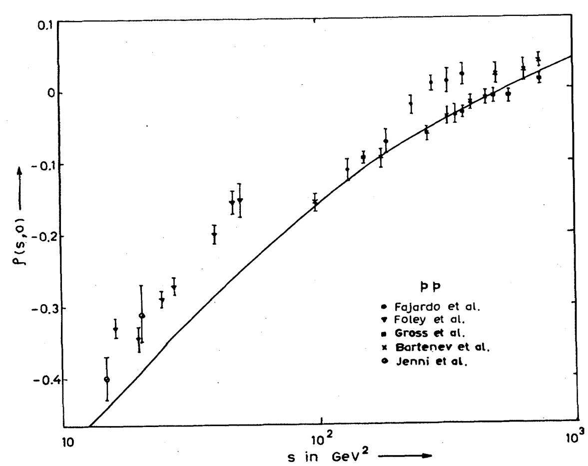

FIG. 5. Ratio of real to imaginary parts of the forward-scattering amplitude for pp.

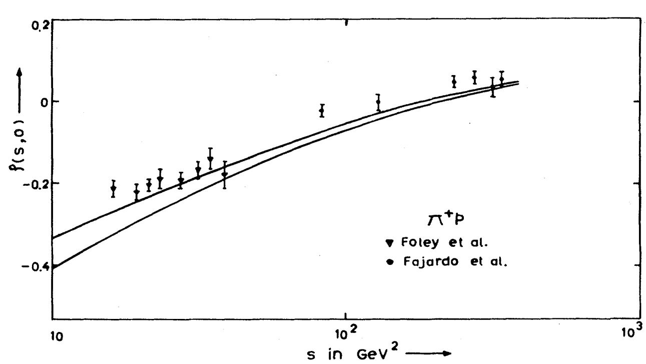

FIG. 7. Ratio of real to imaginary parts of the forward-scattering amplitude for π^+p. The upper curve describes the fit without assuming P'-ρ degeneracy.

TABLE I. Values of the parameters.

Processes	G_R in mb	β	$\alpha_R(0)$	C_{01} in mb	C_{10} in mb	C_{21} in mb	χ^2/DF
pp	29.17	0.99 ± 0.01	0.5	80.	-9.255	-0.529	46/47
πp	36.25	0.12 ± 0.01	0.5	17.022	$-.5539$	-0.1581	30/34
kp	10.8	$1.0\ \pm0.02$	0.5	20.248	-1.4969	-0.2277	27/26

where we have (for $s \geq 4m^2$)

$$z = y - i\pi/2 \tag{19}$$

and

$$y = \ln(x + \sqrt{x^2 - 1}) \ . \tag{20}$$

The corresponding quantities for $p\bar{p}$ can be obtained from these equations by writing $-\beta$ for β. These equations also hold true for πp and Kp with different parameters and different expressions for x.

For πp $[s \geq (m+\mu)^2]$,

$$x = \frac{s-(m+\mu)^2+2m\mu}{2m\mu} \ . \tag{21}$$

For Kp $[s \geq (m+M)^2]$,

$$x = \frac{s-(m+M)^2+2mM}{2mM} \ , \tag{22}$$

where M is the kaon mass. Equations (17) and (18), along with (19)—(22), are the main results of this paper.

At this point it will be worthwhile to note certain features of our analytic parametrization. As our prime suggestion in this paper is that the Pomeron gets dominant contribution from the s-channel branch cuts it is necessary that the contribution should be accurately parametrized. The present parametrization achieves the following objectives.

(a) It is more convergent. This has been discussed earlier and had also been treated by many authors.[7–10]

(b) It retains explicit analyticity in parametrized expression. (One can get a parametrization even without analyticity, but it will be rather *ad hoc* in nature.) As a bonus we obtain the explicit Regge-type energy dependence.

As it incorporates analytic structure explicitly, the result can be easily extended to low energy although our prime motivation has been the high-energy behavior. The reality condition, being independent of analyticity, has to be incorporated separately. This we do by including additional terms with proper analyticity, which restores reality and vanishes at high energy.

Regarding the Regge part of the amplitude, we have used a standard form. It is assumed that the Regge amplitude is well understood and is correct. In fact, our motivation is to check the validity of HF duality with the standard Regge amplitudes. Our model does not contain poles (resonances), but their effect is being described in an average sense by the Regge amplitude through duality.

III. COMPARISON WITH EXPERIMENT

The total cross sections for all the processes are plotted in Fig. 3 (data from Ref. 11). Very-high-energy (including cosmic-ray and CERN SPS) data for pp and $p\bar{p}$ are plotted in Fig. 4 (data from Refs. 11 and 12). For these plots we have used data with center-of-mass energy more than 5 GeV (Ref. 13). For the parameters, the values of $G_R\beta$ are obtained from the $\Delta\sigma_T$ fits. From the particle-particle cross-section fits the values of $G_R(1-\beta)$, C_{01}, C_{10}, and C_{21} are obtained. The values of the parameters are given in Table I. The parameters for the antiparticle cross sections are the same except that β changes sign. With these parameters, the ρ's for the different processes are plotted in Figs. 5—10 (data from Ref. 14).

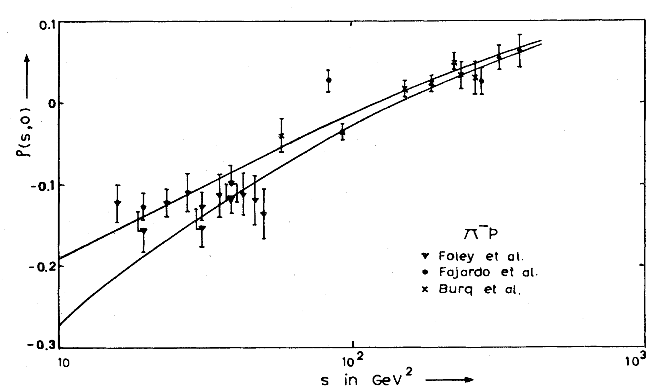

FIG. 8. Ratio of real to imaginary parts of the forward-scattering amplitude for π^-p. The upper curve describes the fit without assuming P'-ρ degeneracy.

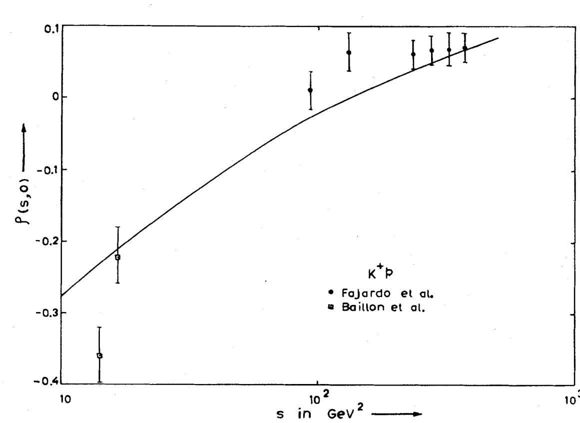

FIG. 9. Ratio of real to imaginary parts of the forward-scattering amplitude for K^+p.

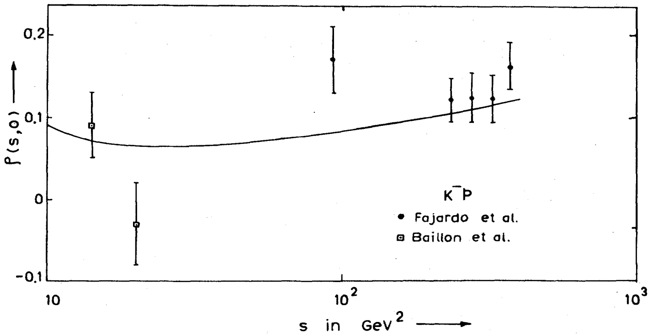

FIG. 10. Ratio of real to imaginary parts of the forward-scattering amplitude for K^-p.

The elastic K^+p and pp channels are exotic channels. As there are no poles or resonances in these channels, duality predicts that the imaginary parts of the Regge amplitudes are zero, implying that $\beta=1$. The best χ^2 fits from our analysis give $\beta=1\pm0.02$ for K^+p and $\beta=0.99\pm0.01$ for pp. Both the processes, within the error limit, agree with duality. For πp, $\beta=0.12\pm0.01$. This is not an exotic channel, so a simple analysis like those for pp and kp cannot be undertaken here.

For these analyses we have taken all the trajectories to be degenerate with $\alpha(0)=0.5$. This value is used by Collins *et al.*[3] and looks most appropriate if one observes the Chew-Frautschi plot of these trajectories.[15] However, charge-exchange scattering suggests $\alpha_p(0)$ to be close to 0.57. Confronted with these alternatives, we first do our analysis with P'-ρ degeneracy. Although this describes data with $s>25$ GeV^2 quite well, the agreement below 25 GeV^2 is rather poor. Then we repeat the πp analysis without P'-ρ degeneracy and fixing $\alpha_\rho(0)$ from $\Delta\sigma$ fits, but keeping $\alpha_{p1}(0)=0.5$ consistent with pp and kp processes. This enables us to get agreement up to $s=9$ GeV^2.

We write

$$\sigma_T^{p\pi^\pm}=\frac{G_{P'}}{\sqrt{s}}\mp\frac{G_\rho}{s^{1-\alpha_\rho(0)}}+\mathrm{Im}f_P(s)\ , \tag{23}$$

where G_ρ and $\alpha_\rho(0)$ are obtained from the $\Delta\sigma_T$ fit and $G_{P'}$ from the $\sigma_T(\pi^+p)$ fit. The parameters are

$$\begin{aligned} &G_P'=23.676,\quad G_\rho=3.683\ ,\\ &C_{01}=30.046\ ,\\ &C_{10}=-3.307\ ,\\ &C_{21}=-0.31517\ , \end{aligned} \tag{24}$$

with $\chi^2/\mathrm{DF}=46/43$. The cross sections are plotted in Fig. 11. The corresponding ρ curves are also shown in Figs. 6 and 7. In Fig. 12, we plot the Regge part of

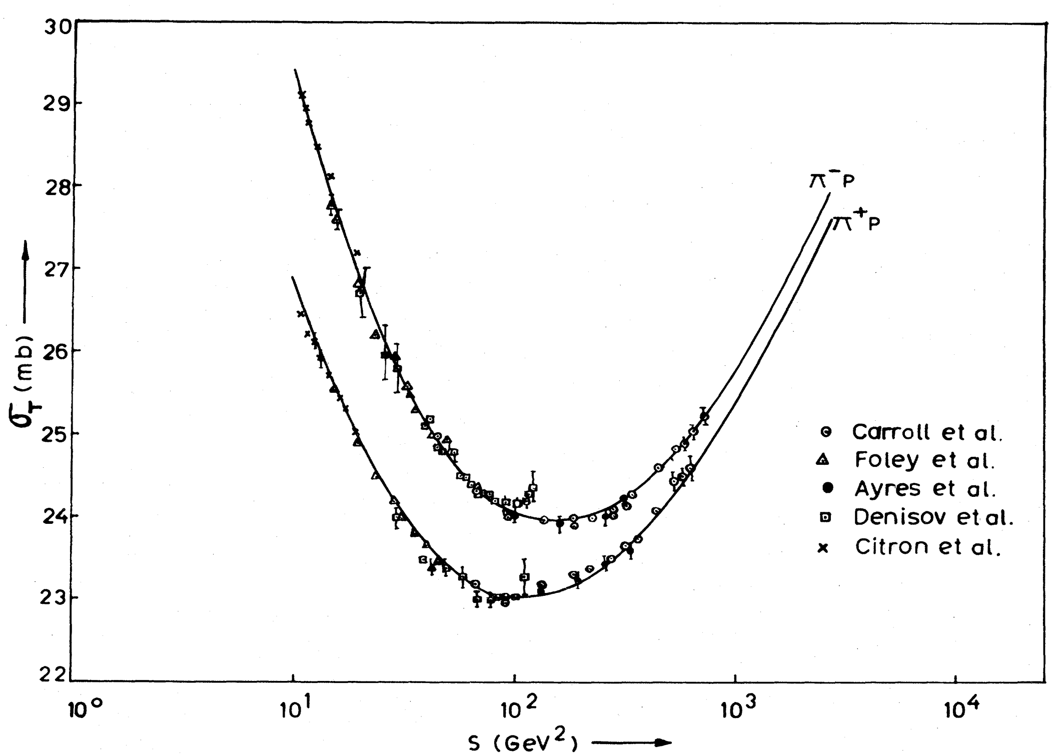

FIG. 11. Total cross sections for $p\pi^+$. Here P' and ρ trajectories are not degenerate.

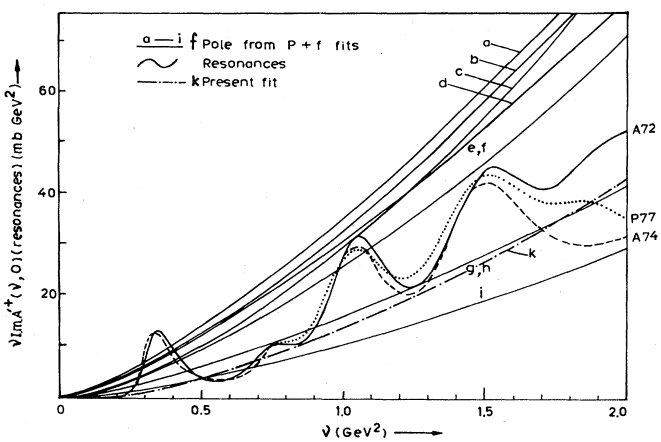

FIG. 12. Comparison of extrapolated Regge amplitudes with resonance amplitudes obtained from phase-shift analyses. All the entries in this diagram except k are reproduced from the first paper of Ref. 2.

$\nu \, \mathrm{Im}A'(\nu,0)$ versus ν where ν is the laboratory pion energy and

$$\mathrm{Im}A'(\nu,0)=\tfrac{1}{2}(\nu^2-\mu^2)^{1/2}[\sigma_T(\pi^+p)+\sigma_T(\pi^-p)] \; . \quad (25)$$

This is compared with the resonance amplitudes as obtained from the several phase-shift analyses.[16–18] Here also we find very good agreement with duality.

IV. CONCLUSION

Assuming the dominant contribution to the Pomeron comes from the multiparticle exchanges in the s channel, we have obtained a new model for the Pomeron. With this our $P+f$ model works very well and has very good agreement with experiment. The model also obeys the Froissart bound. However, the Pomeron, so obtained, represents the totality of the vacuum-exchange phenomena in the t channel and is not merely a simple pole in the complex j plane. We conclude that the HF duality is correct in its present form provided that the Pomeron represents the totality of vacuum-exchange phenomena in the t channel.

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