Gluon "mass" in the bag model

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(Received 23 August 1983)

Bernard has proposed a measure of the effective gluon "mass" in terms of the screening of the linear potential for adjoint sources, and has extracted an estimate of $m_E = 500$–800 MeV from a lattice Monte Carlo calculation. The same quantity is calculated in the bag model, with the result $m_E = 740 \pm 100$ MeV. This arises as an effect of confinement, even though the gluons themselves are massless. The model illustrates several interesting features of this measure of "mass", and these are discussed.

I. INTRODUCTION

The properties of gluonic matter are presently under extensive study by a variety of means. Lattice Monte Carlo estimates yield a strong prediction of the lightest glueball $m(0^{++}) = 700 \pm 300$ MeV, and some weaker indications of excited states in the 1–2.5-GeV range, roughly consistent with other estimates from bag and potential models. Recently, Bernard has suggested another measure of gluonic properties.[1] He notes that the potential between two adjoint sources, which in QCD would correspond to infinitely massive color-octet bosons or fermions, should rise initially but eventually level off due to the screening of the potential by gluons. The physical picture is that, once the energy in the flux tube is great enough, a pair of gluons can be produced from the vacuum which will combine with the octet sources to form two color-singlet states. Further separation of the two color singlets does not require any energy, hence the potential is flat at large distance. Bernard proposes that one half of the energy stored in the flux tube, at the distance where screening sets in, be called the "effective gluon mass." This provides a measure of the minimum energy needed to create a gluon. One can easily see the importance of this measure for models of glueballs. An advantage of this definition of effective mass, which will be called m_E subsequently, is that it is measurable by lattice Monte Carlo methods. Bernard has made estimates of it and claims that values in the range

$$m_E = 500\text{–}800 \text{ MeV} \tag{1}$$

are reasonable.

In the bag model, the gluons are massless. However, there is clearly a minimum energy carried by a confined gluon. This is most easily seen from the glueball spectrum before the spin splittings are included,[2] where ground-state glueballs made of two gluons have a mass of 1 GeV, while those with three gluons have mass 1.5 GeV. At this level, there is an energy cost of 500 MeV per gluon. This occurs because the minimum energy which a massless gluon may have when it is confined to a spherical bag of radius R is

$$E_0 = \frac{2.744}{R} , \tag{2}$$

which for $R = 1$ fm is 550 MeV. A value of about this size is expected just from the uncertainty principle. However, these estimates are not the same as Bernard's definition of m_E. It is the purpose of this paper to calculate the quantity which Bernard has measured.

The existence of Bernard's effective mass does not imply that gluons in the bag model develop a mass. In addition, it is not necessarily true (as some people advocate) that the mass of a glueball is twice m_E. Certainly in the bag model this is not true. If one were able to turn off Coulombic and spin-dependent interactions, then Bernard's definition of mass would be ideal. However, all bound states have masses which include effects of these interactions. For example, in the case of two heavy adjoint sources discussed below, there is an attractive Coulombic energy such that, for a large range of the interparticle separation, the energy $E_{SS} - 2m_S$ is negative. However, since all states have similar Coulomb energies, one does not want to include that binding energy into the definition of the effective mass. For Bernard's effective mass to be a sensible measure, one needs to argue that the effect of the Coulomb interaction cancels out. In the work described below, this appears to be the case. One of the uses of the effective mass is for a comparison of different methods, in this case a comparison between the bag model and lattice Monte Carlo methods. That these methods agree on the effective mass is encouraging for both techniques.

The energy of two static adjoint sources can be obtained from the bag-model studies of heavy-quark systems.[3] The change from sources in the fundamental representation (quarks) to adjoint sources can be made by a modification of the color Casimir factor $4\alpha_s/3 \to 3\alpha_s$. The bag picture generates a flux tube with string tension, for quarks,

$$k_F = \left[\frac{32\pi B \alpha_s(r)}{3} \right]^{1/2} . \tag{3}$$

For adjoint particles the string tension is correspondingly larger:

$$k_A = \tfrac{3}{2} k_F . \tag{4}$$

Likewise the Coulomb forces are greater by a factor of $\frac{9}{4}$. From the heavy-quark studies we can therefore extract the energy of two adjoint sources as a function of r,

$$E_{SS}(r)=2m_S-\frac{3\alpha_s(r)}{r}+\frac{3}{2}k_F r-1.6\alpha_s{}^{3/4}(r)B^{1/4}\ , \qquad (5)$$

where m_S is the mass of the source and the last term is the large-r approximation to the small correction term given by Haxton and Heller.

We wish to compare this to the mass of two isolated color-singlet states each consisting of one source and one gluon. In Sec. II, I calculate this mass, including the Coulomb corrections at $O(\alpha_s)$. As a function of bag size R the result is

$$E_{SG}=m_S+\frac{4\pi}{3}BR^3+\frac{2.744}{R}-\frac{1.75\alpha_s(R)}{R}\ . \qquad (6)$$

The second term is the bag volume energy, the third is the gluon's kinetic energy, and the fourth is the Coulomb attraction of the source and gluon. For the strong coupling constant, I employ a form which has been used many times before in the bag model:[3–5]

$$\alpha_s(R)=\frac{1}{\beta_0\ln(1+1/\Lambda R)}\ , \qquad (7)$$

with $\beta_0=11/2\pi$ in a world without quarks. This reproduces the asymptotic-freedom prediction at small R, and, as it should, only has a pole in α_s when $R\to\infty$. Various estimates of Λ range from $\Lambda=0.2$ GeV (Ref. 3) to $\Lambda=0.47$ GeV (Ref. 4), corresponding to $\alpha_s(1\text{ fm})=1$–2. In presenting numerical values, I will use the experimental value for k_F (≈ 420 MeV). The remaining dependence on α_s (or Λ) is then only in the Coulomb energy. It is important for the utility of Bernard's definition of the effective mass that the Coulomb energy not have too large an effect on the estimate of m_E. It will turn out that the results are very insensitive to the form of α_s and the value of Λ, with identical results being obtained for both $\Lambda=0.2$ and 0.47 GeV. To determine the mass of the asymptotic-source-plus-gluon state, one minimizes E_{SG}. As $m_S\to\infty$, no center-of-mass corrections are needed.

To find the crossover radius at which screening occurs, one equates

$$E_{SS}(r)=2E_{SG}\ , \qquad (8)$$

with the latter calculated at the minimizing value of R. This radius is roughly $r=1.1\pm0.1$ fm and indicates the distance beyond which it becomes energetically favorable to create two gluons and bind them in color singlets to the sources. Bernard's effective mass is then one half the flux-tube energy at this radius,

$$m_E=\tfrac{1}{2}k_A r$$
$$=740\pm100\text{ MeV}\ , \qquad (9)$$

if one uses $k_F=(420\text{ MeV})^2$. The parameter dependence which goes into the quoted errors is discussed in Sec. III.

Within the model we can see that this is not a real mass in that the gluons are massless. However, it may be that there are acceptable uses of the idea of mass for gluons, such as the above example. A similar situation occurs for quarks, where meaning can be made of an effective mass for bagged quarks even though the quarks themselves are nearly massless. These topics are discussed in Sec. IV.

II. MASS OF AN ADJOINT SOURCE PLUS GLUON

The calculation of the mass of a heavy adjoint source with a massless gluon is similar to many calculations of hadronic masses within the bag model,[4–6] but is most directly related to Ref. 5 where mesons with one heavy quark and one light quark are studied. We consider the gluonic case here.

In a bag containing a heavy source plus a gluon, the source will be located at the center of the bag and the gluon will be spread throughout the bag with a wave function given by the minimum-energy mode. The source has a mass m_S and a charge density with unit total color charge, which can be described by the form

$$\rho_A^A=f^{ABC}a_S^{\dagger B}a_S^C\delta^3(x) \qquad (10)$$

with $a_S^{\dagger B}$ and a_S^B being creation and annihilation operators for the source. For the massless transverse gluon, the lowest-energy mode in a spherical cavity is the transverse electric (TE), $l=1$ with wave function

$$\vec{A}(x)=Nj_1(kr)\vec{\epsilon}\times\hat{r}\,e^{ikT} \qquad (11)$$

with $k=2.744/R$, $N=0.63/R$, and $\vec{\epsilon}$ being a polarization vector. This leads to a color current density

$$\rho_{\rm TE}^A=2N^2\omega_E j_1{}^2(kr)f^{ABC}(\vec{\epsilon}^{\,*}\cdot\vec{\epsilon}-\epsilon^*\cdot\hat{r}\,\vec{\epsilon}\cdot\hat{r})a_{\rm TE}^{\dagger B}a_{\rm TE}^C \qquad (12)$$

with $a_{\rm TE}^{\dagger B}$ and $a_{\rm TE}^B$ being the creation and annihilation operators for the TE gluon. For an appropriate combination of spins, the bag pressure will be spherically symmetric and the spherical bag will be stable.

Several terms in the bag energy are immediately obvious,

$$E_{SC}=\frac{4\pi}{3}BR^3+m_S+\frac{2.744}{R}+\cdots\ . \qquad (13)$$

The first term is the volume energy of the bag, while the second and third are the energies of the source and the gluon. In this section I will not include a zero-point-energy term, but will refer to it in Sec. III. The only remaining piece is the interaction energy between the source and the gluon. The spin-spin interaction, mediated by gluon exchange, is unimportant as it goes like $1/m_S$. (It would be absent altogether if the source were spinless.) All that remains is the Coulomb energy

$$H_C=\frac{1}{8\pi}g^2\int d^3x\,d^3y\,\rho^a(x)G(x,y)\rho^a(y)\ , \qquad (14)$$

where $G(x,y)$ is the cavity Coulomb Green's function

$$G(x,y)=\frac{1}{|\vec{x}-\vec{y}|}+\frac{1}{R}\left\{\sum_{l=1}^{\infty}\left[\frac{l+1}{l}\left[\frac{xy}{R^2}\right]^l P_l(\cos\gamma)\right]-1\right\}\ . \qquad (15)$$

The total Coulombic energy, including the lowest-mode self-energy, is then

$$E_C = -3\alpha_s(C_{ES} - \tfrac{1}{2}C_{EE} - \tfrac{1}{2}C_{SS}) \ , \tag{16}$$

where

$$C_{ij} = \int d^3x\, d^3y\, \widetilde{\rho}_i(x) G(x,y) \widetilde{\rho}_j(y)[1+(\hat{x}\cdot\hat{y})^2]\tfrac{3}{4} \tag{17}$$

and

$$\widetilde{\rho}_E(x) = 2N^2 j_1{}^2(kr) \ ,$$
$$\widetilde{\rho}_S(x) = \delta^3(x) \ . \tag{18}$$

In the case of the self-energy of the source, there is an infinite piece independent of the bag size and a finite piece equal to

$$C_{SS} = -1/R \ . \tag{19}$$

The infinite piece is the same value as in free space and goes into the renormalization of the mass of the source. One might wonder why the self-energy of a point source depends on the bag size. This dependence arises because the gluon field cannot extend beyond the bag boundary, and hence the energy contained in the field depends on the size of the bag. The required integrals are integrated numerically[7] with the result

$$C_{ES} = 0.38/R \ ,$$
$$C_{EE} = 0.59/R \ , \tag{20}$$

the total Coulomb energy being given by

$$E_C = -\frac{1.76\alpha_s(R)}{R} \ , \tag{21}$$

and the total bag energy as reported in Sec. I, Eq. (6).

To compute the energy of the state one minimizes the bag energy with respect to the radius. For $\alpha_s(R)$, I take the form of Eq. (7). The minimization then fixes the radius and mass. Because of the dependence of α_s on R this procedure must be carried out numerically. For $\Lambda = 0.20$ [$\alpha_s(1\text{ fm}) = 1$], I find

$$R_{\min} = 4.7\text{ GeV}^{-1} = 0.94\text{ fm} \ ,$$
$$E_{SG} = m_S + 0.46\text{ GeV} \ , \tag{22}$$

while for $\Lambda = 0.47$ [$\alpha_s(1\text{ fm}) = 2$], the result is

$$R_{\min} = 4.7\text{ GeV}^{-1} \ ,$$
$$E_{SG} = m_S + 0.16\text{ GeV} \ . \tag{23}$$

To use this to calculate the effective gluon mass one uses Eq. (5), obtained by modifying the results of Haxton and Heller,[3] with α_s evaluated at $R_{\min}$. I find that the crossover separation is

$$r_s = 1.1\text{ fm} \ ,$$

and the effective gluon mass which results from Bernard's definition is

$$m_E = \tfrac{1}{2} k_A r_s = 0.74\text{ GeV} \ . \tag{24}$$

The above results for r_s and m_E are essentially independent of Λ if the experimental value of k_F is used. It is fairly easy to see why. The Coulomb energy of two sources at the crossover point is almost equal to twice the Coulomb energy of the bound-source-plus-gluon system. Thus the effect of the Coulomb energy and α_s essentially cancels out of the problem. This is fortunate for the reliability of the answer, and it also is a sign that Bernard's definition of the effective mass is a good measure of the cost of adding a gluon to the system.

III. PARAMETER DEPENDENCE

We have seen in the last section that the effective mass is essentially independent of the value of the strong coupling constant, if the bag constant and string tension are extracted from experiment. There are, however, a couple of steps in the procedure which can lead to small shifts in m_E. In this section I discuss these.

In much of the spectroscopy of light hadrons an effective Casimir energy or zero-point energy,

$$E_0 = -Z_0/R \ , \tag{25}$$

has been used.[6] This is supposed to account for the size dependence of the quantum zero-point fluctuations of the fields. Direct calculation[8] of Z_0 in a rigid cavity yields a divergent answer, with the finite part being small and negative (i.e., positive E_0). Phenomenological fits[4] favor $Z_0 = 1$. It is not completely straightforward to incorporate the zero-point energy into the present calculation, as Z_0 should be shape dependent, and therefore neither of the above values would apply for the flux tube. In studies of heavy-quark systems the zero-point energy was not included. However, an estimate of the size of this effect is obtained by including a term $-Z_0/R$ in both E_{SG} and E_{SS}. In the latter case the radius is estimated by the best fit of the flux tube to a spherical shape. A term of this form will lower the effective gluon mass if Z_0 is positive. Physically this occurs because, for positive Z_0, one gains energy by dividing a large bag into two smaller bags if the total volume is unchanged. My estimate of the effect of this modification is to lower m_E by about 80 MeV for $Z_0 = 1$.

The other procedural step which has some effect on m_E is the technique for dealing with the self-energies. For example, if one had two heavy sources in a spherical bag, the interaction energy itself would be

$$E = -3\alpha_s \left[\frac{1}{r} - \frac{1}{R} \right] \ , \tag{26}$$

due to modification of the Coulomb propagator. The standard procedure is to add to this the R dependence of the self-energies which exactly cancels the unusual $1/R$ piece leading to $E = -3\alpha_s/r$. A similar modification occurs in the source-plus-gluon system. There the purely interaction contribution is

$$E = \frac{-1.1\alpha_s}{R} \ ,$$

instead of Eq. (21) which also includes the self-energies. For the gluon, the self-energy was calculated by keeping the gluon in the TE mode. To estimate the effect that these terms produce in m_E, I have redone the calculation without any self-energies. In this case, using $\Lambda=0.2$ GeV, m_E decreases by about 70 MeV. If one were to include self-energies in the state with two sources, but not include them in the gluon-plus-source system, the mass would increase by the same factor.

The uncertainties produced by these procedures are not particularly large. This is reassuring and suggests that the model may be reasonably reliable in this context.

IV. DISCUSSION

From the above calculation we can see that one must be cautious in concluding that Bernard's measurement of the effective mass indicates massive gluons in the usual sense of the word "mass." The bag-model gluons are massless, yet require a net energy cost to produce them because of confinement. One respect in which bag gluons differ from naive massive spin-one particles is that the former are transverse and gauge invariant, while the latter are not. This feature is manifest in the glueball spectrum by the absence of a 1^{-+} two-gluon state. Such a state is forbidden by Yang's theorem for massless gluons, yet occurs with two massive gluons due to the longitudinal component allowed in the latter case. Bernard's measurement does not indicate a naive mass for gluons. Cornwall[9] has proposed that, by adding extra degrees of freedom, a form of mass can be generated for gluons without the loss of gauge invariance. This must, however, differ from the naive idea of mass. Perhaps the bag model is a realization of this idea.

The situation which occurs for gluons is similar to that which occurs for quarks in the bag model. The light quarks in the bag model are treated as nearly massless, with a mass parameter in the equations of motion which is small, often called a "current" mass. Nevertheless when confined to a bag, quarks acquire a minimum energy of about 350 MeV, which is comparable to usual values of "constituent" masses. This energy does behave as a mass in some situations. For example, in the calculation of the magnetic moment for a massless bag quark, the result is very nearly equal to

$$\frac{1}{2E_0}\langle Q\vec{\sigma}\rangle ,$$

which would be the nonrelativistic value if E_0 were identifed with a mass. Likewise, if one defines a mass by the term in the quark propagator which commutes with γ_5, or by the minimum energy in the propagator, these definitions are also close to E_0. However, in some situations,[4] such as the determination of pion and kaon masses, or in the calculation of the σ term, it is the current masses which are the important mass parameters.

The comparison of the bag calculation of m_E with the lattice measurement allows one to draw some relevant conclusions about aspects of the bag model. For example, the question has been raised whether the bag constant for gluons should be the same as for quarks or should be larger (Ref. 10 suggests that it is 10 times larger). While within the standard form of the model the bag constants are equal, it has not been possible to confirm this because of our lack of understanding of the glueball spectrum. The present comparison with the lattice Monte Carlo method clearly favors a single bag constant for quarks and gluons. Likewise, it has been suggested that a large positive self-energy for gluons should be included in order to push the glueball spectrum to higher mass. The present calculation is the cleanest evidence that a large self-energy should not be present and that the bag model's estimate of the energy of a gluon is reasonable.

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